AFRL-VA-WP-TR-2004-3112

LIFE ANALYSIS DEVELOPMENT AND VERIFICATION

Delivery Order 0012: Damage Tolerance Application of Multiple Through Cracks in Plates With and Without Holes

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OCTOBER 2004

Final Report for 25 August 1997 – 31 July 2002

STINFO FINAL REPORT

AIR VEHICLES DIRECTORATE
AIR FORCE MATERIEL COMMAND
AIR FORCE RESEARCH LABORATORY
WRIGHT-PATTERSON AIR FORCE BASE, OH 45433-7542

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THIS TECHNICAL REPORT HAS BEEN REVIEWED AND IS APPROVED FOR PUBLICATION.

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YY)*
October 2004

2. REPORT TYPE
Final

3. DATES COVERED *(From - To)*
08/25/1997 – 07/31/2002

4. TITLE AND SUBTITLE
LIFE ANALYSIS DEVELOPMENT AND VERIFICATION
Delivery Order 0012: Damage Tolerance Application of Multiple Through Cracks in Plates With and Without Holes

5a. CONTRACT NUMBER
F33615-94-D-3212-0012

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER
62201F

5d. PROJECT NUMBER
A04Z

5e. TASK NUMBER

5f. WORK UNIT NUMBER
AG

6. AUTHOR(S)
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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

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8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Air Vehicles Directorate
Air Force Research Laboratory
Air Force Materiel Command
Wright-Patterson Air Force Base, OH 45433-7542

10. SPONSORING/MONITORING AGENCY ACRONYM(S)
AFRL/VASM

11. SPONSORING/MONITORING AGENCY REPORT NUMBER(S)
AFRL-VA-WP-TR-2004-3112

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES
Report contains color.

14. ABSTRACT

This report documents the details of new stress intensity solutions for two independent through-the-thickness cracks in plates with and without holes. The solutions include both curve fits to detailed finite element models, and in some cases, used table lookup solutions for more complex cases. The solutions include the following:

- Two internal through cracks
- Edge crack and an internal crack in a plate
- Unequal edge cracks in a plate with unconstrained bending
- Unequal edge cracks in a plate with constrained bending
- Unequal through cracks at a hole
- Through crack growing toward a hole
- Edge crack growing toward a hole.

15. SUBJECT TERMS
stress intensity factor solutions, multiple through cracks

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT:	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON (Monitor)
a. REPORT	b. ABSTRACT	c. THIS PAGE			James A. Harter
Unclassified	Unclassified	Unclassified	SAR	190	19b. TELEPHONE NUMBER *(Include Area Code)* (937) 904-6771

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std. Z39-18

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LIST OF ACRONYMS

Acronym	Description
ALC	Air Logistic Center
BEM	Boundary Element Model
CI	Contour Integral
FE	Finite Element
FEM	Finite Element Model
LEFM	Linear Elastic Fracture Mechanics
SIF	Stress Intensity Factor

FOREWORD

This report summarizes work performed to develop stress intensity factor solutions for two independent through cracks. The multiple crack cases include cracks in finite plates, cracks growing from holes, and cracks growing toward holes.

The models developed under this effort are being transitioned to end users through the crack growth life prediction software, AFGROW, developed by AFRL/VASM. However, this report contains all of the information required to incorporate these models in any other life prediction code.

The authors would like to thank the Air Force Air Logistic Centers (ALC)s and the Aging Aircraft Office (ASC/SMA) for funding this effort. Thanks also to Alexander Litvinov (Lextech, Inc.), and Scott Prost-Domasky (APES, Inc.) for the exceptional software and finite element modeling (FEM) support.

1.0 BACKGROUND

This work was performed to support the requirements of the Air Force Air Logistic Centers (ALCs) and to advance the current state-of-the-art in damage tolerant life prediction of aircraft structures. Current crack growth life prediction codes are not capable of accurately predicting the life of components with multiple cracks since there are no closed-form stress intensity factor (SIF) solutions for arbitrary, multiple cracks in finite plates. The general form of the equation used to determine the SIF for a given geometry is:

$$K = \sigma\sqrt{\pi X}\,\beta$$

Where,

σ = Applied stress[1]
X = Crack length of interest
β = Factor to account for geometry effects.

It is important to note that the beta (β) term accounts for geometric effects. This term is used throughout this report to account for the geometric differences in the various models being analyzed. Any corrections to the beta value for a given geometry are simply multiplied to the appropriate beta value for a given model.

1.1 Multiple Crack Geometry

Multiple cracks are frequently encountered in structures. The growth of two or more cracks toward each other is much more complex than single (or symmetric) crack growth. The SIF values at the crack tips depend not only on individual crack dimensions but also on their proximity. The coalescence of two cracks increases the complexity of the solutions because of the number of geometric possibilities. In finite geometries, the interaction effect between the crack tips and the effect of specimen edge on crack tip SIF has to be taken into account. Both of these factors affect crack tip SIFs in varying magnitudes, depending on the type of crack geometry.

The SIF solutions for multiple crack situations (certain crack types and geometries) are available in stress intensity handbooks [1-3]. There are several references in the literature on the growth and coalescence of multiple cracks in plates [4-6]. The majority of them deal with crack interaction and growth in infinite plates. The effect of finite geometry is not dealt with in most cases due to the wide range of configurations that need to be tested. A closed-form solution for multiple crack linkup and growth is very difficult to obtain. The degree of difficulty increases with an increasing number of cracks, finite plate effects, and growth of these cracks after linkup. Hence in most cases, finite element modeling (FEM) or boundary element modeling (BEM) is used to obtain SIF values at

[1] Unless otherwise stated, a stress level of 1.0 is used for all SIF calculations.

each crack tip and a fatigue crack growth code is used, in a piece-meal approach, to determine life of a given component.

It was originally hoped that closed-form SIF solutions could be developed for as many as four independent cracks using FEM methods. However, the number of parameters and complexity involved has dictated that only two independent through cracks could be considered for this effort. Literally thousands of FEM analyses were performed for several plate widths, hole diameters, crack lengths, and relative positions. The results of these analyses were used to develop the closed-form SIF solutions described in this report.

The problems addressed in the current work are listed below:

- Two internal through cracks
- Edge crack and an internal crack in a plate
- Unequal edge cracks in a plate with unconstrained bending
- Unequal edge cracks in a plate with constrained bending
- Unequal through cracks at a hole
- Through crack growing toward a hole
- Edge crack growing toward a hole.

In the current work, StressCheck® [7] and FRANC2D/L [8], finite element (FE) programs are used to obtain the SIF values at the crack tips for a range of plate widths. The SIF values, for the crack tips growing toward each other and for crack tips growing toward a hole or specimen edge are tabulated for an infinite geometry. Interaction effects are determined by dividing individual crack SIF values from the FE analysis (above tabulated values) by the appropriate existing SIF solution available in AFGROW [9,10]. Closed-form equations that take the finite plate effect into account are determined utilizing curve fits of these interaction data. The complete approach used to determine a general solution for each geometric case addressed in this report is explained in the following section.

2.0 APPROACH

2.1 Two Internal Through Cracks

The objective of the current work is to develop general SIF solutions to the complex problem of two through cracks in a plate. In order to develop generic solutions for a range of configurations, a large amount of test and/or analytical data are required The interaction effect for two through asymmetric collinear cracks in an infinite plate is available in stress intensity handbooks [2, 11]. In the above references the SIF values at the crack tips are determined using

- Exact solution based on complex stress functions [2]
- Exact solution based on elliptic integrals [11].

The interaction tables in the above references were determined years ago, since then, there have been major advances in techniques to determine SIF at the crack tips. FE analysis methods have proved to be a powerful and accurate tool in fracture mechanics. Most of the commercially available FE tools can now model the stress singularities at the crack tips and accurately predict the SIF for 2-D and 3-D geometries. Another advantage is the use of the J-Integral method of estimating the SIF value. P-version programs like StressCheck [7] provide an option to vary the polynomial degree of individual model elements to obtain better solution convergence. H-version FE tools like FRANC2D/L [8] provide special crack elements and re-meshing algorithms to model stress singularities.

In the current analysis, both the p-version and h-version FE programs are used to obtain the SIF values at each crack tip. The J-Integral method option is selected in both cases for the determination of SIF values.

The approach used for the two through crack problem involves the following four steps:

1) FE modeling
2) Infinite plate solution
3) Finite plate solution
4) Software implementation.

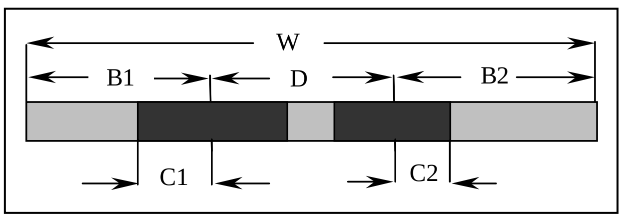
The first three steps are explained in this report, and the last step is covered in the AFGROW Technical Guide and User’s Manual [10]. The first step involves modeling parameters and FE analysis of two cracks in infinite and finite plate geometries. The second step involves obtaining the appropriate solution for the infinite plate, and the third step is the development of corrections to account for finite plate effects. The following sections explain the first three steps in detail.

2.1.1 Modeling Issues

The crack tips are considered as separate individual objects since each is affected by different factors. The problem is modeled by fixing the first crack position and placing the second crack relative to the first. In the current work, the crack on the left is modeled first and is always the short crack and the crack on the right is the long crack. The crack on the left is always non-centered in the plate and the position of the crack on the right depends on crack spacing D. The second crack (crack on the right) can be either centered or non-centered. Changing crack lengths (short or long) will just change the crack length ratio and is equivalent to flipping the plate (viewing plate right to left).

2.1.1.1 Modeling Parameters

It is important to know the definition of variables used to model the two-through-crack problem in infinite and finite geometry. The two through crack model in a finite geometry is shown below in Figure 1. The infinite plate geometry will not include the offsets B1 and B2.



Legend

W – Width of the plate
H – Height of the plate
C1 – Left crack length
C2 – Right crack length
B1 – Offset from the left edge of specimen to the center of left crack
B2 – Offset from the right edge of specimen to the center of right crack
D – Distance between the crack centers

Figure 1: Two Asymmetric Collinear Through Cracks in a Plate

2.1.2 Finite Element Modeling

The infinite and finite plate problem is modeled using both the p-version and the h-version FE programs. The StressCheck [7] (p-version) provides error estimation and convergence output for each polynomial degree of element and, hence, was the preferred code. In all the models, the H/W ratio was set to be equal to 4.

StressCheck provides both p-method and the h-method of mesh refinement to obtain accurate SIF values. Since the data was required to generate curves, a wide range of crack lengths was run for all the cases. Symmetry conditions were used by modeling half of the plate (horizontal line of symmetry) to reduce the number of degrees of freedom. Appropriate boundary conditions to prevent rigid body motion were applied along this symmetry line. A uniaxial tensile stress of $\sigma = 1$ was applied to the top edge of the specimen normal to the crack plane. Geometrically graded elements were used around the region of the two cracks. Large elements were used to model the rest of the plate in order to reduce computational time and memory.

StressCheck uses the contour integral (CI) method to obtain SIF values at the crack tips. The CI method requires the user to input the value of radius of integration path around the crack tip to extract SIF values. For a properly designed mesh, the SIF values must be independent of the radius of integration path. However, the ratio $r/r_c < 0.1$ is desired for best results. This is achieved, where r is the radius of integration path and r_c is the crack length. Several mesh designs were tried to ensure that for different r values the SIF variation was less than a percent. For convergence studies, each problem was run for polynomial degree p ranging from 1 to 8. StressCheck calculates the limiting SIF value for each p and outputs the percentage error between this value and the SIF value for the user-designed mesh. It also outputs convergence and error estimation values for all of the runs in a report. This ensures that the level of accuracy of SIF solutions obtained is high in each case.

In the case of FRANC2D/L, the mesh design included very small quadrilateral elements in the region around the crack and relatively large elements away from it. The element size in the region of crack is about 0.02 percent of the crack length to obtain accurate SIF values. This also ensures good convergence in results. Once the crack is placed in the geometry, FRANC2D/L uses automatic meshing to mesh the area around the crack tip. The FE results for all configurations[2] are shown in Appendix A. Figure 2 shows the FE mesh used in respective FE programs.

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[2] The offset, B, shown in Appendix A is the offset from the left edge of the plate to the leftmost crack (C1). This offset value is equivalent to the modeling parameter, B1.

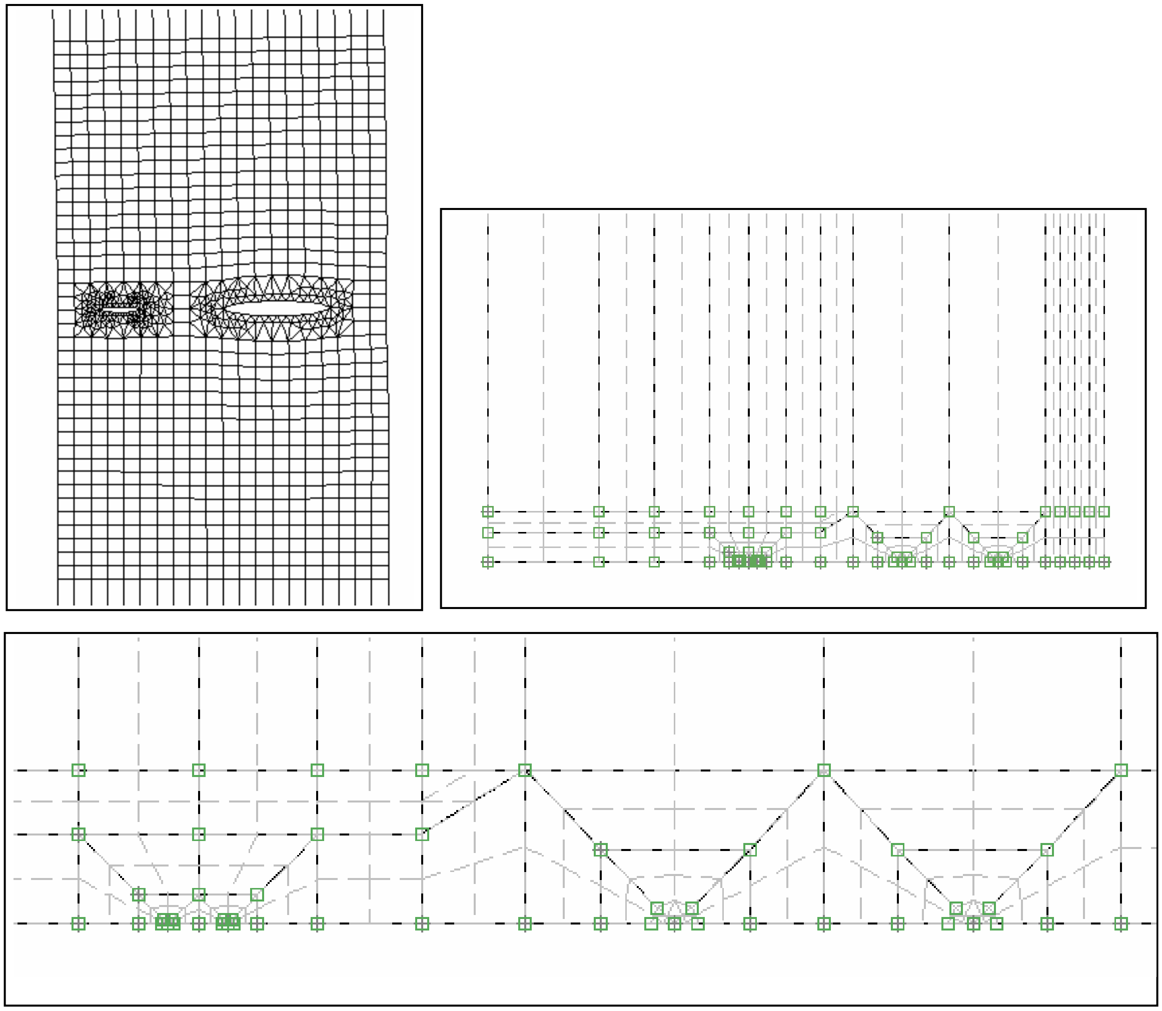


Figure 2: FE Mesh for the Two-Through-Crack Model

2.1.3 Methodology Adopted to Determine the General Solution

The first step is to determine the interaction effect of one crack on another in an infinite plate. The variables involved in an infinite plate problem are shown in Table 1.

Table 1: Infinite Plate Parameters for the Two-Through-Crack Model

Description	**Parameter**
Plate width	W
Left crack length	C1
Right crack length	C2
Distance between cracks	D
Crack length ratio	C1/C2
Crack length to distance ratio	(C1+C2)/D

A 40-inch-wide plate is considered an infinite plate in the current analysis. This assumption is made by taking the crack lengths (either C1 or C2) to be much less than the plate width (W). Combinations of crack length ratio (C1/C2) and crack spacing (D) are modeled using FE analysis and the crack tip SIF values are obtained. The SIF values provide the effect of one crack on the other (effect of adjacent crack tips on each other). Each crack (C1 and C2) is considered separately in AFGROW [9,10] to obtain the SIF value. AFGROW has standard SIF solution for a single internal crack in a plate. The FE determined SIF values for each individual crack tip is divided by the respective single crack tip SIF value obtained from AFGROW. This provides the beta correction tables for multiple crack interaction for various crack length ratios (C1/C2) with respect to crack length distance ratio [(C1+C2)/D]. The beta correction tables for crack tips growing toward the specimen edge and for tips growing toward an adjacent crack tip is provided in Appendix A2. Appendix A3.1 provides the plot of beta correction vs. [(C1+C2)/D] for various C1/C2 ratios.

The Beta Correction for intermediate values of C1/C2 or (C1+C2)/D is obtained using the B-spline interpolation technique. The spline interpolation plots of Beta Correction versus. [(C1+C2)/D] for various C1/C2 ratios is shown in Appendix A3.2.

The next step is to obtain interaction values for the tips in finite geometry. The analysis variables in the finite geometry increase the complexity of the problem. The variables considered in the finite width geometry are shown below.

Table 2: Finite Plate Parameters for the Two-Through-Crack Model

Description	Parameter
Plate width	W
Plate height	H
Left crack length	C1
Right crack length	C2
Left crack offset	B1
Right crack offset	B2
Distance between cracks	D
Height to width ratio	H/W
Crack length ratio	C1/C2
Crack length to distance ratio	(C1+C2)/D

Combinations of crack length ratio (C1/C2) and crack spacing (D) are modeled using FE analysis for finite geometries (W = 24, 20, 16, 8, and 4) and the crack tip SIF values are obtained. Single internal crack SIF values for the crack tips corrected with the infinite plate beta correction are obtained from AFGROW. The FE SIF values are divided by the respective AFGROW SIF values and the ratio indicates the additional correction needed for finite geometry. The additional correction is to take into account the finite plate effect that is due to the influence of the longer crack on the shorter crack in finite geometry. The finite plate effect is not the same as the finite width effect and the existing single crack solutions in AFGROW accounts for finite width effects.

A suitable parameter (single variable or combination of variables) representing the various geometric features, such as, plate widths, crack lengths, crack spacing, and crack offset, is selected. A plot of the parameter versus beta correction required for the finite geometry is obtained, and a fit (closed-form equation) is generated. This closed-form equation provides the finite plate effect for crack tips in the geometry.

2.1.4 Crack Linkup Possibilities

The approach adopted in the current work is based on linear elastic fracture mechanics (LEFM) principles. Crack coalescence occurs when the plastic zones of the adjacent crack tips touch each other. The size of the plastic zone in front of the crack tip will depend on the crack length, material properties of the plate and the state of stress (plane-stress or strain) in the region of the crack tip. This equation is present in AFGROW and is utilized for the current work. In a two through crack problem, crack tips can touch an adjacent crack tip or the edge of the specimen. This leads to any one of the possible cases listed below:

1) An edge crack and an internal crack in a plate
2) Unequal edge crack in a plate
3) A single offset through crack in a plate
4) A single edge crack in a plate.

The SIF solution for possibilities 3 and 4 already exists in AFGROW, and the SIF solutions for the first two cases are developed as part of the current work.

2.1.5 Curve Characteristics

The FE results for the infinite plate case (W= 40 inches) using various combinations of C1/C2 are presented in Appendix A1 (case 1). The beta correction tables and plots for each crack tip are shown in Appendix A2 and Appendix A3.1, respectively. It can be seen from the beta correction plots that the error is high in most of the cases. This is due to the assumption made in the current work regarding the infinite geometry. In the case of a 40-inch wide plate, the plate width is generally much greater than the crack lengths. However, there are cases in which the geometry is not really equivalent to an infinite plate, and high correction terms are the result.

From the FE result, it is obvious that the SIF value for the longer crack tip is higher than for the shorter crack tip. Another point of interest from the plots is that the beta correction values for shorter crack are higher than for the longer crack. As explained earlier, the multiple through cracks beta correction value is developed as an extension to the single-through-crack case in AFGROW. The interaction effect of the longer crack is higher on the shorter crack SIF value; hence there is a higher beta correction for the shorter crack.

The beta correction values have been obtained for a wide range of crack length ratios. Due to innumerable possibilities, a limit was placed on the solution domain. The limits for the problem are shown in the following equation:

$$0.02 < C1/C2 < 50.$$

It was felt that most of the practical problems fall within this solution domain. No extrapolation is done beyond these limits. For crack length ratios (C1/C2) not shown in the tables or plots, no correction was required. Intermediate values are obtained using spline interpolation technique, as its accuracy is higher when compared to linear interpolation. The spline curves are fit to FE results for various C1/C2 cases. The accuracy of the fit is tested by running several intermediate FE multiple crack runs in StressCheck® and comparing it to the values obtained through spline interpolation implemented in AFGROW. The error was less than 1 percent in all the cases.

2.1.6 Closed-Form Equation for the Finite Plate Effect

The crack tip SIF values for finite geometries are given in Appendix A (W = 40, 24, 20, 16, 8, and 4). Closed-form equations are used to account for the finite plate effect. This effect is due to the interaction effects of the cracks in a finite geometry. Two corrections are required in this case, one to account for the effect between the crack tip and specimen edge, and the other to account for the effect between adjacent crack tips.

The first step is to identify certain parameters that may influence the error. Errors lower than 1 percent are eliminated based on the parameters selected. For example, it was seen that for C2/B2 < 0.3 the error was less than a 2 percent; therefore, no correction is used for these cases. The third step is to identify a relationship between these parameters and plot it versus the required beta correction. A fit (closed-form equation) to this plot will provide the correction for finite plate effect.

The closed form correction used for the crack tip facing the specimen edge (left tip correction[3] relative to C1) is shown below.

$$\beta_C = \left(1.012 - \left(0.35\lambda^2\right)\right) \times \left(1.022 - \left(0.4\delta^{2.2}\right)\right)$$

where,

$$\lambda = \left(1 - \frac{2B}{W}\right)\left(\frac{C1}{B1}\right) \text{ , and}$$

[3] This correction is applied only if C2/B2 > 0.3.

$$\delta = \left(1 - \frac{2B}{W}\right)\left(\frac{C2}{B2}\right) .$$

B is defined as the shortest distance from the center of crack to the edge of the specimen (smallest of B1 and B2). A comparison between the curve fit correction and the correction determined from FE analyses is shown in Figure 3 for the left crack tip.

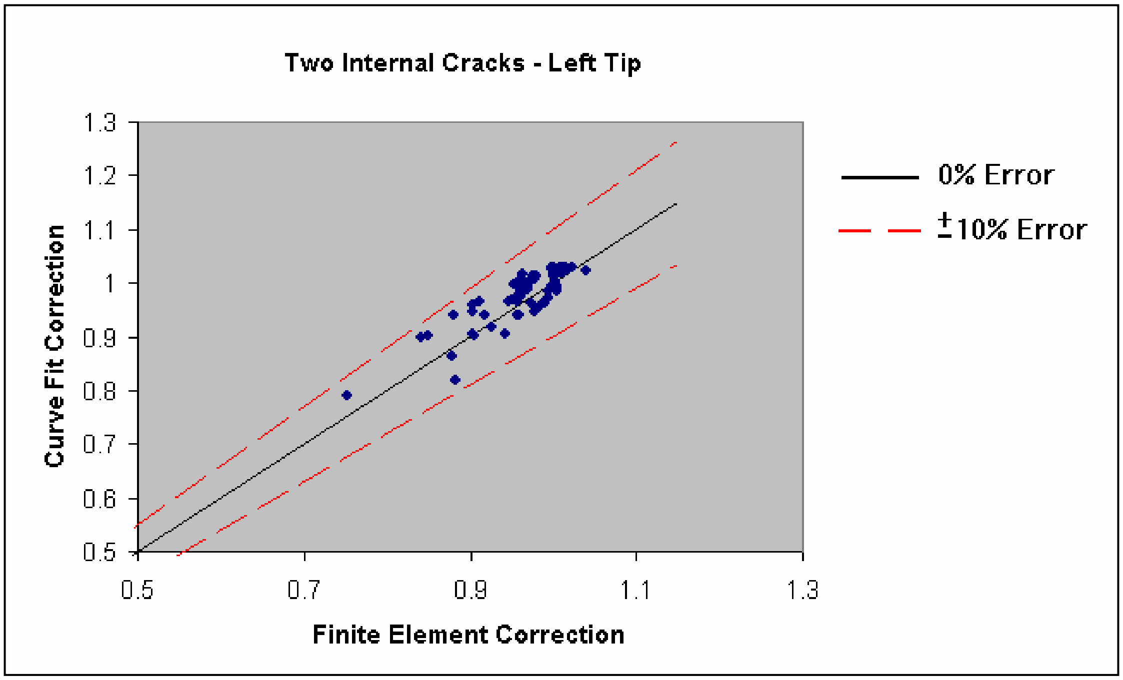


Figure 3: FE Analyses versus Curve Fit Corrections for the Left Crack Tip

The closed-form correction for the crack tip facing an adjacent tip (right tip correction[4] relative to C1) is shown here.

$$\beta_C = \left(TanH\left((\lambda - 0.5) \times \left(10\left(0.25 + 60(1-\lambda)^{28} + 360(1-\lambda)^{30} + \lambda^{35} \right)\right)\right) + 6.333 \right) \Big/ DE$$

where,

$$DE = 6.6667 \times \left(TanH\left((\delta - 0.65) \times 6.6\right) + 11\right)/10 \text{ ,}$$

$$\lambda = \left(\frac{C1 + C2}{D}\right) \text{, and}$$

[4] This correction is applied only if C2/B2 > 0.3.

$$\delta = \left(\frac{D}{W}\right).$$

A comparison between the curve fit correction and the correction determined from FE analyses is shown in Figure 4 for the right crack tip.

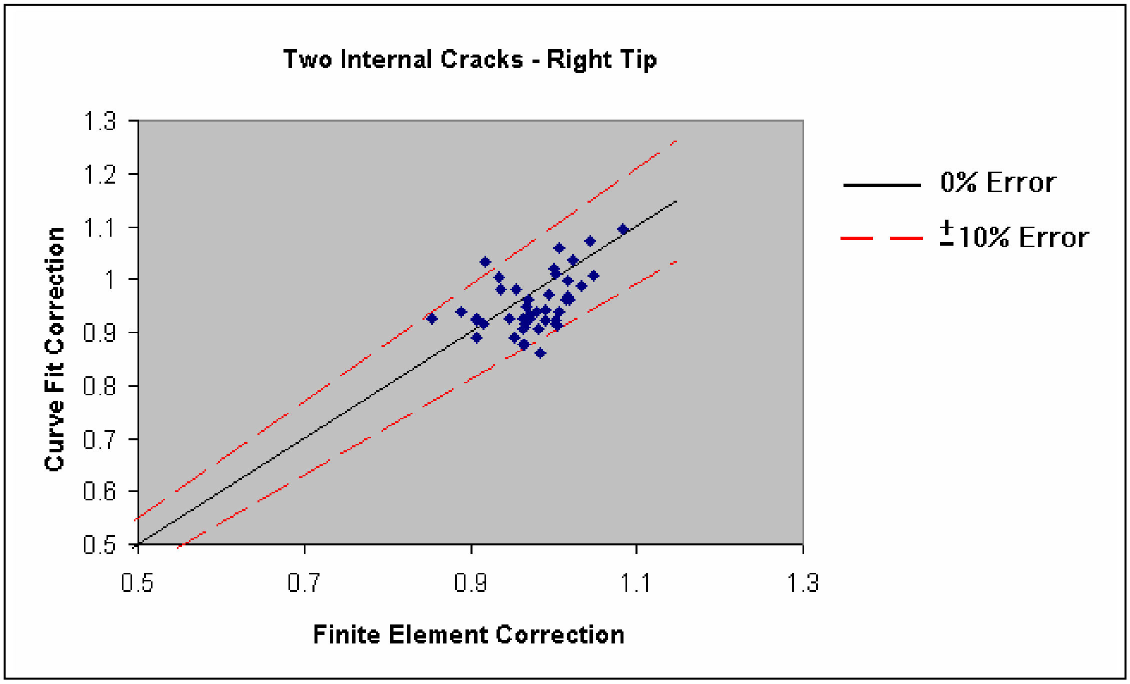


Figure 4: FE Analyses versus Curve Fit Corrections for the Right Crack Tip

2.1.7 Two Internal Through Crack Modeling Summary

A general Mode-I SIF solution to the two-through-crack problem in a plate was obtained using LEFM principles. The interaction values depend on the crack length (C1 and C2), crack spacing (D), width of the plate (W), and loading (σ). For smaller crack lengths and large crack spacing, the interaction is non-existent. Beta correction for shorter crack is higher due to the influence of longer crack. As the crack spacing decreases, the SIF values for the two crack tips approaching each other will increase, and once crack coalescence occurs, the SIF value decreases at the crack fronts.

The two-through-crack problem was implemented as one of the advanced model cases in AFGROW [9]. Crack coalescence occurs when the yield zones of the two cracks touch each other. The correction tables and closed-form equations were tested for certain configuration to determine the range of error in output. The tables in Appendix A4 show the comparison between the SIF values from StressCheck and the AFGROW multiple crack solutions for various configurations. For majority of the cases (> 85 percent) within the solution domain, the error was less than 2 percent. In general, the error is less than 10 percent in most of the cases (> 95 percent) and in some arbitrary cases it is less than 15 percent (about 1 - 2 percent of cases).

It was a very tedious and complex process to obtain a closed-form solution to the two-through-crack problem. It is recommended that for cases where there are more than two cracks, a FE program should be used in conjunction with a fatigue crack growth code like AFGROW.

2.2 Edge Crack and an Internal Crack in a Plate

The objective of the current work is to develop a general SIF solution to the problem of an edge crack and a through crack in a plate. To develop a generic solution for a range of configurations, a large amount of test and/or analytical data are required. The FE analysis codes, StressCheck® [7] and Franc2D/L [8], are used to obtain the SIF values at the crack tips for a range of plate widths. The SIF values for the crack tips growing toward each other and for crack tips growing toward the specimen edge are tabulated for an infinite geometry. Interaction effects are determined by dividing individual crack SIF values from the FE analysis (tabulated values are given in Appendix B1) with the respective single crack SIF solution in AFGROW [9, 10]. A closed-form equation that takes into account the finite plate effect is determined utilizing the existing single-crack SIF solution in AFGROW (finite geometry) and the above developed interaction tables.

No background material for this case could be found in any stress intensity handbooks. In the current analysis, both the p-version and h-version FE programs are used to obtain the SIF values at the tips. The J-integral method option is selected in both the cases for the determination of SIF values

The problem implementation involves the following four steps:

1) FE modeling
2) Infinite plate solution
3) Finite plate solution
4) Software implementation.

The first three steps are explained in this report, and the last step is covered in the AFGROW Technical Guide and User's Manual [10]. The first step involves modeling parameters and FE analysis of two cracks in infinite and finite plate geometries. The second step involves obtaining the appropriate solution for the infinite plate, and the third step is the development of corrections to account for finite plate effects. The following sections explain the first three steps in detail.

2.2.1 Modeling Issues

The crack tips are considered as separate individual objects since each is affected by different factors. The edge crack is considered the first crack and the internal crack is the second crack. In the current work, the crack on the left is modeled first (edge crack) and the crack on the right (internal crack) is placed relative to the left edge of the specimen.

The position of the internal crack depends on crack offset B. The internal crack (crack on the right) can be either centered or noncentered in the plate.

2.2.1.1 Modeling Parameters

It is important to know the definition of variables used to model the edge crack and through crack problem in infinite and finite geometry. The edge and through crack model in a finite geometry is shown in Figure 5. The infinite plate geometry will not include the offset B.

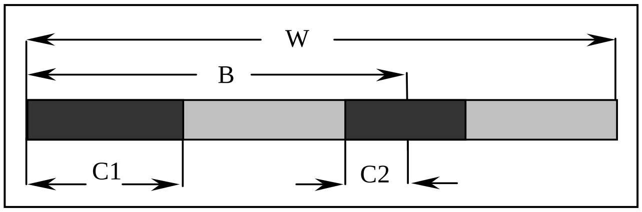


Figure 5: Collinear Edge and Internal Cracks in a Plate

<u>Legend</u>

W – Width of the plate
H – Height of the plate
C1 – Left crack length
C2 – Right crack length
B – Offset from the left edge to the center of the internal crack

2.2.2 Finite Element Modeling

The infinite and finite plate problem is modeled using both the p-version and the h-version FE programs. The StressCheck [7] (p-version) provides error estimation and convergence output for each polynomial degree of element and, hence, was the preferred code. In all of the models, the H/W ratio was set to be equal to 4.

StressCheck provides both the p-method and the h-method of mesh refinement to obtain accurate SIF values. Since the data was required to generate curves, a wide range of crack lengths was run for all the cases. Symmetry conditions were used to model half of the plate (horizontal line of symmetry) to reduce the number of degrees of freedom. Appropriate boundary conditions to prevent rigid body motion were applied along this symmetry line. A uniaxial tensile stress of $\sigma = 1$ was applied to the top edge of the specimen normal to the crack plane. Geometrically graded elements were used around the region of the two cracks. Large elements were used to model the rest of the plate in order to reduce computational time and memory.

StressCheck uses the contour integral (CI) method to obtain SIF values at the crack tips. The CI method requires the user to input the value of radius of integration path around the crack tip to extract SIF values. For a properly designed mesh, the SIF values must be independent of the radius of integration path. For the ratio $r/r_c < 0.1$, this is achieved, where 'r' is the radius of integration path and 'r_c' is the distance of crack tip. Several mesh designs were tried to ensure that for different 'r' values the SIF variation was less than a percent. For convergence studies, each problem was run for polynomial degree 'p' ranging from 1 to 8. StressCheck [16] calculates the limiting SIF value for each 'p' and outputs the percentage error between this value and the SIF value for the user designed mesh. It also outputs convergence and error estimation values for all the runs in a report. This ensures that the level of accuracy of SIF solutions obtained is high in each case.

In the case of FRANC2D/L [8], the mesh design included very small quadrilateral elements in the region around the crack and relatively large elements away from it. The element size in the region of crack is about 0.02 percent of the crack length to obtain accurate SIF values. This also ensures good convergence in results. Once the crack is placed in the geometry, FRANC2D/L uses automatic meshing to mesh the area around the crack tip. The FE runs for all configurations are shown in Appendix B. Figure 6 shows the FE mesh used by FRANC2D/L and StressCheck FE programs, respectively.

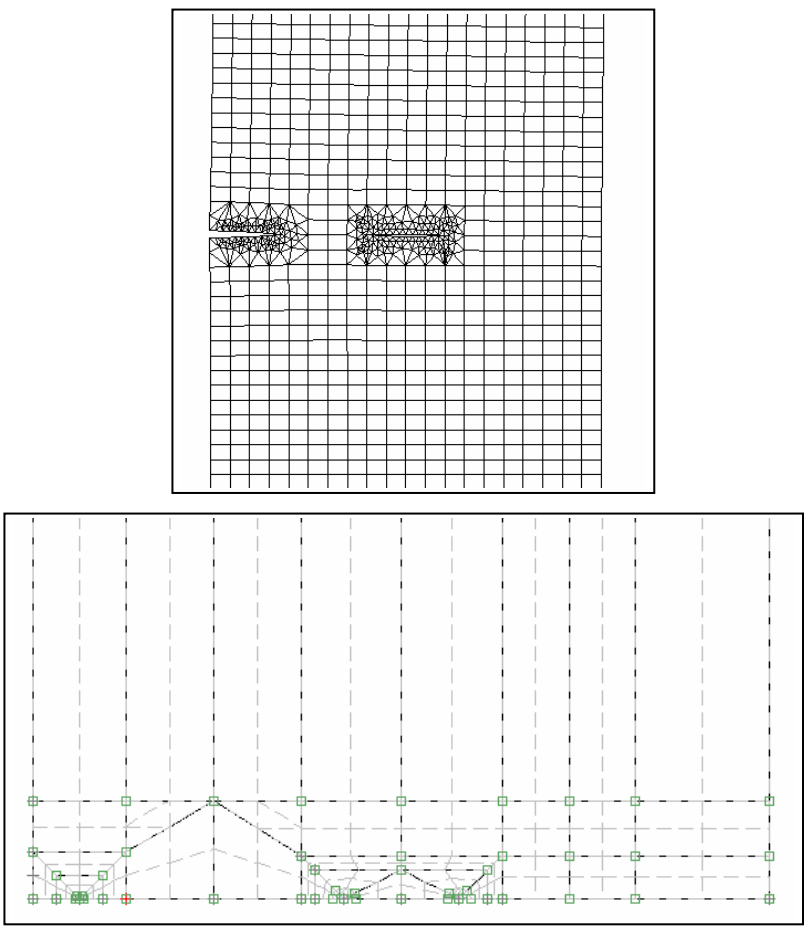


Figure 6: FE Mesh for the Edge and Through Crack

2.2.3 Methodology Adopted to Determine the General Solution

The first step is to determine the interaction effect of one crack on another in an infinite plate. The variables involved in this problem are shown in Table 3.

Table 3: Infinite Plate Parameters for the Edge and Through Crack Model

Description	**Parameter**
Plate width	W
Edge crack length	C1
Internal crack length	C2
Internal crack offset	B
Crack length ratio	C1/C2
Crack length to offset ratio	(C1+C2)/B

A plate width of 40 inches is considered to be an infinite plate in the current analysis. This assumption is reasonable if the crack lengths (C1 and C2) are much less than the plate width (W). Combinations of crack length ratio (C1/C2) are modeled using FE analysis, and the crack tip SIF values are obtained. The SIF values provide the effect of one crack on the other (effect of adjacent crack tips on each other). Each crack (C1 and C2) is considered separately in AFGROW when calculating the SIF value. AFGROW has standard SIF solution for a single internal through crack in a plate and a single edge crack in a plate. The FE determined SIF values for each individual crack tip are divided by the respective single crack tip SIF value obtained from AFGROW. This provides the beta correction tables for multiple crack interaction for various crack length ratios (C1/C2) with respect to crack length to offset ratio [(C1+C2)/B]. The beta correction tables for through crack tip growing toward the specimen edge and for edge and through crack tips growing toward each other are provided in Appendix B2. Appendix B3.1 provides the plot of beta correction vs. [(C1+C2)/B] for various C1/C2 ratios.

The beta correction for intermediate values of C1/C2 or (C1+C2)/B is obtained using the B-spline interpolation technique. The spline interpolation plots of Beta Correction vs. [(C1+C2)/B] for various C1/C2 ratios are shown in Appendix B3.2.

The next step is to obtain interaction values for the tips in a finite geometry. The analysis variables in the finite geometry increase the complexity of the problem. The variables considered in the finite width geometry are shown below.

Table 4: Finite Plate Parameters for the Edge and Through Crack Model

Description	Parameter
Plate width	W
Plate height	H
Edge crack length	C1
Internal crack length	C2
Internal crack offset	B
Height to width ratio	H/W
Crack length ratio	C1/C2
Crack length to offset ratio	(C1+C2)/B

Various crack length ratios (C1/C2) are modeled using FE analysis for finite geometry (W = 24, 20, 16, 8, and 4) and the crack tip SIF values are obtained. Single internal crack and single edge crack SIF values for the crack tips corrected with the infinite plate beta correction is obtained from AFGROW. The FE SIF values are divided by the respective AFGROW SIF values and the ratio indicates the additional correction needed for finite geometry. The additional correction is to take into account the finite plate effect that is due to the influence of the longer crack on the shorter crack in finite geometry. The finite plate effect is not the same as the finite width effect and the existing single crack solutions in AFGROW accounts for finite width effects.

A suitable parameter (single variable or combination of variables) representing the various geometry features such as; plate width, crack lengths, and crack offset, is selected. A plot of the parameter vs. beta correction required for finite geometry is obtained and a fit (closed form equation) is generated. This closed form equation provides the finite plate effect for crack tips in the geometry.

2.2.4 Crack Linkup Possibilities

The approach adopted in the current work is based on LEFM principles. Crack coalescence occurs when the plastic zones of the adjacent crack tips touch each other. The size of the plastic zone in front of the crack tip will depend on the crack length, material properties of the plate and the state of stress (plane stress or strain) in the region of the crack tip. This equation is present in AFGROW and is utilized for the current work. In the current problem, the edge crack tip and the through crack left tip can touch each other and the through crack tip can touch the edge of the specimen. This leads to any one of the following possible cases:

1) Unequal edge cracks in a plate
2) A single edge crack in a plate.

The SIF solution for single edge crack already exists in AFGROW and the SIF solutions for the first case is developed as part of the current work.

2.2.5 Curve Characteristics

The FE results for the infinite plate case (W = 40 inches) using various combinations of C1/C2 are presented in Appendix B1 (case 1). The beta correction tables and plots for the edge and through crack tips are shown in Appendix B2 and B3.1, respectively. It can be seen from the beta correction plots that the error is high in most of the cases. This is due to the assumption made in the current work regarding infinite geometry. In the case of a 40-inch-wide plate, the plate width is generally much greater than the crack lengths. However, there are cases where the geometry is not really equivalent to an infinite plate, and high correction terms are the result.

From the FE result, it is obvious that the SIF value for the longer crack is higher than for the shorter crack. Another point of interest from the plots is that the beta correction values for shorter crack are higher than for the longer crack. As explained earlier, the multiple cracks beta correction value is developed as an extension to the single crack case in AFGROW. The interaction effect of the longer crack is higher on the shorter crack SIF value; hence, there is a higher beta correction for the shorter crack.

The beta correction values have been obtained for a wide range of crack length ratios. Due to innumerable possibilities, a limit was placed on the solution domain. The limits for the problem are shown in the following equation:

$$0.05 < C1/C2 < 20.$$

It was felt that most of the practical problems fall within this solution domain. No extrapolation is done beyond these limits. For crack length ratios (C1/C2) not shown in the tables or plots, no correction was required. Intermediate values are obtained using spline interpolation technique, as its accuracy is higher when compared to linear interpolation. The spline curves are fit to FE results for various C1/C2 cases. The accuracy of the fit is tested by running several intermediate FE multiple crack runs in StressCheck® [7] and comparing it to the values obtained through spline interpolation implemented in AFGROW. The error was less than 1 percent in all the cases.

2.2.6 Closed Form Equation for the Finite Plate Effect

The crack tip SIF values for finite geometry are given in Appendix B (W = 40, 24, and 16). Closed form equations are used to account for the finite plate effect. This effect is due to the crack interaction in a finite geometry. Three corrections are required in this case, one to account for effect on the edge crack approaching an internal crack, one for the internal crack tip approaching the edge crack, and the third to account for the effect on the internal crack tip growing to the specimen edge.

The first step is to identify certain parameters that may influence the error. Errors lower than 1 percent are eliminated based on the parameters selected. For example, it was seen that for B/W = 0.5 the error was less than 1 percent; therefore, no correction is used for that case. The third step is to identify a relation between these parameters and plot it versus beta correction required. A fit (closed-form equation) to this plot will provide the correction for finite plate effect. The closed-form correction for the edge crack tip crack is given below[5]:

$$B_C = 0.4 + 0.74\left(1-(1-\lambda)^{123}\right) - \left(\left(6.68\lambda^{1.1}\right)\times(1-\lambda)^{2}\right) + \left(\left(2.51\lambda^{2.1}\right)\times 22.55\times(1-\lambda)^{5.28}\right)$$

where,

$$\lambda = \left(\frac{C1}{W-B}\right).$$

This correction is shown compared to the corrections determined from the FEM analyses in Figure 7.

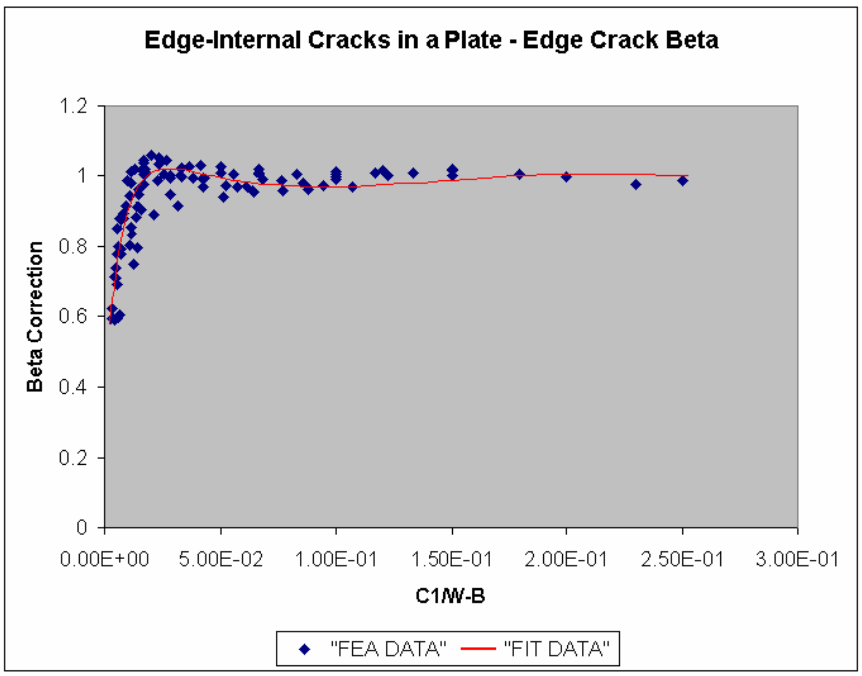


Figure 7: Correction for the Edge Crack Tip

[5] This correction is applied only if B/W ≠ 0.5, C1/B < 0.3 and C2/B > 0.15.

The closed-form correction for the internal crack tip adjacent to the edge crack is given below[6]

$$B_C = TanH\left((\lambda)^{0.25} \times 2.21 \times (1-\lambda)^{2.75} \times 1.98\right) \times 1.06$$

where,

$$\lambda = \left[\frac{C2}{W-B}\right] \times \left[1 - \left(\frac{C1+C2}{B}\right)\right].$$

This correction is shown compared to the corrections determined from the FEM analyses in Figure 8.

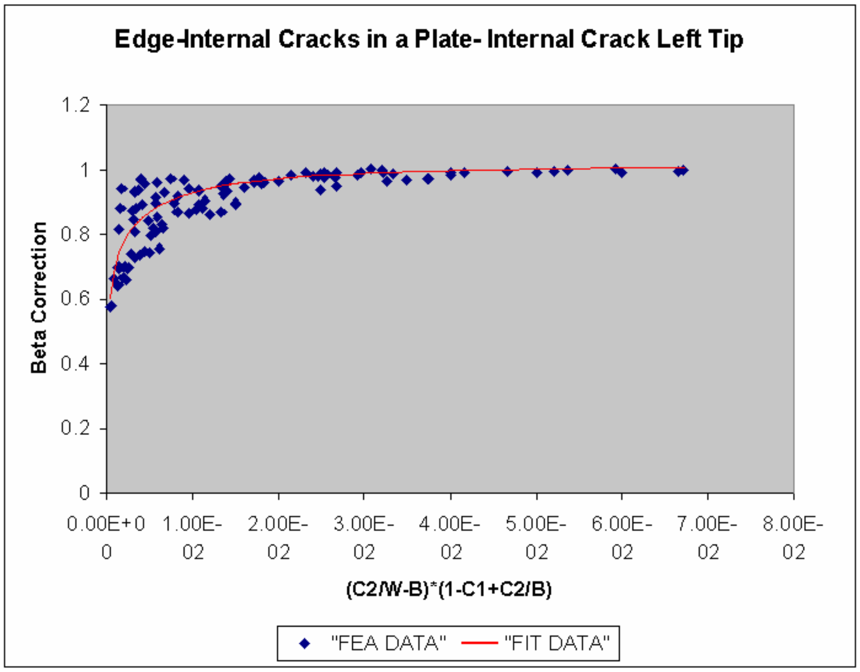


Figure 8: Correction for the Internal Crack Tip Adjacent to the Edge Crack

[6] This correction is applied only if B/W ≠ 0.5, (C1+C2)/B > 0.45 and [(1- 2B/W) (C1/B)] > 0.03.

The closed-form correction for the internal crack tip opposite to the edge crack is given below[7]

$$B_C = TanH\left((\lambda)^{0.38} \times 3.67 \times (1-\lambda)^{3.95} \times 1.96 + 0.568 \times TanH(1-\lambda)^{2.9}\right) \times 1.06$$

Where,

$$\lambda = \left[\frac{C2}{W - B}\right] \times \left[1 - \left(\frac{C1 + C2}{B}\right)\right]$$

This correction is shown compared to the corrections determined from the FEM analyses in Figure 9.

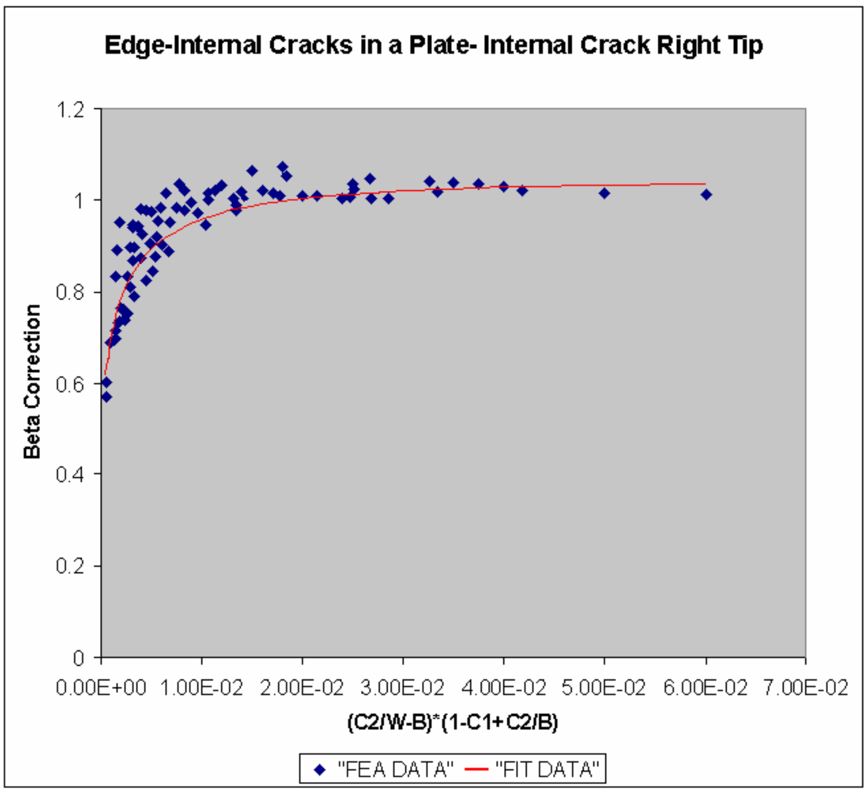


Figure 9: Correction for the Internal Crack Tip Opposite to the Edge Crack

2.2.7 Edge Crack and an Internal Crack in a Plate Modeling Summary

A general Mode-I SIF solution to the edge and through crack problem in a plate was obtained using LEFM principles. The interaction values depend on the crack length (C1 and C2), crack offset (B), width of the plate (W) and loading (σ). For smaller crack

[7] This correction is applied only if $B/W \neq 0.5$, $(C1+C2)/B > 0.45$ and $[(1- 2B/W) (C1/B)] > 0.0625$

lengths and large offset value, the interaction is nonexistent. beta correction for the shorter crack is higher due to the influence of the longer crack. As the through crack offset decreases, the SIF values for the two crack tips approaching each other will increase and once crack coalescence occurs, the SIF value decreases at the crack fronts. The edge crack length also influences the growth of the internal through crack. Long edge crack lengths (C > W/2) cause in-plane bending in the plate and this leads to compressive stresses in the opposite side. Hence, as the length of edge cracks increase the SIF value of through crack tip will decrease.

The edge and through crack problem was implemented as one of the advanced model cases in AFGROW [9]. Crack coalescence occurs when the yield zones of the two cracks touch each other. The correction tables and closed form equations were tested for certain configurations to determine the range of error in the results. The tables in Appendix B4 show the comparison between the SIF values from StressCheck [7] FE program and AFGROW [9]. For the majority of cases (> 90 percent) within the solution domain, the error was less than 2%. In general, the error is less than 10% in most of the cases (> 95 percent) and in some arbitrary cases it is less than 15 percent (about 2 percent of cases).

2.3 Unequal Edge Cracks in a Plate with Unconstrained Bending

The objective of the current work is to develop a general SIF solution to the problem of unequal, collinear edge cracks in a plate with unconstrained in-plane bending. To develop a generic solution for a range of configurations, a large amount of test and/or analytical data are required. The SIF solution for two asymmetric collinear edge cracks in an infinite plate is available in Tada's Stress Intensity Handbook [1]. AFGROW has a SIF solution for two symmetric collinear edge cracks in a plate (infinite and finite).

The solution in the Stress Intensity Handbook was determined years ago. Since then, there have been major advances in techniques to determine SIF at the crack tips. FE analysis methods have proved to be a powerful and accurate tool in fracture mechanics. Most of the commercially available FE tools can now model the stress singularities at the crack tips and accurately predict the SIF for 2-D and 3-D geometries. Another advantage is the use of the J-Integral method of estimating the SIF value. P-version programs like StressCheck [7] provide an option to vary the polynomial degree of individual model elements to obtain better solution convergence. H-version FE tools like FRANC2D/L [8] provide special crack elements and re-meshing algorithms to model stress singularities.

In the current analysis, both the p-version and h-version FE programs are used to obtain the SIF values for each crack. The J-Integral method option is selected in both the cases for the determination of SIF values

The problem implementation involves the following 4 steps:

1) FE modeling
2) Infinite Plate Solution
3) Finite Plate Solution
4) Software Implementation.

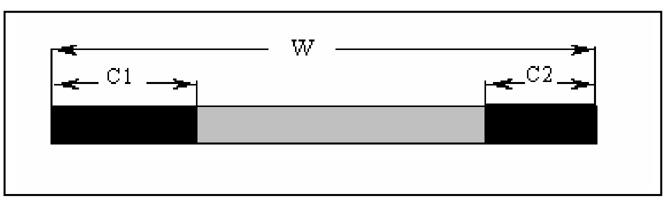
The first three steps are explained in this report and, and the last step is covered in the AFGROW Technical Guide and User's Manual [10]. The first step involves modeling parameters and FE analysis of two cracks in infinite and finite plate geometries. The second step involves obtaining the appropriate solution for the infinite plate, and the third step is the development of corrections to account for finite plate effects. The following sections explain the first three steps in detail.

2.3.1 Modeling Issues

The crack tips are considered as separate individual objects since each is affected by different factors. In the current work, the crack on the left is modeled first and is always the short crack and the crack on the right is the long crack. Changing crack lengths (short or long) will just change the crack length ratio and is equivalent to flipping the plate (viewing plate right to left).

2.3.1.1 Modeling Parameters

It is important to know the definition of variables used to model the problem in infinite and finite geometry. The unequal Edge Crack problem in a finite geometry is shown below in Figure 10.



<u>Legend</u>

W – Width of the plate
H – Height of the plate
C1 – Left crack length
C2 – Right crack length

Figure 10: Two asymmetric collinear edge cracks in a plate

2.3.2 Finite Element Modeling

The infinite and finite plate problem is modeled using both the p-version and the h-version FE programs. The StressCheck [7] (p-version) provides error estimation and convergence output for each polynomial degree of element and hence was the preferred code. In all the models, the H/W ratio was set to be equal to four.

StressCheck provides both p-method and the h-method of mesh refinement to obtain accurate SIF values. Since the data was required to generate curves, a wide range of crack lengths was run for all the cases. Symmetry conditions permitted modeling half of the plate (horizontal line of symmetry) to reduce the number of degrees of freedom. Appropriate boundary conditions to prevent rigid body motion were applied along this symmetry line. The longer edge crack causes in-plane bending in the plate that has a significant effect on the SIF value of the short crack. The boundary conditions are applied such that the in-plane bending is not constrained. A uniaxial tensile stress of $\sigma = 1$ was applied to the top edge of the specimen normal to the crack plane. Geometrically graded elements were used around the region of the two cracks. Large elements were used to model the rest of the plate in order to reduce computational time and memory.

StressCheck uses the CI method to obtain SIF values at the crack tips. The CI method requires the user to input the value of radius of integration path around the crack tip to extract SIF values. For a properly designed mesh, the SIF values must be independent of the radius of integration path. For the ratio $r/r_c < 0.1$, this is achieved, where 'r' is the radius of integration path and r_c is the distance of crack tip. Several mesh designs were tried to ensure that for different r values the SIF variation was less than a percent. For convergence studies, each problem was run for polynomial degree 'p' ranging from 1 to 8. StressCheck calculates the limiting SIF value for each 'p' and outputs the percentage error between this value and the SIF value for the user-designed mesh. It also outputs convergence and error estimation values for all the runs in a report. This ensures that the level of accuracy of SIF solutions obtained is high in each case.

In the case of FRANC2D/L the mesh design included very small quadrilateral elements in the region around the crack and relatively large elements away from it. The element size in the region of crack is about 0.02 percent of the crack length to obtain accurate SIF values. This also ensures good convergence in results. Once the crack is placed in the geometry, FRANC2D/L uses automatic meshing to mesh the area around the crack tip. The FE runs for all configurations are shown in Appendix C. Figure 11 shows the FE mesh used in respective FE programs.

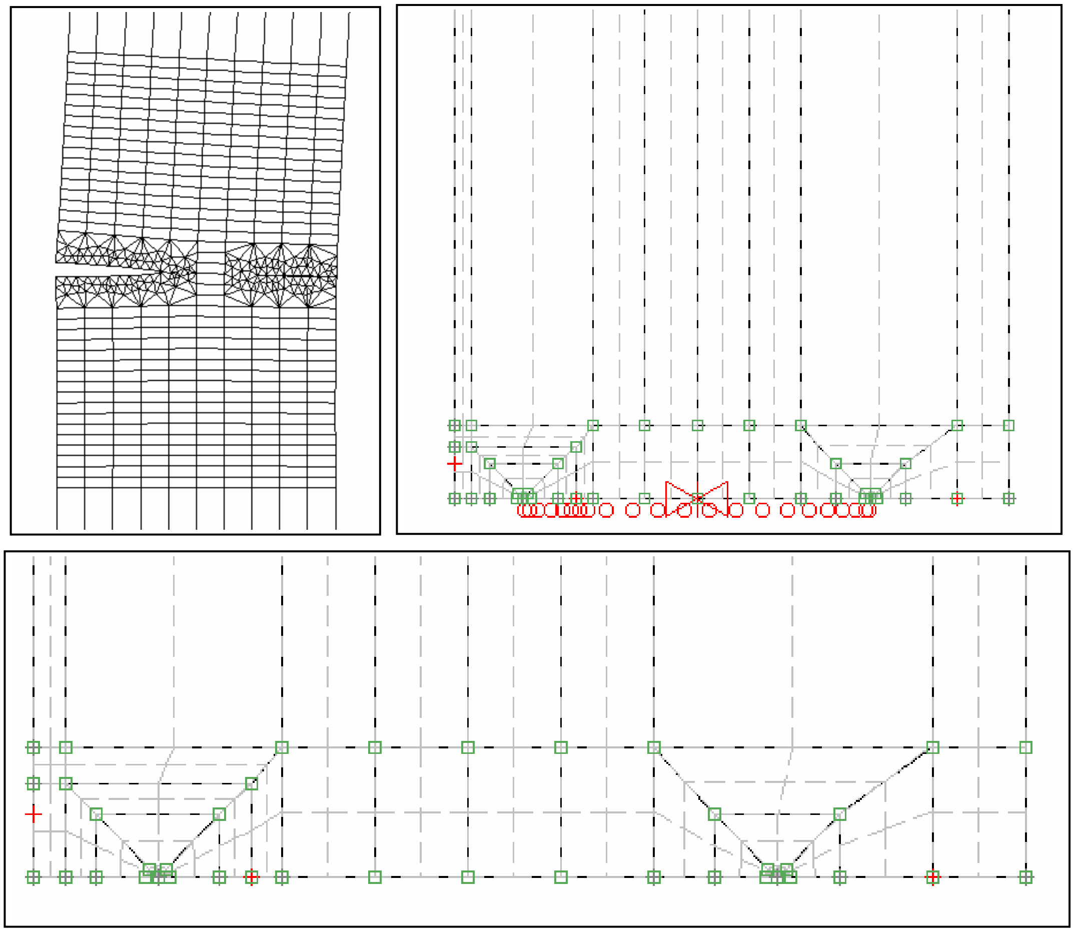


Figure 11: FE Mesh for Two Edge Cracks with Unconstrained Bending

2.3.3 Methodology Adopted to Determine the General Solution

The first step is to determine the interaction effect of one crack on another in an infinite plate. The variables involved in this problem are shown in Table 5.

Table 5: Infinite Plate Parameters for the Unequal Edge Crack Model

Description	Parameter
Plate width	W
Left crack length	C1
Right crack length	C2
Crack length ratio	C1/C2
Crack length to width ratio	(C1+C2)/W

A plate width of 40 inches is considered an infinite plate in the current analysis. This assumption is made by taking the crack lengths (either C1 or C2) to be much less than the plate width (W). A wide range of crack length ratio (C1/C2) is modeled using FE analysis and the crack tip SIF values are obtained. The SIF values provide the effect of one crack on the other (effect of adjacent crack tips on each other). Each crack (C1 and C2) is considered separately in AFGROW to obtain the SIF value. AFGROW has a standard SIF solution for a single edge crack in a plate. The FE determined SIF values for each individual crack tip is divided by the respective single crack tip SIF value obtained from AFGROW. This provides the beta correction tables for multiple crack interaction for various crack length ratios (C1/C2) with respect to crack length width ratio [(C1+C2)/W]. The beta correction tables for the two tips are provided in Appendix C2. Appendix C3.1 provides the plot of beta correction vs. [(C1+C2)/W] for various C1/C2 ratios.

The Beta Correction for intermediate values of C1/C2 or (C1+C2)/W is obtained using a spline interpolation technique. The B-spline interpolation plots of beta correction vs. [(C1+C2)/W] for various C1/C2 ratios are shown in Appendix C3.2.

The next step is to obtain interaction values for the tips in finite geometry. The analysis variables in the finite geometry are the same as in infinite geometry but increase the complexity due to finite geometry effects. The variables considered in the finite width geometry are shown in Table 6.

Table 6: Finite Plate Parameters for the Unequal Edge Crack Model

Description	**Parameter**
Plate width	W
Plate height	H
Left crack length	C1
Right crack length	C2
Height to width ratio	H/W
Crack length ratio	C1/C2
Crack length to width ratio	(C1+C2)/W

A wide range of crack length ratios (C1/C2) are modeled using FE analysis for finite geometries (W = 24, 16, and 4) to determine the crack tip SIF values. Single edge crack SIF values for the crack tips corrected with the infinite plate Beta Correction are obtained from AFGROW. The FE SIF values are divided by the respective AFGROW SIF values and the ratio indicates the additional correction needed for finite geometry. The additional correction is to take into account the finite plate effect that is due to the influence of the longer crack on the shorter crack in finite geometry. The finite plate effect is not the same as the finite width effect and the existing single crack solutions in AFGROW accounts for finite width effects.

A suitable parameter (single variable or combination of variables) representing the various geometry features such as plate width and crack length, is selected. A plot of the parameter versus beta correction required for finite geometry is obtained and a fit (closed form equation) is generated. This closed form equation provides the finite plate effect for crack tips in the geometry.

2.3.4 Crack Linkup Possibilities

The approach adopted in the current work is based on LEFM principles. Crack coalescence occurs when the plastic zones of the adjacent crack tips touch each other. The size of the plastic zone in front of the crack tip will depend on the crack length, material properties of the plate and the state of stress (plane stress or strain) in the region of the crack tip. This equation is present in AFGROW and is utilized for the current work. In a two-edge crack problem, once the yield zones for the crack tips touch each other failure of the geometry occurs. However, it is much more likely that failure will occur when the stress intensity for one, or both, crack tips reaches a critical value.

2.3.5 Curve Characteristics

The FE results for the infinite plate case (W = 40 inches) for various combinations of C1/C2 are presented in Appendix C (case 1). The beta correction tables and plots for the crack tips are shown in Appendix C2 and C3.1, respectively. It can be seen from the beta correction plots that the correction is high in many of the cases. This is due to the assumption made in the current work regarding infinite geometry. In the case of a 40-inch-wide plate, the plate width is generally much greater than the crack lengths. However, there are cases where the geometry is not equivalent to an infinite plate and high correction terms are the result.

Two things can be observed from the beta correction plots based on the length of the longer crack (C2)[8].

First, when the length of the longer crack is greater than or equal to the half width of the plate (C2 >= W/2) bending (in-plane) is seen in the plate. The bending causes high compressive stress to be built up on the other side. This affects the growth of the short crack since its SIF value is greatly affected. If one of the cracks is much longer, relative to the other, the bending effect can be quite large. Hence, the shorter crack SIF is shown as a negative value. The shorter crack will grow only after this residual stress is overcome. The SIF does not have any meaning under compressive loading and most LEFM-based fatigue crack growth life prediction methods do not use negative SIF values. In AFGROW [9], negative beta values are output for these cases, and zero is printed for SIF value.

Second, for cases where the length of the longer crack is lower than half the width of the plate (C2 < W/2), the bending in the plate is not large. Hence, the magnitude of compressive stresses on the other side of plate is not low enough to prevent the growth of the shorter crack. For these cases, the SIF value for the short crack tip is not negative. In AFGROW, both the beta values and SIF values are output.

Another point of interest from the plots is that the beta correction values for shorter crack are higher than for the longer crack. As explained earlier, the two-edge crack beta

[8] For all cases where C1/C2 < 1.

correction value is developed as an extension to the single-edge crack case in AFGROW. The interaction effect of the longer crack is higher on the shorter crack SIF value, hence higher Beta Correction for shorter crack.

The beta correction values have been obtained for a wide range of crack length ratios. Due to innumerable possibilities, a limit was placed on the solution domain. The limits for the problem are: $0.05 < C1/C2 < 20$.

It was felt that most of the practical problems fall within this solution domain. No extrapolation is done beyond these limits. For crack length ratios (C1/C2) not shown in the tables or plots, no correction was required. Intermediate values are obtained using a spline interpolation technique, as its accuracy is higher when compared to linear interpolation. The spline curves are fit to FE results for various C1/C2 cases as shown in Appendix C3.2. The accuracy of the fit was tested by running several intermediate FE multiple crack runs in StressCheck [7] and comparing it to the values obtained through spline interpolation implemented in AFGROW. The error was less than 1 percent in all the cases.

2.3.6 Closed-Form Equation for the Finite Plate Effect

The crack tip SIF values for finite geometry are shown in tables in Appendix C (W= 24, 16, and 4). The closed-form equation is to account for the finite plate effect. The finite plate effect is due to the crack interaction effects in a finite geometry. The interaction effect developed for the infinite plate geometry did an excellent job of accounting for the finite geometry. The error in the longer crack SIF value was less than 2.5 percent for all cases. The short crack SIF error was less than 10 percent in majority of cases (> 95 percent). A few arbitrary cases may give higher error, but were not seen for the configurations run. The beta correction FE value versus fit is shown in Figure 12 for the shorter crack. Due to low errors seen in both long and short crack SIF values, the finite width correction term was set to one: $B_c = 1.0$.

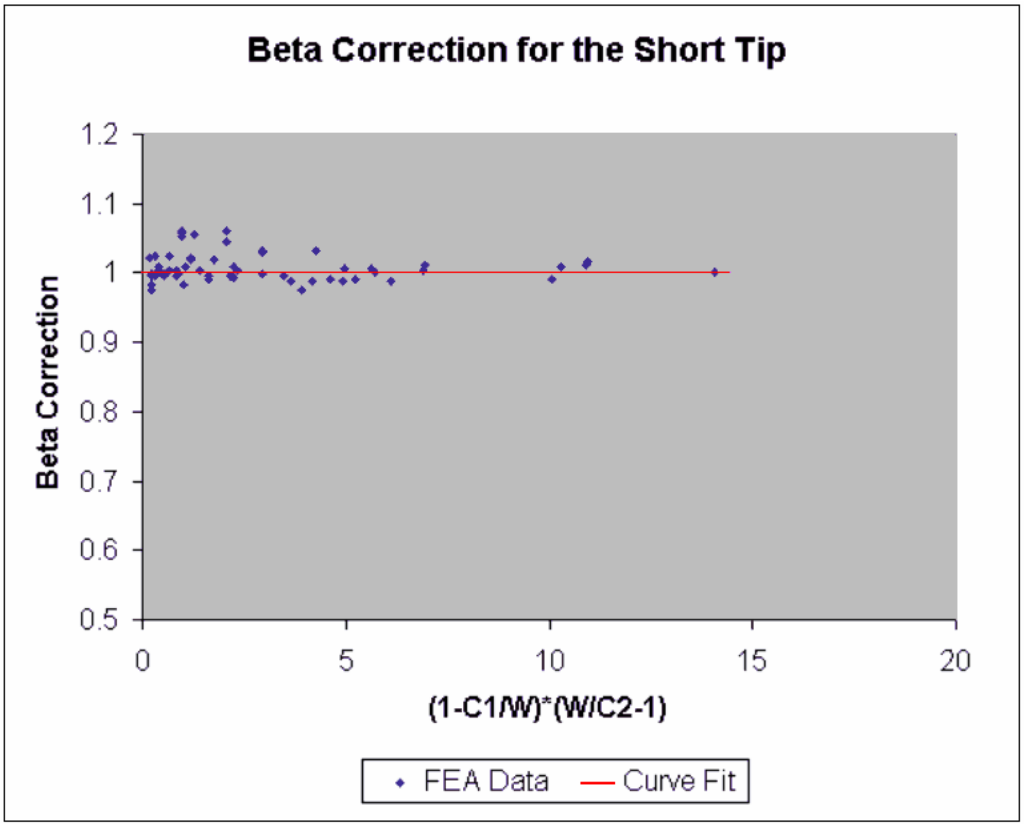


Figure 12: Correction for the Short Edge Crack Tip (Unconstrained)

2.3.7 Edge Cracks in a Plate with Unconstrained Bending Modeling Summary

A general Mode-I SIF solution to the two-edge crack problem in a plate was obtained using LEFM principles. The interaction values depend on the crack length (C1 and C2), width of the plate (W) and loading (σ). When both cracks are relatively short ((C1 + C2)/W << 1), the interaction is nonexistent. The beta correction for the shorter crack becomes increasingly negative when the longer crack length exceeds W/2 due to large in-plane bending. Otherwise, the beta corrections for both cracks are shown to increase rapidly as the two crack tips coalesce (as would be expected).

The two-edge crack problem was implemented as one of the advanced model cases in AFGROW [9]. Crack coalescence occurs when the yield zones of the two cracks touch each other. The correction tables were tested for certain configuration to determine the range of error in output. The tables in Appendix C4 show the comparison between the SIF values from StressCheck [16] FE program and AFGROW [9] for various configurations. For most of the cases (> 95 percent) within the solution domain, the error was less than 2.5 percent for both cracks. In general for the shorter crack, the error is less than 10 percent in most of the cases (> 95 percent), and in some arbitrary cases the error is still less than 15 percent (about 1-2 percent of cases).

2.4 Unequal Edge Cracks in a Plate with Constrained Bending

The objective of the current work is to develop a general SIF solution to the problem of unequal collinear edge cracks in a plate. The in-plane plate bending was constrained by applying appropriate boundary conditions during FE analysis. To develop a generic solution for a range of configurations, a large amount of test and/or analysis data are required. The SIF solution for two asymmetric collinear edge cracks with constrained bending is not available in literature. AFGROW [9,10] has SIF solution for single edge crack in a plate with in-plane bending constrained.

FE analysis methods have proved to be a powerful and accurate tool in fracture mechanics. Most of the commercially available FE tools can now model the stress singularities at the crack tips and accurately predict the SIF for 2-D and 3-D geometries. Another advantage is the use of the J-Integral method of estimating the SIF value. P-version programs like StressCheck [7] provide an option to vary the polynomial degree of individual model elements to obtain better solution convergence. H-version FE tools like FRANC2D/L [8] provide special crack elements and re-meshing algorithms to model stress singularities.

In the current analysis, the FRANC2D/L FE program was used to obtain the SIF values at the tips. The J-integral method option is selected for the determination of SIF values.

The problem implementation involves the following 4 steps:

1) FE modeling
2) Infinite plate solution
3) Finite plate solution
4) Software implementation.

The first three steps are explained in this report and, and the last step is covered in the AFGROW Technical Guide and User’s Manual [10]. The first step involves modeling parameters and FE analysis of two cracks in infinite and finite plate geometries. The second step involves obtaining the appropriate solution for the infinite plate, and the third step is the development of corrections to account for finite plate effects. The following sections explain the first three steps in detail.

2.4.1 Modeling Issues

The crack tips are considered as separate individual objects since each is affected by different factors. In the current work, the crack on the left is modeled first and is always the short crack and the crack on the right is the long crack. Changing crack lengths (short or long) will just change the crack length ratio and is equivalent to flipping the plate (viewing the plate from right to left).

2.4.1.1 Modeling Parameters

It is important to understand the definition of variables used to model the problem in infinite and finite plate geometries. The unequal edge crack problem in a finite geometry is shown in Figure 10.

2.4.2 Finite Element Modeling

The infinite and finite plate problem is modeled using the h-version FE program (FRANC2D/L). In all the models, the H/W ratio was set to be equal to 4. The complete geometry was modeled to apply appropriate constraints.

The longer edge crack causes in-plane bending in the plate that may affect the SIF value of the short crack. To prevent bending, the mid-nodes along the center of the plate were constrained. The constraints were applied sufficiently away from the crack plane region. In addition, nodes on the bottom edge of the specimen were constrained to prevent rigid body motion. The constraint used to prevent bending in the plate due to the longer edge crack is shown in Figure 13.

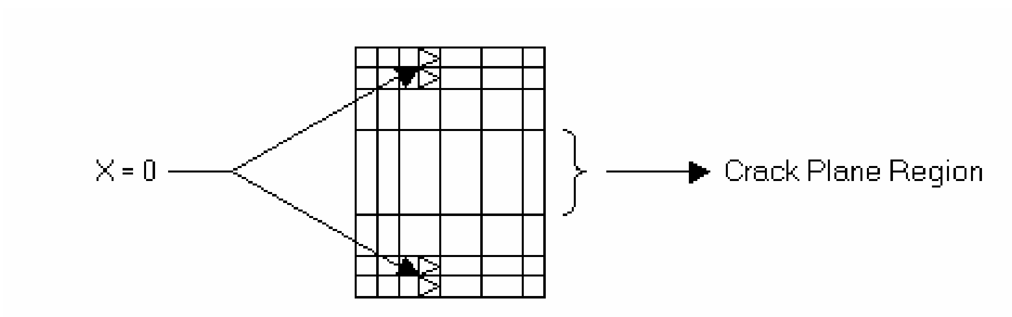


Figure 13: FEM Boundary Conditions for In-Plane Bending Constraint

The X-displacements of internal nodes were constrained along the y-symmetry line a sufficient distance from the crack region. A uniaxial tensile stress of $\sigma = 1$ was applied to the top edge of the specimen normal to the crack plane. Geometrically graded elements were used around the region of the two cracks. Large elements were used to model the rest of the plate in order to reduce computational time and memory.

In FRANC2D/L the mesh design included very small quadrilateral elements in the region around the crack and relatively large elements away from it. The element size in the region of crack is about 0.02 percent of the crack length to obtain accurate SIF values. This also ensures good convergence in results. Once the crack is placed in the geometry, FRANC2D/L uses automatic meshing to mesh the area around the crack tip. The FE runs for all configurations are shown in Appendix D. The FE mesh used to solve the current problem is shown in Figure 14.

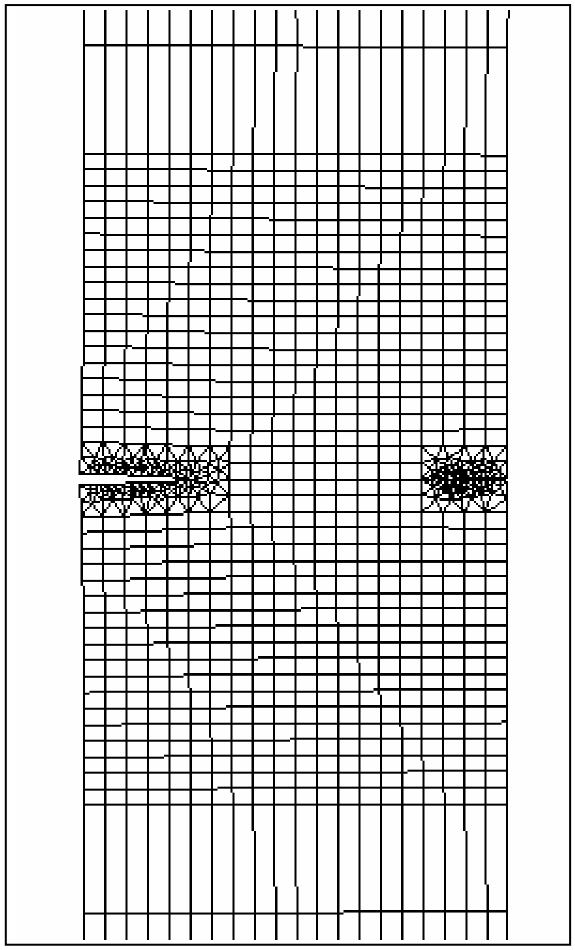


Figure 14: FE Mesh for Asymmetric Edge Crack

2.4.3 Methodology Adopted to Determine the General Solution

The first step is to determine the interaction effect of one crack on another in an infinite plate. The variables involved in this problem are shown in Table 7.

Table 7: Infinite Plate Parameters for the Constrained Unequal Edge Crack Model

Description	Parameter
Plate width	W
Left crack length	C1
Right crack length	C2
Crack length ratio	C1/C2
Crack length to width ratio	(C1+C2)/W

A plate width of 40 inches is considered to be an infinite plate in the current analysis. This assumption is made reasonable by taking the crack lengths (either C1 or C2) to be much less than the plate width (W). A wide range of crack length ratios (C1/C2) was modeled using FE analysis, and the crack tip SIF values were obtained. The SIF values include the effect of one crack on the other (effect of adjacent crack tips). Each crack (C1 and C2) is considered separately in AFGROW to obtain the SIF value. AFGROW has standard SIF solution for the constrained single edge crack in a plate. The FE determined SIF values for each individual crack tip was divided by the respective single crack tip SIF value obtained from AFGROW. This provided the beta correction tables for multiple crack interaction for various crack length ratios (C1/C2) with respect to crack length width ratio [(C1+C2)/W]. The beta correction tables for the two tips are provided in Appendix D2. Appendix D3.1 provides the plot of Beta Correction versus [(C1+C2)/W] for various C1/C2 ratios.

The beta correction for intermediate values of C1/C2 or (C1+C2)/W is obtained using a B-spline interpolation technique. The interpolation, plots of beta correction vs. [(C1+C2)/W] for various C1/C2 ratios are shown in Appendix D3.2.

The next step was to obtain interaction values for the crack tips in finite width geometries. The analysis variables used in the finite width geometry are nearly the same as shown above, but the complexity is increased due to finite geometry effects. The variables considered in the finite width geometry are shown in Table 8.

Table 8: Finite Plate Parameters for the Constrained Unequal Edge Crack Model

Description	**Parameter**
Plate width	W
Plate height	H
Left crack length	C1
Right crack length	C2
Height to width ratio	H/W
Crack length ratio	C1/C2
Crack length to width ratio	(C1+C2)/W

A wide range of crack length ratio (C1/C2) was modeled using FE analysis for several plate widths (W = 24, 16, 8 and 4 inches), and the corresponding crack tip SIF values were obtained. Single edge crack SIF values for the crack tips corrected with the infinite plate Beta Correction are obtained from AFGROW. The FE SIF values were divided by the respective AFGROW SIF values and the ratio indicates the additional correction needed for finite geometry. The additional correction is needed to account for the finite plate effect due to the influence of the longer crack on the shorter crack in a finite geometry. The finite plate effect is not the same as the finite width effect used in the existing AFGROW single crack solutions (AFGROW accounts for finite width effects).

A suitable set of parameters representing the various geometry features, such as plate width and crack lengths, were selected. A plot of this parameter versus the beta correction required for finite geometry was obtained, and a fit (closed-form equation) was generated. This closed form equation provides the finite plate effect for the crack tips in this geometry.

2.4.4 Crack Linkup Possibilities

The approach adopted in the current work is based on LEFM principles. Crack coalescence occurs when the plastic zones of adjacent crack tips touch each other. The size of the plastic zone in front of the crack tip will depend on the crack length, material properties of the plate, as well as the level and state of stress (plane stress or strain) in the region of the crack tip. This determination is made by AFGROW and is utilized for the current work. In a two-edge crack problem, once the crack tips touch each other, failure of the geometry occurs.

2.4.5 Curve Characteristics

The FE results for the infinite plate case (W = 40 inches) for various combinations of C1/C2 are presented in Appendix D1 (case 1). The beta correction tables and plots for each crack tip is shown in Appendix D2 and Appendix D3.1, respectively. It can be seen from the beta correction plots that the error is high in most of the cases. This is due to the assumption made in the current work regarding infinite geometry. In the case of a 40-inch-wide plate, the plate width is generally much greater than the crack lengths. However, there are cases where the geometry is not really equivalent to an infinite plate, and relatively high correction terms were required.

There are three things to note in comparison to the unconstrained case:

1. Since bending is prevented, the SIF value of the longer crack tip will be lower than that in the unconstrained problem. In AFGROW, the SIF value for an unconstrained single edge crack case is higher than constrained case for same crack length and stress level.
2. The SIF value of the shorter crack is not negative as in the case of unconstrained two-edge crack case. This means that the compressive stresses acting on the shorter crack are not high enough to prevent growth.
3. In some cases where the longer crack does not induce bending in plate (for $C2 \ll W/2$), the SIF value at the short crack tip will be lower when compared to unconstrained case due to the additional boundary condition applied.

The constrained multiple crack case is not currently implemented in AFGROW.

Another point of interest from the plots is that the beta correction values for the shorter crack are higher than for the longer crack. As explained earlier, the two-edge crack beta correction value is developed as an extension to the single-edge crack case in AFGROW. The interaction effect of the longer crack is higher on the shorter crack SIF value, hence higher beta correction for shorter crack.

The beta correction values have been obtained for a wide range of crack length ratios. Due to innumerable possibilities, a limit was placed on the solution domain. The limits for the problem are $0.05 < C1/C2 < 20$.

It was felt that most of the practical problems fall within this solution domain. No extrapolation is done beyond these limits. For crack length ratios (C1/C2) not shown in the tables or plots, no correction was required. Intermediate values are obtained using linear interpolation technique. The curves are fit to FE results for various C1/C2 cases. The accuracy of the fit is tested by running several intermediate FE multiple crack runs in FRANC2D/L [17] and comparing it to the values obtained through linear interpolation partially implemented in a test version of AFGROW [14,15]. The error was less than 1 percent in all the cases.

2.4.6 Closed-Form Equation for the Finite Plate Effect

The crack tip SIF values for finite geometry are shown in tables in Appendix D (W= 24 and 16). Closed-form equations are used to account for the finite plate effect. The finite plate effect is due to the crack interaction effects in finite geometries. The interaction effect developed for the infinite plate geometry did a relatively good job of accounting for finite geometry effect. The error in the longer crack SIF value was less than 3 percent for all cases. The short crack SIF value was less than 10 percent in majority of cases (> 95 percent). A few arbitrary cases may give higher error, but were not seen for the configurations run. The beta correction[9] curve fit is shown below for the shorter crack. Since errors less than 3% were seen in long crack SIF values, the correction term was set to one (B_c = 1.0) for that case.

$$B_C = \left(194.27A^4 - 482.31A^3 + 399.16A^2 - 110.74A + 1.5 + 6.43 \times (1 - A)^{4.8} + 0.43A^{82}\right),$$

where,

$$A = \left(\left(1 + \frac{1}{(W - (C1 + C2))}\right) \Big/ \left(1 + \frac{C2}{W}\right)\right).$$

The beta correction determined from FEM analysis versus the curve fit is shown in Figure 15 for the shorter crack.

[9] The correction is applied only if C1/B1 <= 0.15

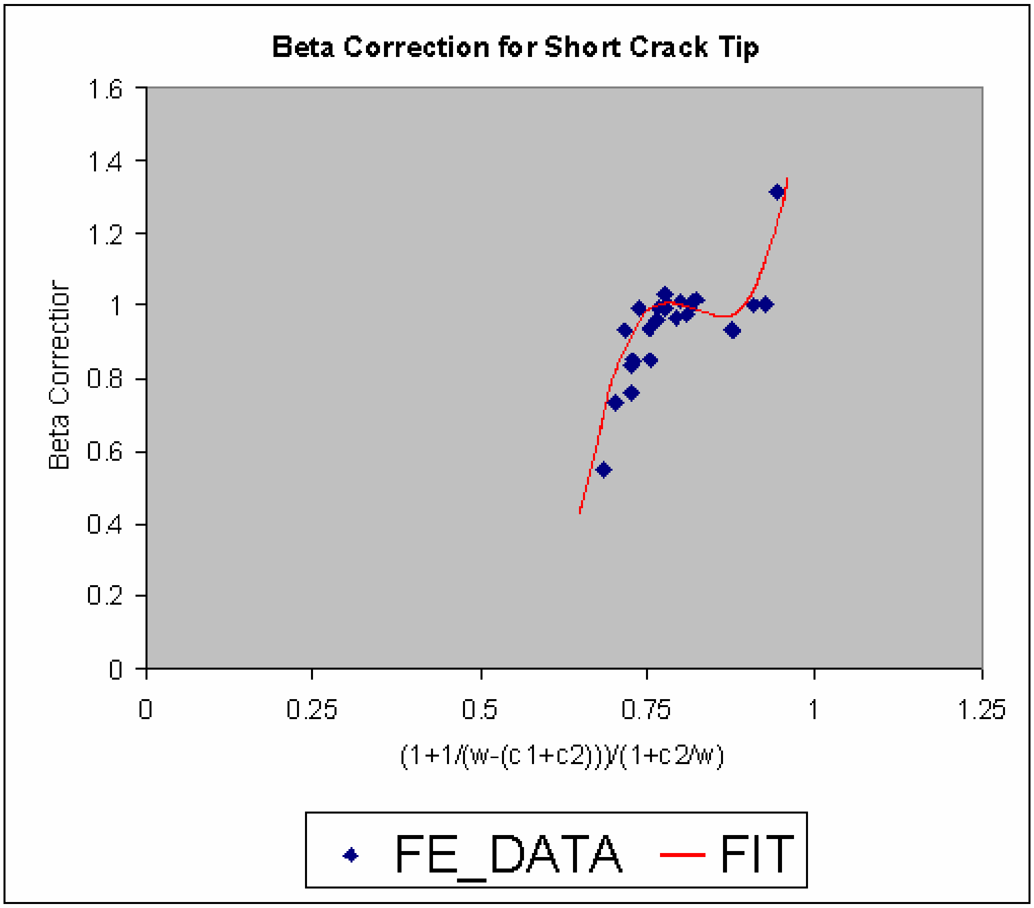


Figure 15: Correction for the Short Edge Crack Tip (Constrained)

2.4.7 Edge Cracks in a Plate with Constrained Bending Modeling Summary

A general Mode-I SIF solution to the two-edge crack problem in a plate with constrained bending was obtained using LEFM principles. The interaction values depend on the crack length (C1 and C2), width of the plate (W) and loading (σ). When both cracks are relatively short ((C1 + C2)/W << 1), the interaction is minor. The beta correction for the shorter crack does not become negative when the longer crack length exceeds W/2 due to in-plane bending constraint. Otherwise, the beta corrections for both cracks are shown to increase rapidly as the two crack tips coalesce (as would be expected).

The general SIF solution obtained was implemented as one of the advanced model cases in AFGROW [9].

2.5 Unequal Through Cracks at a Hole

The objective of the current work is to develop a general SIF solution to the problem of unequal through cracks at a hole. To develop a generic solution for a range of configurations, a large amount of test and/or analytical data are required. The SIF solutions for asymmetric through cracks at a hole in an infinite plate are available in stress intensity handbooks [1-3]. In most of the references, the SIF values at the crack tips are determined using an approximate solution based on body force methods [12, 13]. Solutions for the finite plate case could not be found in the literature. These solutions are very complex since the general solution must account for possible hole offset in addition to the finite plate effect.

The solution in the above references was determined years ago. Since then, there have been major advances in techniques to determine SIF at the crack tips. FE analysis methods have proved to be a powerful and accurate tool in fracture mechanics. Most of the commercially available FE tools can now model the stress singularities at the crack tips and accurately predict the SIF for 2-D and 3-D geometries. Another advantage is the use of the J-Integral method of estimating the SIF value. P-version programs like StressCheck [7] provide an option to vary the polynomial degree of individual model elements to obtain better solution convergence.

In the current analysis, the StressCheck FE program was used to obtain the SIF values at the tips. The J-Integral method option is selected in both the cases for the determination of SIF values.

The problem implementation involves the following 4 steps:

1) FE modeling
2) Infinite Plate Solution
3) Finite Plate Solution
4) Software Implementation.

The first three steps are explained in this report, and the last step is covered in the AFGROW Technical Guide and User's Manual [10]. The first step involves modeling parameters and FE analysis of the problem in infinite and finite plate geometry. The second step involves obtaining a solution for the infinite plate, and the third step is the development of corrections to account for finite plate effects[10]. The following sections explain the first three steps in detail.

2.5.1 Modeling Issues

The crack tips are considered as separate individual objects since each is affected by different factors. A hole of specific diameter is first located at a given distance from the left edge of the plate. The crack on the left was modeled first and was given the name, C1. Changing crack lengths (short or long) will just change the crack length ratio and is equivalent to flipping the plate (viewing plate right to left).

[10] The finite plate effects include the effect of hole offset.

2.5.1.1 Modeling Parameters

It is important to understand the definition of variables used to model the problem in infinite and finite plate geometries. The two-through-cracked-hole model in a finite geometry is shown below in Figure 16.

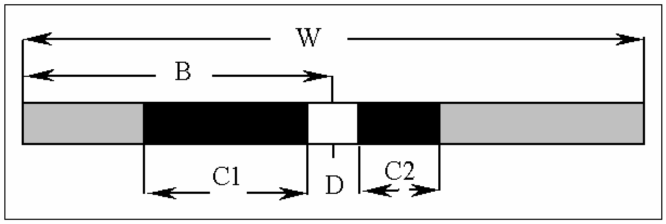


Figure 16: Asymmetric Collinear Through Cracks at a Hole

Legend

W – Width of the plate
H – Height of the plate
C1 – Left crack length
C2 – Right crack length
B – Offset of hole from left edge of specimen
D – Diameter of the hole

2.5.2 Finite Element Modeling

The infinite and finite plate problem is modeled using both the p-version FE program. The StressCheck (p-version) provides error estimation and convergence output for each polynomial degree of element and hence was the preferred code. A detailed account on StressCheck can be found in an earlier section. In all the models, the H/W ratio was set to be equal to 4.

StressCheck provides both the p-method and h-method of mesh refinement to obtain accurate SIF values. Since several data points were required to develop the solutions, a wide range of crack lengths was run for several geometric cases. Symmetry conditions permitted modeling half of the plate (horizontal line of symmetry) to reduce the number of degrees of freedom. Appropriate boundary conditions to prevent rigid body motion were applied along this symmetry line. A uniaxial tensile stress of $\sigma = 1$ was applied to the top edge of the specimen normal to the crack plane. Geometrically graded elements were used around the region of the two cracks. This helps to obtain accurate SIF value especially when the crack length is less than the diameter of the hole. Large elements were used to model the rest of the plate in order to reduce computational time and memory

StressCheck uses the CI method to obtain SIF values at the crack tips. The CI method requires the user to input the value of radius of integration path around the crack tip to extract SIF values. For a properly designed mesh, the SIF values must be independent of the radius of integration path. For the ratio $r/r_c < 0.1$, this is achieved, where r is the radius of integration path and r_c is the distance of crack tip. Several mesh designs were tried to ensure that for different r values the SIF variation was less than a percent. For convergence studies, each problem was run for polynomial of degree p ranging from 1 to 8. StressCheck [16] calculates the limiting SIF value for each p-level and outputs the percentage error between this value and the SIF value for the user-designed mesh. It also outputs convergence and error estimation values for all the runs in a report. This ensures that the level of accuracy of SIF solutions obtained is high in each case.

The FE results for a centered and non-centered hole are given in Appendix E1 and E2, respectively. The FE mesh used in the current case is shown in Figure 17.

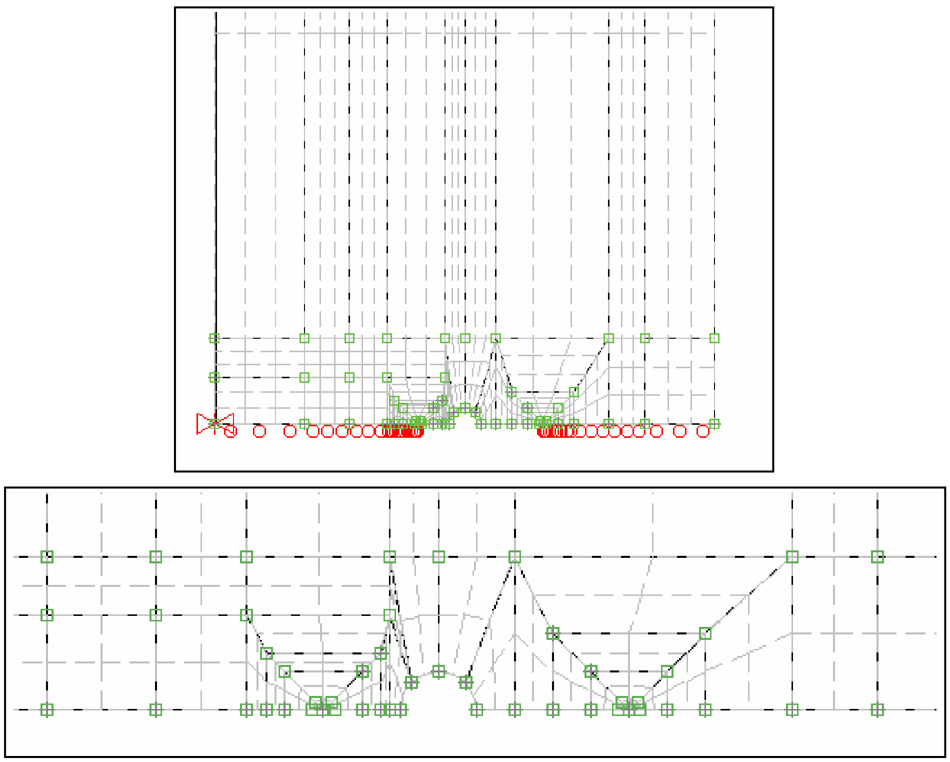


Figure 17: FE Mesh for Two-Through-Cracks at a Hole

2.5.3 Methodology Adopted to Determine the General Solution

The first step is to determine the crack interaction effect in an infinite plate. The variables involved in an infinite plate problem are shown in Table 9.

Table 9: Infinite Plate Parameters for the Two-Through-Cracked-Hole Model

Description	**Parameter**
Left crack length	C1
Right crack length	C2
Radius of the hole	R = D/2

A general solution for the infinite plate case is given in reference [13], and is considered accurate within 10 percent (5 percent for most cases) for crack lengths up to the value of the hole radius (D/2). Since this solution was available, there was no need to develop a curve fit solution based on the FE results. The referenced solution is given below.

$$K_{C1} = \sigma\sqrt{\pi\, C1} * F_{\lambda}\sqrt{\frac{R + 0.5(C1 + C2)}{R + C1}} \ ,$$

where,

$$F_{\lambda} = 3.3645\left[\frac{1}{3} + \frac{1}{6}\left(\frac{1}{(1+\lambda)^2} + \frac{3}{(1+\lambda)^4}\right)\right] * \left[1 + 0.2238\lambda - 0.1643\lambda^2\right] \text{, and}$$

$$\lambda = \frac{C1}{R} \ .$$

The above solution is made applicable for the C2 dimension by simply switching the dimensions (C1 and C2) used in the equations. A plate width of 40 inches is considered as an infinite plate in the FE analyses. This assumption is valid when the crack lengths (either C1 or C2) and hole diameter (D) are much less than the plate width (W).

The term, F_{λ}, used in the above equations is equivalent to the beta (β) value for a symmetric, double through crack at a hole in an infinite plate. The square root term is the correction used to account for crack asymmetry. As noted above, the solution is considered accurate for crack lengths up to the value of the hole radius (D/2). A beta solution for the symmetric, double cracked hole [10] is compared to F_{λ} for various C/R values in Table 10 for an infinite plate.

Table 10: Beta Values for a Double, Symmetric Through Crack at a Hole

C/R	Beta	Fλ	C/R	Beta	Fλ	C/R	Beta
0.000	3.365	3.365	1.000	1.450	1.448	6.000	1.080
0.050	3.056	3.045	1.250	1.360	1.328	8.000	1.060
0.100	2.807	2.791	1.500	1.300	1.212	10.00	1.049
0.150	2.595	2.582	1.750	1.250	1.088	100.0	1.005
0.200	2.425	2.411	2.000	1.225	0.952	1000	1.0005
0.300	2.158	2.149	2.500	1.180	0.628		
0.400	1.967	1.962	3.000	1.150	0.224		
0.500	1.824	1.824	3.500	1.131	-		
0.625	1.686	1.694	4.000	1.115	-		
0.750	1.590	1.596	5.000	1.095	-		

The agreement between the beta values and F_λ begins to diverge for C/R values greater than 1.0 (as may be expected based on the limit published in reference [13]). Since the tabular betas are applicable for all C/R values, the tabular beta values are being used in place of the term, F_λ. However, the solution given in reference [13] will be limited to individual crack lengths not to exceed a C/R value of 1.0. The beta value for each crack tip (C1 and C2) is calculated separately using B-spline interpolation of the points given in Table 10. Finally, the SIF for each crack tip is determined as indicated below using the square root term to account for crack asymmetry.

$$K_{C1} = \sigma\sqrt{\pi\, C1} * Beta_{C1} \sqrt{\frac{R + 0.5(C1 + C2)}{R + C1}}\ \text{, and}$$

$$K_{C2} = \sigma\sqrt{\pi\, C2} * Beta_{C2} \sqrt{\frac{R + 0.5(C1 + C2)}{R + C2}}\ .$$

As crack lengths increase, the hole will have less influence on the SIF, and the solution will converge to the solution for an equivalent through crack with a half-length (C_{eq}): Ceq = (C1 + C2 + D)/2.

The SIF solution for the equivalent through crack in an infinite plate is given below.

$$K = \sigma\sqrt{\pi\, C_{eq}}$$

The issue to be addressed is the minimum crack length for which the equivalent through crack solution matches the solution for the cracked hole case. Crack lengths (C1 and C2) are measured from the edge of the hole, and the equivalent through crack (C_{eq}) is measured from the center of the hole. The SIF solutions for the two cases must be based on the same crack length definition to determine the crack length at which the two SIF solutions converge. The beta values for symmetric, through cracks at a hole are given in

Table 10 based on normalized crack lengths (C/R). The equivalent beta values for through cracks (without a hole) were calculated as shown below.

$$Beta_{eq} = \frac{K_{eq}}{\sqrt{\pi C}} = \frac{\sqrt{\pi C_{eq}}}{\sqrt{\pi C}} = \sqrt{\frac{C_{eq}}{C}} = \sqrt{\frac{C+R}{C}} = \sqrt{\frac{\left(\frac{C}{R}+1\right)}{\left(\frac{C}{R}\right)}}$$

Based on the information given in Table 11, the cracked hole solution converges with the through crack solution for C/R = 5.0 and higher. For crack lengths between these limits, the solution is obtained using linear interpolation between the solutions with and without a hole for any individual crack length.

Table 11: Symmetric Cracked Hole and an Equivalent Through Crack Beta Values

C/R	Beta	$Beta_{eq}$	C/R	Beta	$Beta_{eq}$	C/R	Beta	$Beta_{eq}$
0.000	3.365	-	1.000	1.450	1.414	6.000	1.080	1.080
0.050	3.056	4.583	1.250	1.360	1.342	8.000	1.060	1.060
0.100	2.807	3.317	1.500	1.300	1.291	10.00	1.049	1.049
0.150	2.595	2.769	1.750	1.250	1.254	100.0	1.005	1.005
0.200	2.425	2.449	2.000	1.225	1.225	1000	1.0005	1.0005
0.300	2.158	2.082	2.500	1.180	1.183			
0.400	1.967	1.871	3.000	1.150	1.155			
0.500	1.824	1.732	3.500	1.131	1.134			
0.625	1.686	1.612	4.000	1.115	1.118			
0.750	1.590	1.528	5.000	1.095	1.095			

2.5.4 Crack Linkup Possibilities

Crack linkup was not considered for this geometry since the cracks are diametrically opposed.

2.5.5 Curve Characteristics

No curve fitting methods were required for this geometry.

2.5.6 Closed Form Solutions for the Finite Plate Effect

The solution for the finite plate uses the method described in Section 2.5.3. If the normalized crack length (C/R) for either crack is ≤ 1.0, then the solution for that crack tip is determined by modifying the beta value for the appropriate[11] double, symmetric cracked hole case [10] by the appropriate term to account for the crack asymmetry. If the normalized crack length (C/R) for either crack is ≥ 5.0, then the solution for that crack tip is determined for the appropriate through crack case [10] where the crack offset is located at the center of the hole. In cases where C/R is between 1.0 and 5.0, both methods are used for the given case[12], and the final solution is determined by linear interpolation based on the actual C/R value for the given crack tip, as shown below:

$$K = \left(\frac{5 - C/R}{4}\right) K_{AssymetricCrackedHole} + \left(\frac{C/R - 1}{4}\right) K_{ThroughCrack} \ .$$

This method provides for a smooth SIF transition when C/R is between 1.0 and 5.0 since the asymmetric cracked hole solution is not valid for C/R > 1.0, and the through crack solution is not accurate until C/R ≥ 5.0.

The crack tip SIF values obtained using FE models for finite width geometries (W= 20, 8, and 4) are given in Appendix E1 and E2 for centered and offset holes. These data are provided as additional information for the reader.

2.5.7 Unequal Through Cracks at a Hole Modeling Summary

A general Mode-I SIF solution for asymmetric cracks at a hole in a plate was obtained using LEFM principles. The SIF values depend on the crack length (C1 and C2), hole diameter (D), and plate width (W). Beta Correction for shorter crack is higher due to the load transfer from the longer crack side. For equal crack lengths (C1=C2), the standard symmetric double cracked hole SIF solution in AFGROW [10] is used. The infinite plate solution was compared with data from an independent literature [12] source. This comparison is shown in Appendix E3.

2.6 Through Crack Growing Toward a Hole

The objective of the current work is to develop a general Stress Intensity Factor SIF solution to the problem of an internal through crack growing toward a hole. To develop a generic solution for a range of configurations, a large amount of test and/or analytical data are required. The SIF solution for a through crack growing toward a centered hole in an infinite plate is available in the open literature [3]. This solution was developed many years ago and is valid only for the crack tip growing toward the hole. Since that time, there have been major advances in the technology used to determine the SIF for finite

[11] For the given plate width, hole radius, and crack length (C1 or C2)
[12] Both methods (asymmetric cracked hole and through crack) are used for a given C/R value

geometries where the hole and crack are arbitrarily offset in a finite width plate. FE analysis methods have proved to be a powerful and accurate tool in fracture mechanics. Most of the commercially available FE tools can now model the stress singularities at the crack tips and accurately predict the SIF for 2-D and 3-D geometries. Another advantage is the use of the J-Integral method of estimating the SIF value. P-version programs like StressCheck [7] provide an option to vary the polynomial degree of individual model elements to obtain better solution convergence.

In the current analysis, the StressCheck FE program is used to obtain the SIF values at both crack tips. The J-integral method option is selected in both the cases for the determination of SIF values. The approach used to determine the general solution for through crack growing toward a hole in a plate (infinite and finite) is explained in the next section.

The problem implementation involves the following 4 steps:

1) FE modeling
2) Infinite Plate Solution
3) Finite Plate Solution
4) Software Implementation.

The first three steps are explained in this report and, and the last step is covered in the AFGROW Technical Guide and User's Manual [10]. The first step involves modeling parameters and FE analysis of the problem in infinite/finite plate geometries. The second step involves obtaining the correction for an infinite plate width, and the third step is the development of a solution to account for the finite plate effect. The following sections explain the first three steps in detail.

2.6.1 Modeling Issues

The crack tips are considered as separate individual objects since the hole effect will be different for each crack tip.

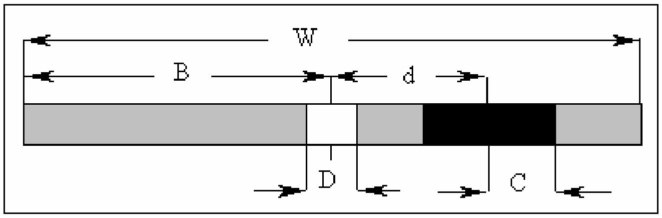
First, a hole of specific diameter was located in the center of a plate. The crack was modeled by positioning it relative to the center of the hole. The hole was always centered (center of hole = W/2) in the plate. This problem was modeled using a wide range of plate width and relative distance between the crack and hole centers.

Second, an offset through crack was modeled with an offset hole located at various distances from the crack. This case was modeled for a limited number of geometric combinations since it was used to verify the solution for more extreme geometric cases.

The main issue in this case is whether or not the effect of a hole a given through-the-thickness crack may be affected by the relative positions of the hole and crack in a finite plate.

2.6.1.1 Modeling Parameters

It is important to know the definition of variables used to model the problem in infinite and finite geometry. The model of a crack growing toward a hole in a finite width plate is shown below in Figure 18.



Legend:

W – Width of the plate
C – Crack length
d – Distance from the center of hole to the center of crack
B – Offset of hole from left edge of specimen
D – Diameter of the hole (R = D/2)
Cmax – Maximum crack length (Cmax = d-R)

Figure 18: Through Crack Growing Toward a Hole

2.6.2 Finite Element Modeling

The infinite and finite plate problems are modeled using the p-version FE program, StressCheck. StressCheck provides a very user-friendly interface and provides error estimation and convergence output for each polynomial degree of freedom and, hence, was the preferred code. A more detailed description of StressCheck can be found in an earlier section. In all models, the H/W ratio was set equal to 4 in order to obtain a more uniform remote gross stress condition.

StressCheck provides both p-method and the h-method of mesh refinement to obtain accurate SIF values. To cover all possible geometric conditions, a wide range of radius of hole to center distance (R/d) ratios was modeled. Symmetry conditions were used by modeling half of the plate (horizontal line of symmetry) to reduce the number of degrees of freedom for the FE models. Appropriate boundary conditions to prevent rigid body motion were applied along this line of symmetry. A uniaxial tensile stress of one ($\sigma = 1$) was applied to the top edge of the specimen normal to the crack plane. Geometrically graded elements were used around the region of the crack. This helps to obtain accurate SIF value especially when the crack tip is very close to the hole. Large elements were used to model the rest of the plate in order to reduce computational time and memory requirements.

StressCheck uses the CI method to obtain SIF values at the crack tips. The CI method requires the user to input the value of radius of integration path around the crack tip to extract SIF values. For a properly designed mesh, the SIF values must be independent of the radius of integration path. For the ratio $r/r_c < 0.1$, this is achieved, where r is the radius of integration path and r_c is the distance of crack tip. Several mesh designs were tried to ensure that for different r values the SIF variation was less than a percent. For convergence studies, each problem was run for polynomial degree p ranging from 1 to 8. StressCheck calculates the limiting SIF value for each p and outputs the percentage error between this value and the SIF value for the user designed mesh. It also outputs convergence and error estimation values for all the runs in a report. This ensures that the level of accuracy of SIF solutions obtained is high in each case.

The FE runs for all configurations are given in Appendix F. Figure 19 shows the FE mesh used for this case.

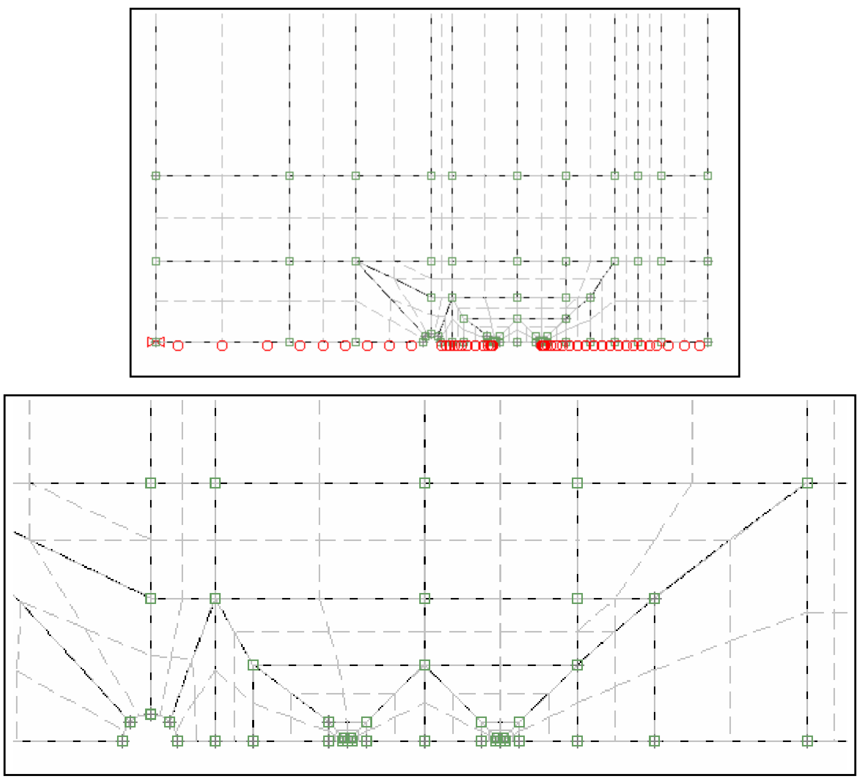


Figure 19: FE Mesh for a Through Crack Growing to a Hole

2.6.3 Methodology Adopted to Determine the General Solution

The first step is to determine the interaction effect of a hole on the crack tip in an infinite plate. The variables involved in an infinite plate problem are shown in Table 12.

Table 12: Infinite Plate Parameters for a Through Crack Growing to a Hole

Description	**Parameter**
Plate width	W
Crack length	C
Diameter of the hole	D (R = D/2)
Radius to center distance ratio	R/d
Crack length ratio	C/Cmax

A 40-inch-wide plate is considered as an infinite plate for the current analysis. This assumption is valid as long as the crack length (C) and hole diameter (D) are much smaller than the plate width (W). A wide range of radius to center distance ratio (R/d) was modeled using FE analysis and the crack tip SIF values were obtained as a function of normalized crack length (C/Cmax). The resulting SIF values provide the effect of the hole for each geometric case. Each crack tip is considered separately in AFGROW to obtain the SIF value. AFGROW has a standard SIF solution for the single internal crack. The FE determined SIF values for each individual crack tip is divided by the respective single crack tip SIF value assuming that the hole is absent. This provides the beta correction that accounts for the effect of hole on each crack tip. Beta corrections for various hole radius to center distance ratios (R/d) with respect to crack length ratios [C/Cmax] are provided in Appendix F2. Appendix F3 provides the plot of beta correction versus C/Cmax for various R/d ratios.

2.6.4 Crack Linkup Possibilities

Crack linkup was not considered for this geometry since only a single crack is being considered.

2.6.5 Curve Characteristics

The beta correction values have been obtained for a wide range of R/d ratios. Due to numerical considerations, a limit was placed on the FEM solution domain. The limits for the FEM solutions are $0.0625 \leq R/d \leq 0.9$ and $0.1 \leq C/Cmax \leq 0.98$.

It was discovered that the beta correction, as the crack length goes to zero, approaches the value of stress concentration (Kt) at the appropriate distance from the hole edge for an infinite plate. In addition, it is obvious that the SIF will go to infinity as the crack actually touches the hole edge. AFGROW normally applies a very large numerical value in cases that are infinite in the analog world. Therefore, the numerical limits placed on the FEM solutions do not limit the ability to reasonably determine appropriate SIF solutions for all possible geometries.

2.6.5.1 Parameters Used to Determine the Influence of a Hole

The amount of influence the hole has on the crack tip SIF value depends on variables, such as hole diameter, crack length, distance from the crack center to hole center, and load. The first three variables define the geometry and play an important role. It was found that for large center distance (d) values, the influence of hole is nonexistent and the problem can be treated as a single through crack in a plate. AFGROW has the SIF solution for both centered and offset through cracks that are used for large center distances when the hole effect is insignificant.

To determine the parameters required to model the hole effect, FE analyses were performed using the StressCheck program. Combinations of center distance (d), crack length (C) and hole diameter (D) were modeled in an infinite plate (W = 40). The influence of hole on crack tip SIF value was studied. Based on these FE results, the parameters shown below were determined to be adequate to model the hole effect:

$$P_l = \frac{C}{C\max} = \frac{C}{(d-R)} \ ,$$

$$P_d = \frac{R}{d} \ .$$

The first parameter, P_l, can be thought of as the normalized crack length (relative to C_{max}). The maximum crack length is defined as the half crack length at the point that the crack would intersect the edge of the hole. The second parameter, P_d, is simply the ratio of the hole radius to the distance from the centers of the crack and hole. It is clear that the hole effect will increase as P_d increases since a larger hole will have an influence over a greater distance from its center. It is also obvious that the hole effect will be greater for the crack tip closest to the hole and less for the crack tip growing away from the hole. These parameters are used to determine the hole effect for both crack tips. It was not possible to develop closed-form beta correction equations for each crack tip, so a table lookup approach was used. The corrections for each crack tip are shown as a function of P_d and P_l in Appendix F2 and F3.

It is interesting to note that the correction for the crack tip growing away from the hole shows an initial decrease as C/Cmax increases, but then begins to increase as the crack continues to grow. At first glance, this appears strange since that crack tip is continually growing away from the effect of the hole. However, as the crack tip growing toward the hole gets closer to the hole, a portion of the stress in the plate will begin to shift to the other crack tip.

It should also be noted that the beta correction for each crack tip should actually be equal for the same R/d when C/Cmax = 0. Since a B-spline method is being used to interpolate the correction in AFGROW, the tabular values were adjusted to ensure that the interpolated curves are well behaved. An example comparison between the B-spline fit and the FEM results are shown in Figure 20.

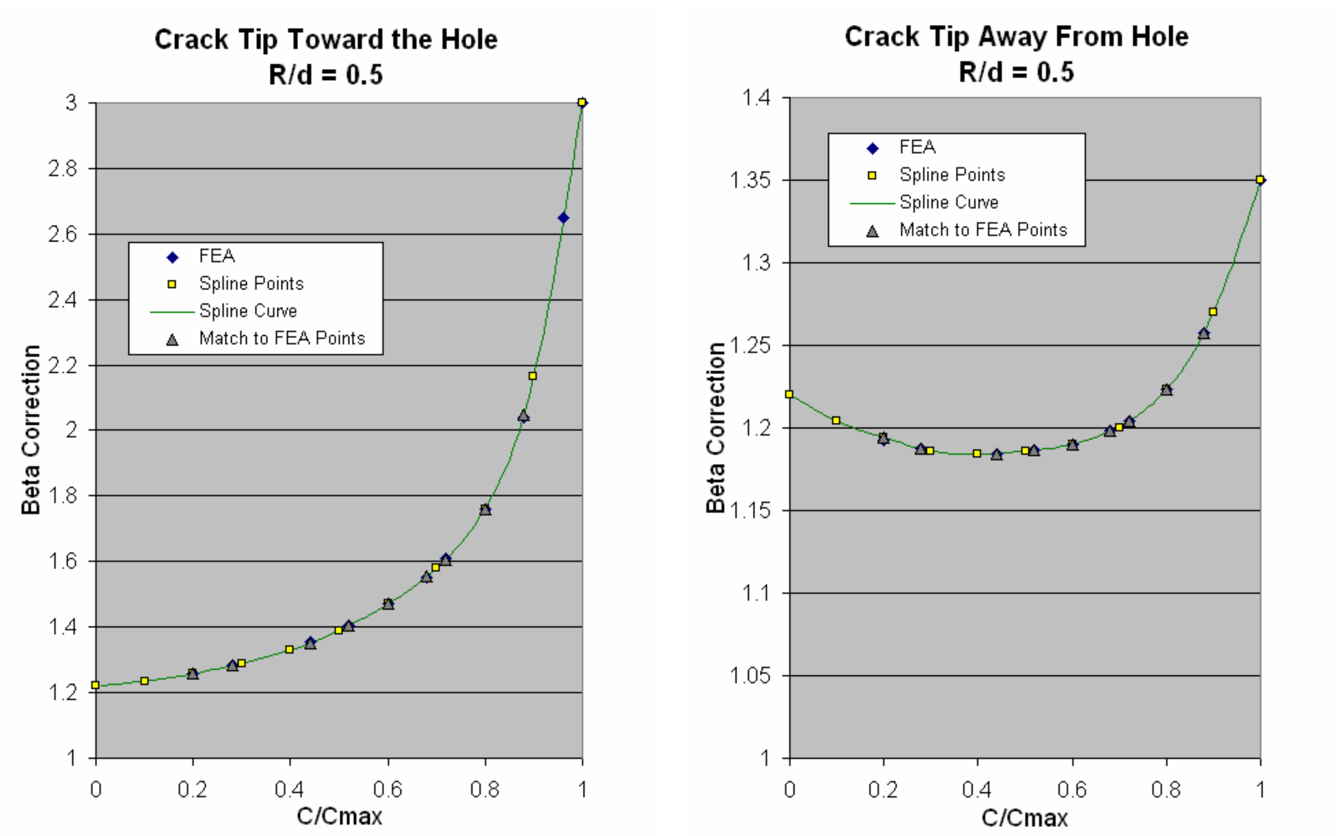


Figure 20: Beta Correction versus FEM Data

2.6.6 Correction for the Finite Plate Effect

In the previous section, the effect of plate width was not considered since the initial corrections were determined for a wide plate, which was assumed for practical purposes to be equivalent to an infinite plate. The plate width has to be considered in finite geometry. The FE runs for various finite plate widths (W = 20, 16, 8, and 4) are shown in Appendix F. These results were used to obtain the additional correction required to account for finite plate effect.

The parameter used to determine the finite plate effect was determined based on the error remaining after application of the initial correction for the hole effect. After looking at several parameters and sorting the results based on the magnitude of the remaining error for the various plate widths, the following parameter was used to provide the additional finite width correction:

$$P_{fw} = \left(\frac{C}{C_{\max}}\right)\left(\frac{D}{W - D - 2C}\right).$$

The finite plate correction is provided in Table 13 as a function of P_{fw}.

Table 13: Finite Width Correction for a Through Crack Growing to a Hole

P_{fw}	Correction	P_{fw}	Correction	P_{fw}	Correction
0.000	1.0000	0.100	1.0433	0.300	1.0625
0.010	1.0100	0.125	1.0472	0.400	1.0685
0.025	1.0200	0.150	1.0500	1.400	1.1243
0.050	1.0310	0.200	1.0551	2.000	1.1570
0.075	1.0382	0.250	1.0589	10.000	1.5930

This correction is only required for the crack tip growing toward the hole since the additional finite width error for the other crack tip was insignificant. AFGROW uses B-spline interpolation to determine the finite width correction and does not extrapolate values beyond the limits of the table. However, for all practical cases, P_{fw} values will not exceed the limits of the table.

2.6.7 Through Crack Growing Toward a Hole Modeling Summary

A general Mode-I SIF solution for the case of a through crack growing toward a hole in a plate was obtained using existing solutions for a through crack without a hole and applying corrections to account for both the hole and plate geometries. The crack tip SIF values depend on the proximity of the crack to the hole and the influence of the plate width. The parameters that define the influence were obtained from FE modeling of a large number of geometric combinations (plate width, hole and crack locations, and crack lengths). Beta correction tables for the two crack tips have been obtained for the infinite width geometry. The SIF values for the crack tips in finite geometries have been tabulated for a range of plate widths. This solution has been implemented as one of the advanced model cases in AFGROW [9]. Stress intensity values obtained using AFGROW for a 1,000-inch-wide plate were compared to handbook solutions [3] for an infinite plate (for the crack tip growing toward a centered hole). In addition, a comparison between the solution implemented in AFGROW and the results from FE analyses for an offset crack growing toward an offset hole. The results of these comparisons are given in Appendix F4.

2.7 Edge Crack Growing Toward a Hole

The objective of the current work is to develop a general SIF solution to the problem of an edge crack growing toward a hole. To develop a generic solution for a range of configurations, a large amount of test and/or analytical data are required. In the current analysis, the StressCheck FE program was used to obtain the SIF value for the edge crack. The J-Integral method option was used for the determination of SIF values

The problem implementation involves the following 4 steps:

1) FE modeling
2) Infinite plate solution
3) Finite plate solution
4) Software implementation.

The first three steps are explained in this report and, and the last step is covered in the AFGROW Technical Guide and User's Manual [10]. The first step involves modeling parameters and FE analysis of the problem in infinite/finite plate geometries. The second step involves obtaining the correction for an infinite plate, and the third step is the development of a solution to account for the effect of the finite plate. The following sections explain the first three steps in detail.

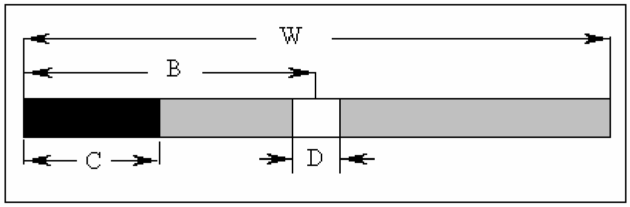
2.7.1 Modeling Issues

The first step in the development of the SIF solution for an edge crack approaching a hole was to apply the correction for the crack tip growing toward a hole that was developed for the through crack (see section 2.7). In the through crack case, the infinite plate case was modeled using a plate width of 40 inches and a centered hole where the crack, hole, and center distance dimensions were much smaller than the width. The edge crack case (by definition) may only be considered to be semi-infinite at best, since the crack origin occurs at a free edge. Therefore, it was expected that the through crack correction for a through crack in an infinite plate would not be adequate to model the hole effect for an edge crack in a semi-infinite plate. The StressCheck [7] FEM code was used to model the edge crack for a 40-inch-wide plate for a large number of geometric combinations of crack length and hole placement to fully assess the effect of the semi-infinite plate.

It is also important to consider any differences between the hole effect for through and edge cracks in finite width plates. Numerous FEM analyses were performed for combinations of hole diameter, placement, and crack length for several plate widths.

2.7.1.1 Modeling Parameters

It is important to know the definition of variables used to model the problem in infinite and finite geometry. The model of a crack growing toward a hole in a finite width plate is shown below in Figure 21.



Legend:

W – Width of the Plate
C – Crack Length
B – Offset of Hole from Left Edge of Specimen
D – Diameter of the Hole (R = D/2)
Cmax – Maximum Crack Length (Cmax = B-R)

Figure 21: Edge Crack Growing Toward a Centered Hole

2.7.2 Finite Element Modeling

The semi-infinite and finite plate problems are modeled using the p-version FE program, StressCheck. StressCheck provides a very user friendly interface and provides error estimation and convergence output for each polynomial degree of freedom and hence was the preferred code. A more detailed description of StressCheck can be found in an earlier section. In all models, the H/W ratio was set equal to four (or greater) to obtain a more uniform remote gross stress condition.

StressCheck provides both the p-method and the h-method of mesh refinement to obtain accurate SIF values. In order to cover all possible geometric conditions, a wide range of plate width to hole diameter (W/D) and hole offset to plate width (B/W) ratios was modeled. Symmetry conditions were used by modeling half of the plate (horizontal line of symmetry) to reduce the number of degrees of freedom. Appropriate boundary conditions to prevent rigid body motion were applied along this line of symmetry. A uniaxial tensile stress of one ($\sigma = 1$) was applied to the top edge of the specimen normal to the crack plane. Geometrically graded elements were used around the region of the crack. This helps to obtain accurate SIF value especially when the crack tip is very close to the hole. Large elements were used to model the rest of the plate in order to reduce computational time and memory requirements.

StressCheck uses the CI method to obtain SIF values at the crack tips. The CI method requires the user to input the value of radius of integration path around the crack tip to extract SIF values. For a properly designed mesh, the SIF values must be independent of the radius of integration path. For the ratio $r/r_c < 0.1$, this is achieved, where r is the radius of integration path and r_c is the distance of crack tip. Several mesh designs were tried to ensure that for different r values the SIF variation was less than a percent. For convergence studies, each problem was run for polynomial degree p ranging from 1 to 8.

StressCheck calculates the limiting SIF value for each *p* and outputs the percentage error between this value and the SIF value for the user designed mesh. It also outputs convergence and error estimation values for all the runs in a report. This ensures that the level of accuracy of SIF solutions obtained is high in each case.

The FE results for all configurations are given in Appendix G. Figure 22 shows the FE mesh used for this case.

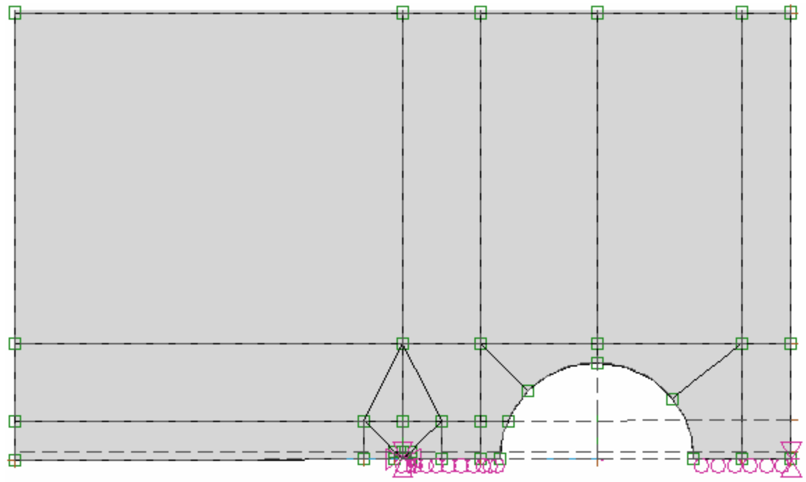


Figure 22: FE Mesh for an Edge Crack Growing to a Hole

2.7.3 Methodology Adopted to Determine the General Solution

The first step is to determine the interaction effect of hole on the crack tip in an infinite plate. The variables involved in an infinite plate problem are shown below (See Figure 21).

Table 14: Infinite Plate Parameters for an Edge Crack Growing to a Hole

Description	**Parameter**
Plate width	W
Crack length	C
Diameter of the hole	D (R = D/2)
Center distance (assuming symmetry)	d = B
Radius to center distance ratio	R/d
Crack length ratio	C/Cmax

A 40-inch-wide plate is considered as a semi-infinite plate for the current analysis. This assumption is valid as long as the crack length (C) and hole diameter (D) are much smaller than the plate width (W). A constant hole diameter of 0.25 inch was used. A wide range of hole offset to plate width (B/W) ratios was modeled using FE analysis and the crack tip SIF values were obtained as a function of normalized crack length (C/Cmax). These SIF values provide the effect of the hole on edge crack. AFGROW has a standard SIF solution for the edge crack as well as an initial correction for a through crack (for the tip growing toward a hole). The FE determined SIF values for each individual crack tip is divided by the resultant edge crack SIF value. This provides the beta correction that accounts for the effect of the semi-infinite plate. Beta corrections for various hole offset values with respect to crack length ratios [C/Cmax] are provided in Appendix G1[13].

2.7.4 Crack Linkup Possibilities

Crack linkup was not considered for this geometry since only a single edge crack is being considered.

2.7.5 Curve Characteristics

The beta correction values have been obtained for a wide range of hole offset values for the 40-inch-wide semi-infinite plate. These corrections are in addition to the hole effect itself, which are described in Section 2.7 for the crack tip growing to a hole (where the center distance, d, is set equal to the hole offset). The additional correction for the edge crack in a semi-infinite plate actually reduce the hole effect for low values of C/Cmax and increase the hole effect as C/Cmax increases. The limits for the FEM solutions are: $0.1 \leq C/Cmax \leq 0.98$. In the case of the 40-inch-wide, semi-infinite plate, a constant hole diameter (0.25 inch.) was placed at the following offset distances: 0.25, 0.5, 1, 2, 5, 10, 20, 30, 35, 38, 39, 39.5, and 39.75 inches.

The additional semi-infinite plate correction became insignificant for hole offset values greater than 10 inches. The SIF error was less than 3 percent based on the FEM data. This level of error is within the typical scatter observed for this type of modeling. The semi-infinite plate correction is shown in Figure 23 as a function of hole offset and normalized crack length (C/Cmax). The correction is well behaved, and a polynomial curve fit is also shown to further demonstrate how quickly this correction becomes insignificant as the hole offset increases. Data for all of the FEM analyses are given in Appendix G1.

[13] These corrections were developed for the through crack approaching a hole, but the correction developed for the crack tip approaching a hole is also used for the edge crack case.

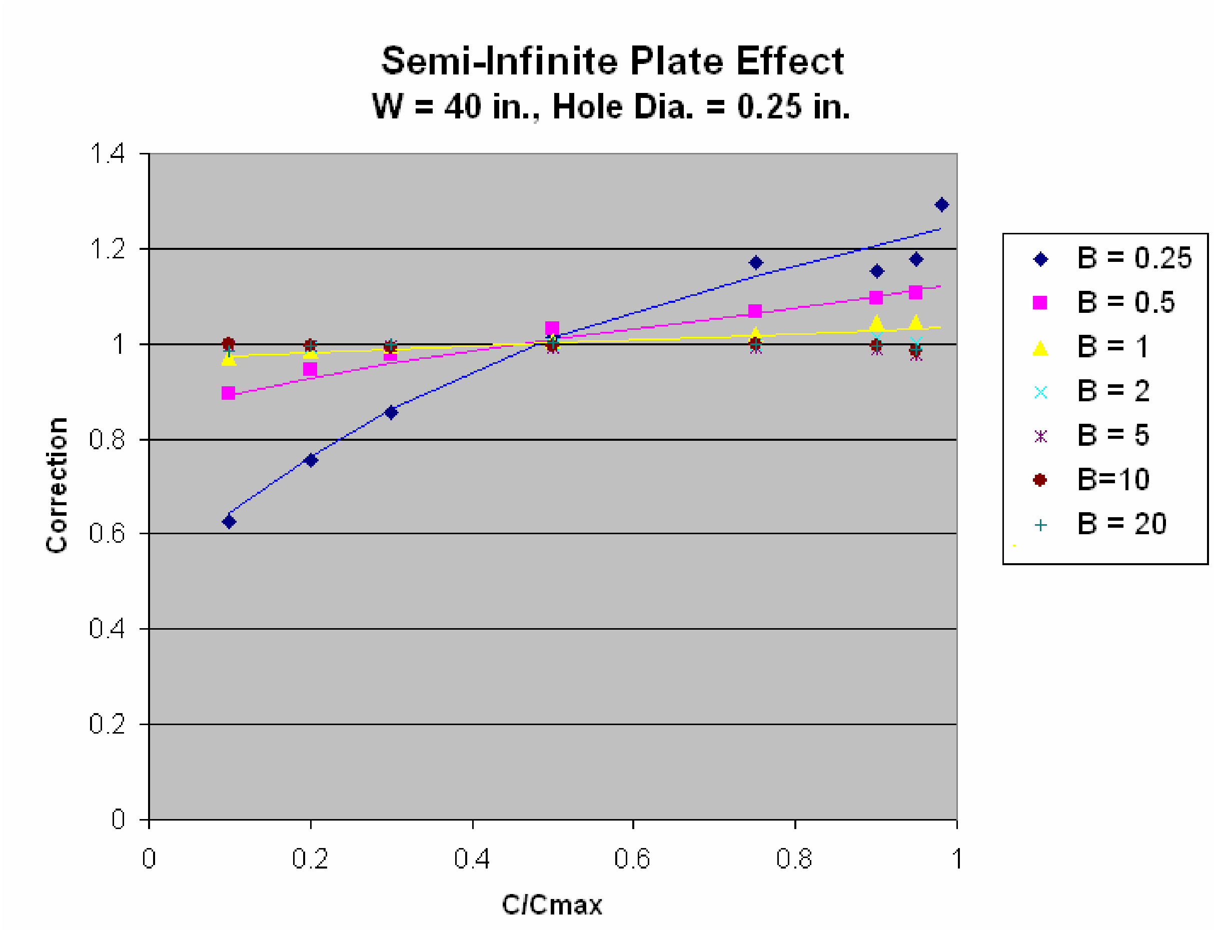


Figure 23: Semi-Infinite Plate Correction for an Edge Crack Growing Toward a Hole

2.7.5.1 Parameters to Determine the Semi-Infinite Plate Effect

The FEM data in Figure 23 shows a strong relationship between hole offset and the semi-infinite plate correction. The hole diameter for each case was 0.25 inch. Since any correction developed as part of this effort must be valid for any hole size and offset, the first parameter considered was the nondimensional hole diameter to offset distance ratio (D/B). It is intuitive that this was the logical parameter that would have a major affect on the correction. It was also clear that some combination of plate width and hole offset would also be a player in the development of a curve fitting parameter for the semi-infinite plate correction. During the process of manually curve fitting the data shown in Figure 23, a polynomial equation as a function of C/Cmax was seen as the obvious choice. As a result, the parameter of choice was the projected value of the semi-infinite plate correction at C/Cmax = 0. After several trials, the parameter was determined as follows:

$$P_0 = \left[\frac{SIN\left(\left(\frac{D}{B} \right) \left(\frac{(W - 2B)}{W} \right) \right)}{\left(\frac{D}{B} \right) \left(\frac{(W - 2B)}{W} \right)} \right]^{4.1} .$$

The use of this parameter raises a few issues that merit further discussion. First, the use of the normalized hole diameter to offset ratio was assumed based on experience gained over the course of the effort to characterized the SIF for a crack approaching a hole. This assumption was also verified for a few cases that will be discussed in Section 2.7.6. The other issue is the use of the plate width when this is a semi-infinite plate correction. The semi-infinite plate correction accounts for the free edge and makes it difficult to remove the width from the equation. It was also hoped that a single equation could be used for any plate width. The latter was not the case, and details of the finite plate correction are given in Section 2.7.6.

The polynomial equation used to determine the semi-infinite plate effect is given below:

$$P_{correction} = P_0 + 3.1(1-P_0)\left(\frac{C}{C_{\max}}\right) - 2.6(1-P_0)\left(\frac{C}{C_{\max}}\right)^2 + (1-P_0)(P_0+0.5)\left(\frac{C}{C_{\max}}\right)^3 .$$

As shown in Figure 23, the equation used for the semi-infinite plate correction does a good job of fitting the data obtained using FE analysis. The largest error was approximately 5 percent for a case with a very small offset relative to the hole diameter (B/D = 1). In practice, hole offset values are rarely less than 2.0, so the error for most practical cases will be negligible.

2.7.6 Correction for a Finite Width Plate

The correction used for the semi-infinite plate are applied as an additional correction to the solution developed for a through crack tip approaching a hole in an infinite plate. It was hoped that this correction would be applicable for all plate widths for the edge crack growing toward a hole. Unfortunately, the FEM results for relatively narrow plates were extremely complex. It was discovered that it was impossible to use the correction for the semi-infinite plate and impractical to develop new corrections for the hole effect developed for the through crack case. The effect of hole offset was very complex for narrow plate widths. It was decided that a set of correction tables would be required, and these corrections would be applied directly to the edge crack solution in AFGROW instead of being additional corrections to the solution developed for a through crack tip approaching a hole in an infinite plate.

A large number of FEM runs were performed for an edge crack growing toward a 0.25-inch diameter hole for plate widths ranging from 0.5 to 4 inches. It was noted that the SIF results for the 4-inch plate were generally within 5 percent of the results obtained using the semi-infinite plate correction. However, the error diverged quickly as the plate width decreased. Some FEM runs were performed using a 0.5-inch diameter hole to verify that the results were equivalent to the results for the same normalized crack length (C/Cmax), normalized edge distance (B/W), and plate width to hole diameter ratio (W/D). These data are provided in Appendix G3.

The FEM results for the narrow plates clearly showed that as the plate width to hole diameter ratio (W/D) increased, the SIF results converged to the results obtained using the solution developed for a through crack tip approaching a hole in an infinite plate along with the semi-infinite plate correction given in Section 2.8.5. As noted earlier, the error observed for the 4-inch-wide plate with a 0.25-inch diameter hole (W/D = 16) was within 5 percent of the solution obtained using the semi-infinite plate correction for all normalized edge distances (B/W) and crack lengths (C/Cmax). As the plate width increases, it is necessary to model an increasing number of B/W values to allow for accurate interpolation of the finite width correction. Since the solutions for W/D = 16 was within 5 percent for all cases and was clearly converging as W/D increased from 1.5 to 16, the existing semi-infinite plate correction was used to calculate a finite plate correction for W/D = 32[14]. This was used as the lower limit for the semi-infinite plate correction given in Section 2.8.5. For edge crack cases in which W/D < 32, a tabular lookup and B-spline double interpolation is used to determine the appropriate finite plate correction (which is applied directly to the standard edge crack solution without a hole). Tabular solutions were developed for the following W/D values: 32, 16, 8, 4, 2, and 1.5. The correction values for C/Cmax = 0.0 were determined using standard curve fitting methods based on data obtained for the low C/Cmax values. These tables are given in Appendix G2.

As stated earlier, these corrections for an edge crack growing toward a hole are applied to the standard edge cracked plate solution without a hole.

One of the complications encountered with these solutions was the fact that the correction for C/Cmax = 0 was highest when the hole was in the center of the plate. This was not expected since it seemed logical that the correction would increase as the hole was moved closer to the cracked edge. However, the reason the correction decreases is simply because the stresses in the plate will tend to shift to the side of the plate with more solid material (stiffer side). This effect is seen for all plate widths, but as the plate becomes wider, the effect becomes more gradual until the hole is very close to the crack origin. It is actually the consequence of the geometry of the edge crack and is also evident in the semi-infinite plate correction.

2.7.7 Edge Crack Growing Toward a Hole Modeling Summary

A general Mode-I SIF solution to the case of an edge crack growing toward a hole in a semi-infinite plate was obtained using existing solutions for the through crack tip growing toward hole with a center distance (d) equal to the hole offset (B) as shown in Section 2.7.5. This value is then corrected for the effect of the semi-infinite plate using the correction given in Section 2.8.5. As the plate width decreases, the problem becomes more complex, and tabulated edge crack correction tables are applied to the existing solution for an edge crack in a plate without a hole. The crack tip SIF values depend on the proximity of the crack tip to the hole, the hole offset, and the influence of the plate width. The parameters that define the influence were obtained from FE modeling of a

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[14] The resulting table was converted to a correction that is applied to an edge crack without a hole. Therefore, these corrections include a hole effect, unlike the semi-infinite plate correction explained in Section 2.8.5.

large number of geometric combinations (plate width, hole and crack locations, and crack lengths). The FE results used to develop the edge crack corrections are given in Appendix G.

3.0 Summary and Conclusions

SIF solutions for multiple through cracks have been published for a few geometric configurations over the years [1-3]. The problem with these solutions is that they are generally applicable for infinite plates and/or cases in which a geometric pattern is repeating. These solutions have been used for practical cases, but it is very difficult to assess the error involved in this type of over simplification. This report attempts to provide a building block approach to the development of a series of solutions for a few typical structural geometries with two, independent through cracks.

3.1 Solution Accuracy

For the majority of cases in this report, the difference between the FEM results and the resulting curve fit solutions is less than 1 percent. However, there are a few cases in which the error was approximately 10 percent. The cases with the highest error are cases in which the crack is very close to a free boundary where the SIF is increasing very rapidly. Also, in cases involving a hole, a normalized hole dimension (R/d, D/B, etc.) is used to make the solution applicable for any hole diameter. The differences between FEM analyses for various hole diameters with the same normalized dimensions were typically within 3 percent of each other. Since it isn't possible to analyze all possible combinations of geometric possibilities, it was necessary to make the assumption that the solutions would be applicable for all practical hole diameters. For this reason, 3 percent differences between the FEM results and the resulting curve fits were considered to be a reasonable target for this effort.

The accuracy of the FEM results is also a real concern. The StressCheck software has been well documented and provides information on the numerical accuracy of a given analysis. However, there are many other sources of error involved in this type of analysis. There are many types of modeling errors to be considered. Great effort was expended in an attempt to minimize these errors, and many cases were double and sometimes triple checked by StressCheck experts. In addition, known solutions were also modeled whenever possible using the same mesh design as the unknown cases. For example, the FE mesh used to for two unequal cracks at a hole was used to model the known solution for two equal length cracks at a hole. This provided more confidence in the validity and accuracy of the FEMs.

3.2 Lessons Learned

At the beginning of this effort, it was thought that as many as three, independent through cracks could be modeled using this building block approach. As time progressed, it became clear that this approach was barely adequate to address two, independent through cracks. There are so many combinations of geometric possibilities and many of the controlling parameters were very difficult to discern. In several cases, parameters were chosen based on limited information or anecdotal data. One could easily argue for the use of other parameters to fit the FEM data. It is felt that the parameters used to develop the solutions in this report capture the major geometric effects for the cases under consideration.

This effort required a great deal more time that was originally estimated. The solutions were not only complex, but were also difficult to implement. In some cases, inconsistencies between models became apparent after implementation which forced the development of new and/or improved solutions. As a result, these solutions are a significant improvement in the ability to more accurately predict the crack growth lives for complex structures.

3.3 Future Work in this Area

The major lesson learned during this effort was that more innovative techniques will be required to perform life analyses for more complex structures or cases with more than two independent through cracks.

SIF solutions for many cases can be stored in tables for use by a life prediction program. As the possible number of geometric combinations increase, the size of the table can become very large. This also requires all possible solutions to be calculated in advance – unless the required solutions are known in advance. This is only possible for relatively simple cases in which the cracking options can be predetermined.

As computing power increases, the option to determine SIF values during a life prediction becomes a realistic possibility. Until recently, it was not practical to consider this option because of the time required to calculate the SIF for a given crack geometry. As advances in computing technology reduce this time, a life prediction program can send geometry and crack length information to another program (i.e., FEM code) to obtain the current SIF value(s) as required during a life analysis. This has tremendous potential to permit extremely accurate life predictions for very complex cases.

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Appendix A
Two Through Cracks in a Plate

A1. Cases

Case 1: Beta Values for W: 40.0 in., D: 8.0 in. and B: 16.0 in.

C1	C2	C1/C2	C1+C2/D	FE-K1	FE-K2	FE-K3	FE-K4
0.06	1.2	0.05	0.1575	0.4408	0.4408	1.946	1.951
0.11	2.2	0.05	0.28875	0.616	0.6168	2.657	2.659
0.15	3	0.05	0.39375	0.7479	0.7505	3.124	3.132
0.2	4	0.05	0.525	0.9322	0.9403	3.654	3.673
0.25	5	0.05	0.65625	1.165	1.188	4.156	4.198
0.3	6	0.05	0.7875	1.515	1.592	4.655	4.729
0.31	6.2	0.05	0.81375	1.611	1.721	4.76	4.836
0.32	6.4	0.05	0.84	1.725	1.866	4.866	4.933
0.343	6.857	0.05	0.9	2.072	2.361	5.169	5.213
0.12	1.2	0.1	0.165	0.6234	0.6237	1.946	1.951
0.22	2.2	0.1	0.3025	0.8701	0.8723	2.659	2.661
0.3	3	0.1	0.4125	1.057	1.064	3.129	3.135
0.4	4	0.1	0.55	1.311	1.334	3.664	3.677
0.5	5	0.1	0.6875	1.636	1.705	4.186	4.205
0.55	5.5	0.1	0.75625	1.846	1.97	4.455	4.467
0.6	6	0.1	0.825	2.115	2.345	4.756	4.748
0.65	6.5	0.1	0.89375	2.49	2.977	5.161	5.035
0.15	1.05	0.1428	0.15	0.6942	0.6947	1.823	1.823
0.3	2.1	0.1428	0.3	1.009	1.014	2.598	2.599
0.4	2.8	0.1428	0.4	1.203	1.212	3.016	3.02
0.5	3.5	0.1428	0.5	1.409	1.431	3.412	3.415
0.6	4.2	0.1428	0.6	1.634	1.683	3.786	3.791
0.7	4.9	0.1428	0.7	1.9	1.999	4.168	4.164
0.8	5.6	0.1428	0.8	2.249	2.5	4.599	4.539
0.9	6.3	0.1428	0.9	2.764	3.446	5.252	4.948
0.25	1.25	0.2	0.1875	0.9002	0.9011	1.993	1.99
0.375	1.875	0.2	0.28125	1.119	1.123	2.452	2.452
0.5	2.5	0.2	0.375	1.328	1.337	2.847	2.846
0.6	3	0.2	0.45	1.493	1.512	3.142	3.14
0.7	3.5	0.2	0.525	1.664	1.699	3.427	3.421
0.8	4	0.2	0.6	1.847	1.914	3.705	3.692
1	5	0.2	0.75	2.297	2.51	4.309	4.231
1.1	5.5	0.2	0.825	2.584	2.977	4.675	4.508
1.2	6	0.2	0.9	2.973	3.808	5.289	4.824
0.25	1	0.25	0.15625	0.8957	0.8963	1.779	1.778
0.5	2	0.25	0.3125	1.301	1.306	2.536	2.536
0.7	2.8	0.25	0.4375	1.592	1.611	3.037	3.03
0.9	3.6	0.25	0.5625	1.899	1.958	3.503	3.482
1	4	0.25	0.625	2.065	2.16	3.738	3.703
1.2	4.8	0.25	0.75	2.449	2.697	4.259	4.146
1.3	5.2	0.25	0.8125	2.686	3.095	4.584	4.38

C1	C2	C1/C2	C1+C2/D	FE-K1	FE-K2	FE-K3	FE-K4
1.4	5.6	0.25	0.875	2.974	3.697	5.044	4.618
0.3	0.9	0.3333	0.15	0.979	0.9796	1.689	1.688
0.6	1.8	0.3333	0.3	1.415	1.42	2.406	2.404
0.9	2.7	0.3333	0.45	1.797	1.826	2.991	2.979
1.2	3.6	0.3333	0.6	2.191	2.286	3.549	3.503
1.5	4.5	0.3333	0.75	2.646	2.934	4.184	4.017
1.6	4.8	0.3333	0.8	2.829	3.257	4.455	4.198
1.8	5.4	0.3333	0.9	3.29	4.341	5.325	4.581
0.5	1	0.5	0.1875	1.264	1.267	1.783	1.782
1	2	0.5	0.375	1.84	1.856	2.561	2.551
1.5	3	0.5	0.5625	2.358	2.439	3.243	3.189
1.8	3.6	0.5	0.675	2.687	2.883	3.689	3.559
2	4	0.5	0.75	2.926	3.259	4.052	3.809
2.2	4.4	0.5	0.825	3.199	3.843	4.52	4.07
2.4	4.8	0.5	0.9	3.534	4.772	5.321	4.367
0.6	0.9	0.6666	0.1875	1.383	1.386	1.694	1.692
1.2	1.8	0.6666	0.375	2.003	2.02	2.438	2.424
1.6	2.4	0.6666	0.5	2.364	2.417	2.882	2.839
1.8	2.7	0.6666	0.5625	2.548	2.635	3.107	3.04
2	3	0.6666	0.625	2.73	2.874	3.349	3.237
2.4	3.6	0.6666	0.75	3.124	3.477	3.928	3.644
2.8	4.2	0.6666	0.875	3.594	4.61	4.923	4.094
0.7	0.7	1	0.175	1.492	1.492	1.492	1.491
1.8	1.8	1	0.45	2.461	2.494	2.494	2.46
2.2	2.2	1	0.55	2.765	2.842	2.841	2.765
2.5	2.5	1	0.625	2.988	3.129	3.127	2.996
3	3	1	0.75	3.385	3.711	3.718	3.385
3.5	3.5	1	0.875	3.845	4.807	4.77	3.845
3.6	3.6	1	0.9	3.957	5.145	5.143	3.956
0.9	0.6	1.5	0.1875	1.692	1.694	1.386	1.383
1.8	1.2	1.5	0.375	2.425	2.437	2.02	2.003
2.4	1.6	1.5	0.5	2.841	2.884	2.419	2.366
2.7	1.8	1.5	0.5625	3.036	3.101	2.633	2.546
3	2	1.5	0.625	3.236	3.341	2.871	2.732
3.6	2.4	1.5	0.75	3.643	3.927	3.468	3.125
4.2	2.8	1.5	0.875	4.096	4.926	4.539	3.592
1	0.5	2	0.1875	1.782	1.783	1.267	1.264
2	1	2	0.375	2.551	2.562	1.856	1.839
3	1.5	2	0.5625	3.19	3.246	2.441	2.358
3.6	1.8	2	0.675	3.558	3.698	2.879	2.687
4	2	2	0.75	3.802	4.019	3.264	2.926
4.4	2.2	2	0.825	4.069	4.521	3.788	3.203
4.8	2.4	2	0.9	4.368	5.323	4.702	3.534
0.9	0.3	3	0.15	1.688	1.689	0.9796	0.979
1.8	0.6	3	0.3	2.404	2.406	1.421	1.415
2.7	0.9	3	0.45	2.978	2.99	1.824	1.798

C1	C2	C1/C2	C1+C2/D	FE-K1	FE-K2	FE-K3	FE-K4
3.6	1.2	3	0.6	3.504	3.55	2.286	2.191
4.5	1.5	3	0.75	4.019	4.202	2.936	2.645
4.8	1.6	3	0.8	4.195	4.46	3.255	2.829
5.4	1.8	3	0.9	4.587	5.333	4.339	3.289
1	0.25	4	0.15625	1.778	1.779	0.8963	0.8957
2	0.5	4	0.3125	2.536	2.537	1.306	1.301
2.8	0.7	4	0.4375	3.03	3.037	1.611	1.592
3.6	0.9	4	0.5625	3.482	3.503	1.958	1.898
4	1	4	0.625	3.703	3.738	2.16	2.065
4.8	1.2	4	0.75	4.15	4.286	2.696	2.45
5.2	1.3	4	0.8125	4.38	4.585	3.094	2.687
5.6	1.4	4	0.875	4.618	5.044	3.695	2.971
1.25	0.25	5	0.1875	1.99	1.993	0.9011	0.9002
1.875	0.375	5	0.28125	2.452	2.452	1.123	1.119
2.5	0.5	5	0.375	2.846	2.848	1.337	1.327
3	0.6	5	0.45	3.14	3.142	1.511	1.492
3.5	0.7	5	0.525	3.421	3.427	1.7	1.663
4	0.8	5	0.6	3.692	3.707	1.914	1.847
5	1	5	0.75	4.231	4.309	2.508	2.294
5.5	1.1	5	0.825	4.508	4.675	2.977	2.584
6	1.2	5	0.9	4.824	5.29	3.817	2.975
1.05	0.15	7	0.15	1.823	1.823	0.6947	0.6942
2.1	0.3	7	0.3	2.599	2.598	1.014	1.009
2.8	0.4	7	0.4	3.019	3.017	1.214	1.205
3.5	0.5	7	0.5	3.416	3.411	1.43	1.409
4.2	0.6	7	0.6	3.791	3.787	1.684	1.635
4.9	0.7	7	0.7	4.163	4.168	2.011	1.903
5.6	0.8	7	0.8	4.539	4.597	2.501	2.252
6.3	0.9	7	0.9	4.951	5.251	3.445	2.766
1.2	0.12	10	0.165	1.951	1.946	0.6237	0.6234
2.2	0.22	10	0.3025	2.661	2.659	0.8723	0.8701
3	0.3	10	0.4125	3.135	3.129	1.063	1.057
4	0.4	10	0.55	3.677	3.665	1.337	1.313
5	0.5	10	0.6875	4.205	4.195	1.705	1.634
5.5	0.55	10	0.75625	4.467	4.455	1.966	1.843
6	0.6	10	0.825	4.748	4.756	2.346	2.115
6.5	0.65	10	0.89375	5.033	5.166	2.975	2.489
1.2	0.06	20	0.1575	1.951	1.946	0.4408	0.4408
2.2	0.11	20	0.28875	2.659	2.657	0.6168	0.616
3	0.15	20	0.39375	3.133	3.124	0.7503	0.7481
4	0.2	20	0.525	3.673	3.654	0.9398	0.9319
5	0.25	20	0.65625	4.198	4.156	1.189	1.165
6	0.3	20	0.7875	4.73	4.655	1.592	1.514
6.2	0.31	20	0.81375	4.836	4.76	1.712	1.612
6.4	0.32	20	0.84	4.933	4.866	1.866	1.725
6.857	0.343	20	0.9	5.211	5.169	2.35	2.073

C1	C2	C1/C2	C1+C2/D	FE-K1	FE-K2	FE-K3	FE-K4
0.025	1.25	0.02	0.15937	0.2861	0.2861	1.99	1.99
0.04	2	0.02	0.255	0.3693	0.3695	2.529	2.53
0.05	2.5	0.02	0.31875	0.4209	0.4214	2.841	2.845
0.065	3.25	0.02	0.41437	0.502	0.5028	3.261	3.272
0.08	4	0.02	0.51	0.5905	0.5926	3.654	3.677
0.09	4.5	0.02	0.57375	0.6602	0.6635	3.904	3.937
0.1	5	0.02	0.6375	0.7404	0.7463	4.148	4.194
0.11	5.5	0.02	0.70125	0.84	0.8503	4.39	4.46
0.12	6	0.02	0.765	0.9682	0.9875	4.628	4.725
0.13	6.5	0.02	0.82875	1.152	1.19	4.874	4.997
0.136	6.8	0.02	0.867	1.305	1.365	5.026	5.162
0.14	7	0.02	0.8925	1.4706	1.561	5.124	5.27
1.25	0.025	50	0.15937	1.99	1.99	0.2861	0.2861
2	0.04	50	0.255	2.53	2.529	0.3695	0.3693
2.5	0.05	50	0.31875	2.845	2.841	0.4214	0.4209
3.25	0.065	50	0.41437	3.272	3.261	0.5028	0.502
4	0.08	50	0.51	3.672	3.654	0.5924	0.5913
4.5	0.09	50	0.57375	3.937	3.904	0.6633	0.6604
5	0.1	50	0.6375	4.196	4.148	0.7473	0.7418
5.5	0.11	50	0.70125	4.458	4.39	0.8503	0.84
6	0.12	50	0.765	4.725	4.628	0.9875	0.9682
6.5	0.13	50	0.82875	4.998	4.874	1.188	1.149
6.8	0.136	50	0.867	5.163	5.026	1.367	1.304
7	0.14	50	0.8925	5.27	5.124	1.561	1.4706

Case 2: W: 40.0 in., D: 12.0 in. and B: 14.0 in

C1	C2	(C1+C2)/D	FEA-K1	FEA-K2	FEA-K3	FEA-K4
0.3	6	0.525	1.161	1.173	4.692	4.861
0.31	6.2	0.5425	1.198	1.211	4.788	4.98
0.343	6.857	0.6	1.329	1.352	5.131	5.414
6	0.3	0.525	4.862	4.691	1.171	1.16
6.2	0.31	0.5425	4.981	4.787	1.211	1.198
6.857	0.343	0.6	5.415	5.13	1.351	1.328
2.5	2.5	0.4167	2.925	2.952	2.953	2.924
3	3	0.5	3.263	3.321	3.322	3.262
3.6	3.6	0.6	3.671	3.8	3.797	3.671

Case 3: W: 40.0 in., D: 16.0 in. and B: 12.0 in.

C1	C2	(C1+C2)/D	FEA-K1	FEA-K2	FEA-K3	FEA-K4
0.3	6	0.3938	1.066	1.071	4.804	5.085
0.31	6.2	0.4069	1.088	1.095	4.914	5.235
0.343	6.857	0.45	1.177	1.187	5.288	5.774
6	0.3	0.3938	5.086	4.803	1.071	1.066
6.2	0.31	0.4069	5.236	4.913	1.095	1.088
6.857	0.343	0.45	5.774	5.288	1.187	1.178
2.5	2.5	0.3125	2.904	2.912	2.913	2.904
3	3	0.375	3.227	3.243	3.243	3.227
3.6	3.6	0.45	3.621	3.655	3.656	3.622

Case 4: W: 40.0 in., D: 20.0 in. and B: 10.0 in.

C1	C2	(C1+C2)/D	FEA-K1	FEA-K2	FEA-K3	FEA-K4
0.3	6	0.315	1.002	1.007	5.001	5.497
0.31	6.2	0.3255	1.023	1.028	5.133	5.708
0.343	6.857	0.36	1.087	1.095	5.605	6.511
6	0.3	0.315	5.498	5.001	1.008	1.004
6.2	0.31	0.3255	5.712	5.138	1.028	1.023
6.857	0.343	0.36	6.513	5.608	1.093	1.085
2.5	2.5	0.25	2.906	2.903	2.904	2.905
3	3	0.3	3.233	3.228	3.229	3.233
3.6	3.6	0.36	3.622	3.614	3.615	3.622

Case 5: W: 40.0 in., D: 24.0 in. and B: 8.0 in.

C1	C2	(C1+C2)/D	FEA-K1	FEA-K2	FEA-K3	FEA-K4
0.3	6	0.2625	0.9382	0.9441	5.4	6.501
0.31	6.2	0.2713	0.9476	0.9538	5.592	6.9
0.343	6.857	0.3	0.9781	0.9879	6.314	8.711
6	0.3	0.2625	6.499	5.404	0.9435	0.9388
6.2	0.31	0.2713	6.898	5.593	0.9538	0.9476
6.857	0.343	0.3	8.713	6.311	0.988	0.9785
2.5	2.5	0.2083	2.918	2.903	2.903	2.918
3	3	0.25	3.264	3.234	3.234	3.263
3.6	3.6	0.3	3.687	3.628	3.628	3.687

Case 6: W: 40.0 in., D: 12.0 in. and B: 4.0 in.

C1	C2	(C1+C2)/D	FEA-K1	FEA-K2	FEA-K3	FEA-K4
0.1	2	0.175	0.576	0.5763	2.533	2.531
0.15	3	0.2625	0.7269	0.7279	3.134	3.124
0.18	3.6	0.315	0.8145	0.8172	3.465	3.448
2	0.1	0.175	2.786	2.695	0.577	0.5768
3	0.15	0.2625	4.165	3.615	0.7344	0.7333
3.6	0.18	0.315	5.882	4.328	0.8373	0.8358
2.5	2.5	0.4167	3.494	3.28	3.001	2.94
3	3	0.5	4.359	3.876	3.433	3.302
3.6	3.6	0.6	6.331	4.849	4.089	3.774

Case 7: W: 40.0 in., D: 16.0 in. and B: 4.0 in.

C1	C2	(C1+C2)/D	FEA-K1	FEA-K2	FEA-K3	FEA-K4
0.1	2	0.1313	0.5691	0.5693	2.528	2.528
0.15	3	0.1969	0.7072	0.7078	3.119	3.118
0.18	3.6	0.2363	0.7821	0.7845	3.439	3.439
2	0.1	0.1313	2.793	2.694	0.5696	0.5694
3	0.15	0.1969	4.165	3.615	0.7107	0.71
3.6	0.18	0.2363	5.882	4.328	0.794	0.7938
2.5	2.5	0.3125	3.428	3.204	2.91	2.879
3	3	0.375	4.244	3.733	3.259	3.195
3.6	3.6	0.45	6.074	4.556	3.717	3.576

Case 8: W: 40.0 in., D: 20.0 in. and B: 10.0 in.

C1	C2	(C1+C2)/D	FEA-K1	FEA-K2	FEA-K3	FEA-K4
0.15	0.1	0.0125	0.6878	0.6883	0.5622	0.5623
0.20	0.15	0.0175	0.7943	0.7945	0.6878	0.6883
0.25	0.2	0.0225	0.889	0.8895	0.7944	0.7945
0.30	0.25	0.0275	0.9725	0.9732	0.889	0.8896
0.35	0.3	0.0325	1.046	1.048	0.9725	0.9732
0.40	0.35	0.0375	1.123	1.123	1.046	1.048
0.45	0.4	0.0425	1.192	1.192	1.123	1.123
0.50	0.45	0.0475	1.258	1.258	1.192	1.192
0.55	0.5	0.0525	1.318	1.318	1.258	1.258
0.60	0.55	0.0575	1.377	1.377	1.318	1.318
0.65	0.6	0.0625	1.433	1.433	1.377	1.377
0.70	0.65	0.0675	1.481	1.483	1.433	1.433
0.75	0.7	0.0725	1.537	1.539	1.481	1.484
0.80	0.75	0.0775	1.591	1.591	1.537	1.539
0.85	0.8	0.0825	1.641	1.641	1.591	1.591
0.90	0.85	0.0875	1.689	1.689	1.641	1.641

Case 9: W: 40.0 in., D: 2.0 in. and B: 19.0 in

C1	C2	(C1+C2)/D	FEA-K1	FEA-K2	FEA-K3	FEA-K4
0.15	0.1	0.125	0.6879	0.689	0.5633	0.5631
0.2	0.15	0.175	0.7956	0.7962	0.691	0.6902
0.25	0.2	0.225	0.8917	0.8929	0.8009	0.7998
0.3	0.25	0.275	0.9789	0.9816	0.8948	0.8932
0.35	0.3	0.325	1.06	1.064	0.9878	0.9844
0.4	0.35	0.375	1.138	1.146	1.077	1.069
0.45	0.4	0.425	1.212	1.225	1.161	1.147
0.5	0.45	0.475	1.283	1.303	1.242	1.218
0.55	0.5	0.525	1.349	1.38	1.327	1.293
0.6	0.55	0.575	1.421	1.467	1.414	1.367

Case 10: W: 40.0 in., D: 8.0 in. and B: 8.0 in.

C1	C2	(C1+C2)/D	FEA-K1	FEA-K2	FEA-K3	FEA-K4
0.3	6	0.7875	1.581	1.657	4.76	4.628
0.31	6.2	0.8138	1.684	1.79	4.879	4.724
0.343	6.857	0.9	2.182	2.481	5.35	5.045
6	0.3	0.7875	6.511	5.442	1.79	1.694
6.2	0.31	0.8138	6.91	5.645	1.954	1.828
6.857	0.343	0.9	8.717	6.492	2.863	2.495
2.5	2.5	0.625	3.133	3.233	3.178	3.015
3	3	0.75	3.639	3.907	3.828	3.432
3.6	3.6	0.9	4.437	5.52	5.432	4.046

Case 11: W: 24.0 in., D: 12.0 in. and B: 6.0 in.

C1	C2	(C1+C2)/D	FEA-K1	FEA-K2	FEA-K3	FEA-K4
0.15	3	0.2625	0.7015	0.7039	3.39	3.572
0.2	4	0.35	0.8263	0.832	4.235	4.849
0.25	5	0.4375	0.9511	0.9664	5.387	7.384
3	0.15	0.2625	3.569	3.389	0.7024	0.7006
4	0.2	0.35	4.85	4.236	0.8318	0.8266
5	0.25	0.4375	7.385	5.387	0.9662	0.9512
2.5	2.5	0.4167	3.103	3.094	3.093	3.102
3	3	0.5	3.578	3.559	3.559	3.578
3.6	3.6	0.6	4.259	4.225	4.226	4.258

Case 12: W: 24.0 in., D: 16.0 in. and B: 4.0 in.

C1	C2	(C1+C2)/D	FEA-K1	FEA-K2	FEA-K3	FEA-K4
0.1	2	0.1313	0.5504	0.5505	2.738	2.855
0.15	3	0.1969	0.6484	0.6513	3.751	4.461
0.18	3.6	0.2363	0.673	0.6772	4.682	6.9
2	0.1	0.1313	2.856	2.738	0.5503	0.5505
3	0.15	0.1969	4.473	3.758	0.6518	0.6487
3.6	0.18	0.2363	6.902	4.682	0.6776	0.6729
2.5	2.5	0.3125	3.349	3.152	3.153	3.342
3	3	0.375	4.11	3.65	3.641	4.108
3.6	3.6	0.45	5.911	4.43	4.43	5.909

Case 13: W: 24.0 in., D: 8.0 in. and B: 8.0 in.

C1	C2	(C1+C2)/D	FEA-K1	FEA-K2	FEA-K3	FEA-K4
0.07	3.5	0.4463	0.5379	0.539	3.587	3.731
0.08	4	0.51	0.6034	0.6065	3.926	4.164
0.1	5	0.6375	0.7779	0.7863	4.64	5.254
0.12	6	0.765	1.07	1.096	5.51	6.967
0.13	6.5	0.8288	1.327	1.41	6.051	8.362
0.15	3	0.3938	0.7554	0.7579	3.259	3.334
0.2	4	0.525	0.9524	0.9618	3.929	4.165
0.25	5	0.6563	1.221	1.253	4.65	5.257
0.3	6	0.7875	1.775	1.673	5.542	6.974
0.31	6.2	0.8138	1.94	1.805	5.758	7.466
0.343	6.857	0.9	2.876	2.492	6.675	9.857
0.3	3	0.4125	1.067	1.073	3.262	3.336
0.4	4	0.55	1.343	1.372	3.943	4.17
0.5	5	0.6875	1.712	1.803	4.69	5.27
0.55	5.5	0.7563	1.973	2.136	5.13	6.014
0.6	6	0.825	2.326	2.632	5.664	6.999
0.65	6.5	0.8938	2.859	3.519	6.414	8.433
3.5	0.07	0.4463	3.73	3.588	0.539	0.5382
4	0.08	0.51	4.165	3.927	0.606	0.6037
5	0.1	0.6375	5.255	4.648	0.7861	0.7779
6	0.12	0.765	6.966	5.51	1.095	1.069
6.5	0.13	0.8288	8.359	6.05	1.374	1.322
3	0.15	0.3938	3.337	3.257	0.7571	0.7534
4	0.2	0.525	4.167	3.928	0.9614	0.9498
5	0.25	0.6563	5.256	4.657	1.254	1.222
6	0.3	0.7875	6.975	5.543	1.776	1.674
6.2	0.31	0.8138	7.467	5.759	1.941	1.807
6.857	0.343	0.9	9.858	6.676	2.878	2.493
3	0.3	0.4125	3.337	3.262	1.075	1.067
4	0.4	0.55	4.171	3.942	1.372	1.343
5	0.5	0.6875	5.267	4.688	1.805	1.71
5.5	0.55	0.7563	6.013	5.131	2.137	1.973
6	0.6	0.825	7.007	5.664	2.635	2.327
6.5	0.65	0.8938	8.428	6.414	3.521	2.863
2.5	2.5	0.625	3.13	3.254	3.256	3.128
3	3	0.75	3.626	3.955	3.954	3.622
3.5	3.5	0.875	4.257	5.226	5.223	4.258
3.6	3.6	0.9	4.416	5.677	5.665	4.419

Case 14: W: 24.0 in., D: 20.0 in. and B: 2.0 in.

C1	C2	(C1+C2)/D	FEA-K1	FEA-K2	FEA-K3	FEA-K4
0.05	1	0.0525	0.3919	0.3917	1.896	1.964
0.06	1.2	0.063	0.4263	0.4262	2.136	2.279
1.5	1.5	0.15	2.794	2.446	2.447	2.795

Case 15: W: 24.0 in., D: 4.0 in. and B: 4.0 in.

C1	C2	(C1+C2)/D	FEA-K1	FEA-K2	FEA-K3	FEA-K4
0.05	1	0.2625	0.4144	0.4151	1.79	1.787
0.1	2	0.525	0.6736	0.6786	2.601	2.581
0.15	3	0.7875	1.116	1.172	3.361	3.26
1	0.05	0.2625	1.824	1.816	0.4154	0.4148
2	0.1	0.525	2.856	2.74	0.6871	0.6806
3	0.15	0.7875	4.481	3.793	1.254	1.189
1	1	0.5	1.897	1.923	1.9	1.858
1.5	1.5	0.75	2.557	2.753	2.701	2.426

Case 16: W: 24.0 in., D: 8.0 in. and B: 2.0 in.

C1	C2	(C1+C2)/D	FEA-K1	FEA-K2	FEA-K3	FEA-K4
0.05	1	0.13125	0.4031	0.4029	1.784	1.784
0.06	1.2	0.1575	0.4441	0.4444	1.958	1.958
0.08	1.6	0.21	0.5203	0.5204	2.277	2.274
1	0.05	0.1313	1.965	1.895	0.403	0.4034
1.2	0.06	0.1575	2.276	2.137	0.4451	0.4447
1.6	0.08	0.21	3.221	2.676	0.5237	0.5239
1	1	0.25	1.989	1.925	1.815	1.807
1.5	1.5	0.375	2.989	2.627	2.305	2.268

Case 17: W: 24.0 in., D: 8.0 in. and B: 4.0 in.

C1	C2	(C1+C2)/D	FEA-K1	FEA-K2	FEA-K3	FEA-K4
0.05	2.5	0.3188	0.4294	0.4296	2.883	2.883
0.06	3	0.3825	0.4869	0.488	3.198	3.198
0.065	3.25	0.4144	0.5178	0.5185	3.348	3.35
0.1	2	0.2625	0.5893	0.5896	2.555	2.555
0.12	2.4	0.315	0.6597	0.6611	2.821	2.821
0.15	3	0.3938	0.7682	0.7712	3.198	3.196
0.16	3.2	0.42	0.8066	0.8096	3.32	3.319
0.2	2	0.275	0.8324	0.8349	2.555	2.555
0.25	2.5	0.3438	0.9577	0.9617	2.886	2.885
0.3	3	0.4125	1.086	1.094	3.204	3.201
0.32	3.2	0.44	1.141	1.15	3.326	3.321
2	0.1	0.2625	2.856	2.739	0.5925	0.5918
2.4	0.12	0.315	3.367	3.118	0.666	0.6645
3	0.15	0.3938	4.465	3.762	0.7868	0.7823
3.2	0.16	0.42	5.021	4.027	0.8369	0.8312
2	0.2	0.275	2.857	2.74	0.8381	0.8358
2.5	0.25	0.3438	3.52	3.218	0.9703	0.9646

3	0.3	0.4125	4.47	3.765	1.118	1.107
3.2	0.32	0.44	5.029	4.034	1.189	1.171
2.5	0.05	0.3188	3.517	3.215	0.4338	0.4333
2.8	0.056	0.357	4.031	3.534	0.4717	0.4708
3	0.06	0.3825	4.464	3.761	0.4986	0.4977
3.25	0.065	0.4144	5.186	4.093	0.5363	0.5348
2.5	2.5	0.625	3.773	3.61	3.329	3.094
3	3	0.75	5.005	4.62	4.184	3.581
3.5	3.5	0.875	7.548	6.613	6.005	4.247
3.6	3.6	0.9	8.526	7.36	6.732	4.436

Case 18: W: 20.0 in., D: 2.0 in. and B: 9.0 in.

C1	C2	(C1+C2)/D	FEA-K1	FEA-K2	FEA-K3	FEA-K4
0.15	0.1	0.125	0.6847	0.6847	0.5626	0.5625
0.2	0.15	0.175	0.7959	0.7964	0.691	0.6904
0.3	0.25	0.275	0.9792	0.982	0.8995	0.8968
0.4	0.35	0.375	1.139	1.147	1.078	1.069
0.5	0.45	0.475	1.287	1.307	1.248	1.228
0.55	0.5	0.525	1.358	1.389	1.334	1.302
0.6	0.55	0.575	1.428	1.472	1.422	1.374

Case 19: W: 16.0 in., D: 4.0 in. and B: 6.0 in.

C1	C2	(C1+C2)/D	FEA-K1	FEA-K2	FEA-K3	FEA-K4
0.05	1	0.2625	0.4122	0.4127	1.793	1.796
0.1	2	0.525	0.6658	0.672	2.634	2.677
0.15	3	0.7875	1.101	1.159	3.435	3.62
1	0.05	0.2625	1.802	1.797	0.4136	0.4133
2	0.1	0.525	2.677	2.636	0.6716	0.6659
3	0.15	0.7875	3.619	3.436	1.155	1.095
1	1	0.5	1.864	1.899	1.898	1.863
1.5	1.5	0.75	2.445	2.68	2.681	2.444

Case 20: W: 16.0 in., D: 8.0 in. and B: 4.0 in.

C1	C2	(C1+C2)/D	FEA-K1	FEA-K2	FEA-K3	FEA-K4
0.1	2	0.2625	0.5719	0.5737	2.766	2.915
0.15	3	0.3938	0.7258	0.7332	3.864	4.768
0.18	3.6	0.4725	0.8197	0.8386	4.95	7.866
2	0.1	0.2625	2.915	2.767	0.5745	0.5736
3	0.15	0.3938	4.77	3.864	0.7329	0.7254
3.6	0.18	0.4725	7.866	4.95	0.8383	0.8197
2.5	2.5	0.625	3.637	3.61	3.606	3.64
3	3	0.75	4.721	4.664	4.667	4.721
3.6	3.6	0.9	7.763	7.837	7.837	7.763

Case 21: W: 16.0 in., D: 12.0 in. and B: 2.0 in.

C1	C2	(C1+C2)/D	FEA-K1	FEA-K2	FEA-K3	FEA-K4
0.05	1	0.0875	0.3894	0.3896	1.916	1.993
0.06	1.2	0.105	0.4214	0.4217	2.171	2.331
0.08	1.6	0.14	0.4686	0.4698	2.765	3.379
1	0.05	0.0875	1.993	1.917	0.3895	0.3895
1.2	0.06	0.105	2.337	2.171	0.4217	0.4214
1.6	0.08	0.14	3.382	2.767	0.4697	0.4688
1	1	0.1666667	1.945	1.885	1.88	1.945
1.5	1.5	0.25	2.832	2.491	2.491	2.833

Case 22: W: 16.0 in., D: 4.0 in. and B: 4.0 in.

C1	C2	(C1+C2)/D	FEA-K1	FEA-K2	FEA-K3	FEA-K4
0.05	1	0.2625	0.414	0.4147	1.795	1.795
0.1	2	0.525	0.67	0.6761	2.613	2.609
0.15	3	0.7875	1.116	1.171	3.393	3.372
1	0.05	0.2625	1.834	1.824	0.4146	0.4143
2	0.1	0.525	2.919	2.771	0.6856	0.6794
3	0.15	0.7875	4.776	3.899	1.27	1.198
1	1	0.5	1.904	1.929	1.904	1.86
1.5	1.5	0.75	2.575	2.77	2.713	2.434

Case 23: W: 8.0 in., D: 2.0 in. and B: 3.0 in.

C1	C2	(C1+C2)/D	FEA-K1	FEA-K2	FEA-K3	FEA-K4
0.15	0.1	0.125	0.6878	0.6879	0.5634	0.5632
0.2	0.15	0.175	0.7978	0.7982	0.6924	0.6918
0.3	0.25	0.275	0.9841	0.9854	0.9024	0.9005
0.4	0.35	0.375	1.151	1.158	1.088	1.081
0.5	0.45	0.475	1.308	1.327	1.267	1.246
0.55	0.5	0.525	1.386	1.415	1.359	1.328
0.6	0.55	0.575	1.463	1.507	1.454	1.406
0.65	0.6	0.625	1.543	1.606	1.557	1.488
0.7	0.65	0.675	1.623	1.716	1.67	1.573
0.75	0.7	0.725	1.707	1.841	1.801	1.66
0.8	0.75	0.775	1.795	1.995	1.956	1.75

Case 24: W: 4.0 in., D: 2.0 in. and B: 1.0 in.

C1	C2	(C1+C2)/D	FEA-K1	FEA-K2	FEA-K3	FEA-K4
0.03	0.6	0.315	0.317	0.3184	1.58	1.741
0.04	0.8	0.42	0.3782	0.3832	2.081	2.717
0.06	0.6	0.33	0.448	0.4521	1.584	1.742
0.08	0.8	0.44	0.5322	0.5462	2.083	2.716
0.08	0.56	0.32	0.5146	0.5198	1.501	1.616
0.13	0.91	0.52	0.6879	0.7368	2.535	4.126
0.56	0.08	0.32	1.621	1.504	0.5199	0.5148
0.91	0.13	0.52	4.127	2.535	0.7365	0.6877
0.6	0.06	0.33	1.742	1.584	0.452	0.4481
0.8	0.08	0.44	2.716	2.083	0.546	0.5319
0.9	0.09	0.495	3.927	2.476	0.6011	0.5743
0.6	0.03	0.315	1.742	1.583	0.3183	0.317
0.8	0.04	0.42	2.717	2.081	0.3829	0.3779
0.6	0.6	0.6	1.739	1.715	1.723	1.738
0.7	0.7	0.7	2.109	2.085	2.088	2.112
0.8	0.8	0.8	2.685	2.66	2.659	2.688
0.9	0.9	0.9	3.872	3.932	3.904	3.873

A2. Beta Interaction Tables for Crack Tips in an Infinite Plate

Table 1: Beta Interaction values for Crack Tips growing towards the Specimen Edge

(C1+C2)/D	C1/C2=1/50	C1/C2=1/20	C1/C2=1/10	C1/C2=1/7	C1/C2=1/5	C1/C2=1/4	C1/C2=1/3	C1/C2=1/2
0	1	1	1	1	1	1	1	1
0.1	1.015	1.013	1.0114	1.01	1.0084	1.0073	1.0058	1.004
0.2	1.032	1.0295	1.026	1.023	1.02	1.017	1.014	1.01
0.3	1.057	1.052	1.046	1.041	1.036	1.0325	1.028	1.021
0.4	1.104	1.093	1.085	1.076	1.069	1.062	1.052	1.041
0.5	1.173	1.156	1.137	1.122	1.107	1.096	1.082	1.063
0.6	1.271	1.245	1.217	1.19	1.163	1.146	1.126	1.093
0.7	1.428	1.387	1.323	1.28	1.24	1.217	1.18	1.132
0.8	1.684	1.586	1.48	1.416	1.35	1.306	1.254	1.187
0.9	2.3	1.995	1.763	1.64	1.526	1.45	1.37	1.27
1	3.8	2.8	2.2	2.01	1.85	1.72	1.58	1.43

Table 1: Continued

(C1+C2)/D	C1/C2=2/3	C1/C2=1	C1/C2=1.5	C1/C2=2	C1/C2=3	C1/C2=4	C1/C2=5	C1/C2>5
0	1	1	1	1	1	1	1	1
0.1	1.0028	1.002	1.0016	1.0012	1.0008	1.0005	1.0002	1.0002
0.2	1.008	1.0058	1.004	1.003	1.002	1.001	1.0005	1.0005
0.3	1.0155	1.011	1.008	1.0056	1.0031	1.0017	1.00085	1.00085
0.4	1.031	1.022	1.014	1.0093	1.0045	1.0025	1.0014	1.0014
0.5	1.048	1.033	1.0205	1.0141	1.007	1.0037	1.0023	1.0023
0.6	1.072	1.046	1.03	1.0208	1.01	1.006	1.00326	1.00326
0.7	1.102	1.066	1.042	1.0284	1.015	1.0092	1.0047	1.0047
0.8	1.146	1.093	1.058	1.0405	1.0227	1.013	1.0075	1.0075
0.9	1.212	1.1416	1.092	1.0649	1.0382	1.025	1.018	1.018
1	1.35	1.25	1.155	1.107	1.074	1.055	1.04	1.04

Table 2: Beta Interaction values for Crack Tips growing towards each other

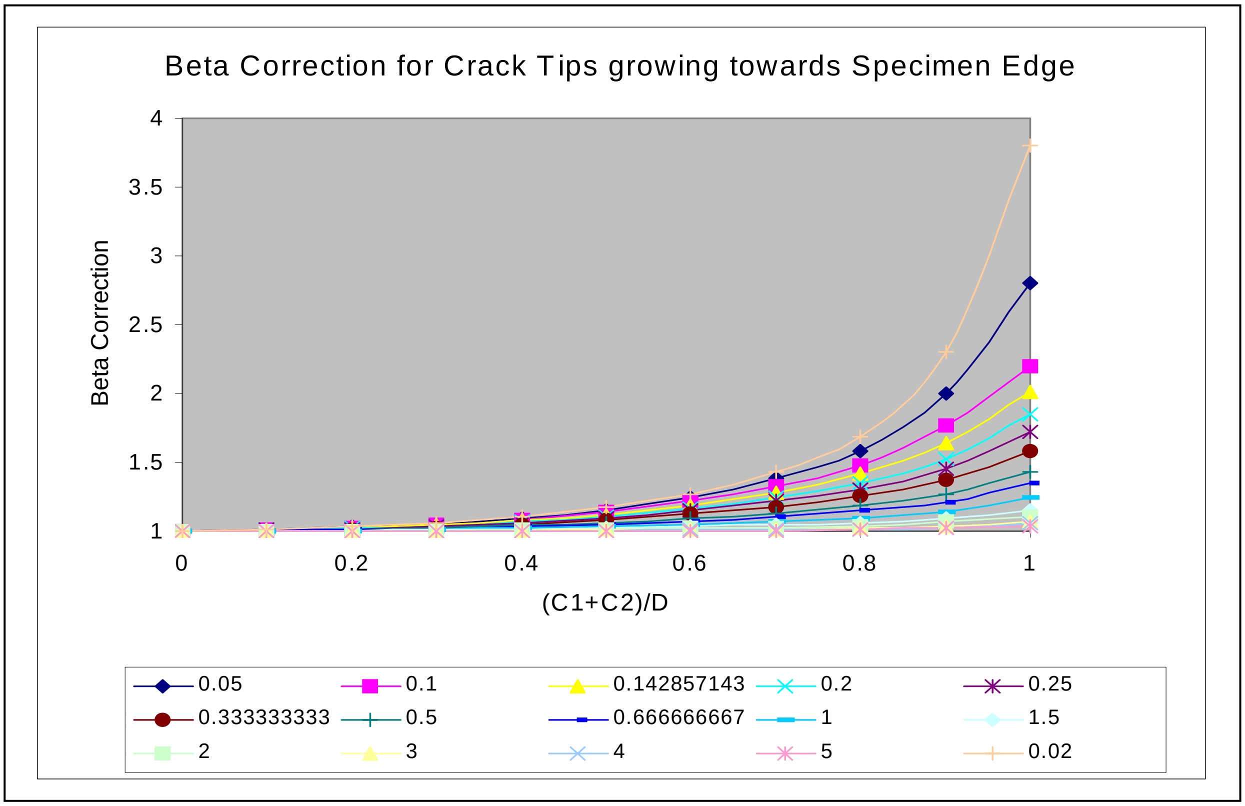
(C1+C2)/D	C1/C2=1/20	C1/C2=1/10	C1/C2=1/7	C1/C2=1/5	C1/C2=1/4	C1/C2=1/3	C1/C2=1/2	C1/C2=2/3
0	1	1	1	1	1	1	1	1
0.1	1.016	1.013	1.01	1.009	1.008	1.0065	1.006	1.005
0.2	1.038	1.033	1.028	1.025	1.022	1.02	1.018	1.014
0.3	1.063	1.059	1.052	1.045	1.04	1.036	1.03	1.026
0.4	1.1	1.091	1.085	1.074	1.069	1.063	1.051	1.044
0.5	1.16	1.153	1.142	1.129	1.117	1.109	1.091	1.073
0.6	1.273	1.243	1.223	1.208	1.188	1.18	1.146	1.124
0.7	1.428	1.38	1.347	1.318	1.306	1.28	1.231	1.2
0.8	1.72	1.61	1.577	1.531	1.492	1.449	1.385	1.34
0.9	2.43	2.14	2.048	1.96	1.87	1.818	1.723	1.63
1	3.8	3.1	2.84	2.7	2.55	2.43	2.26	2.1

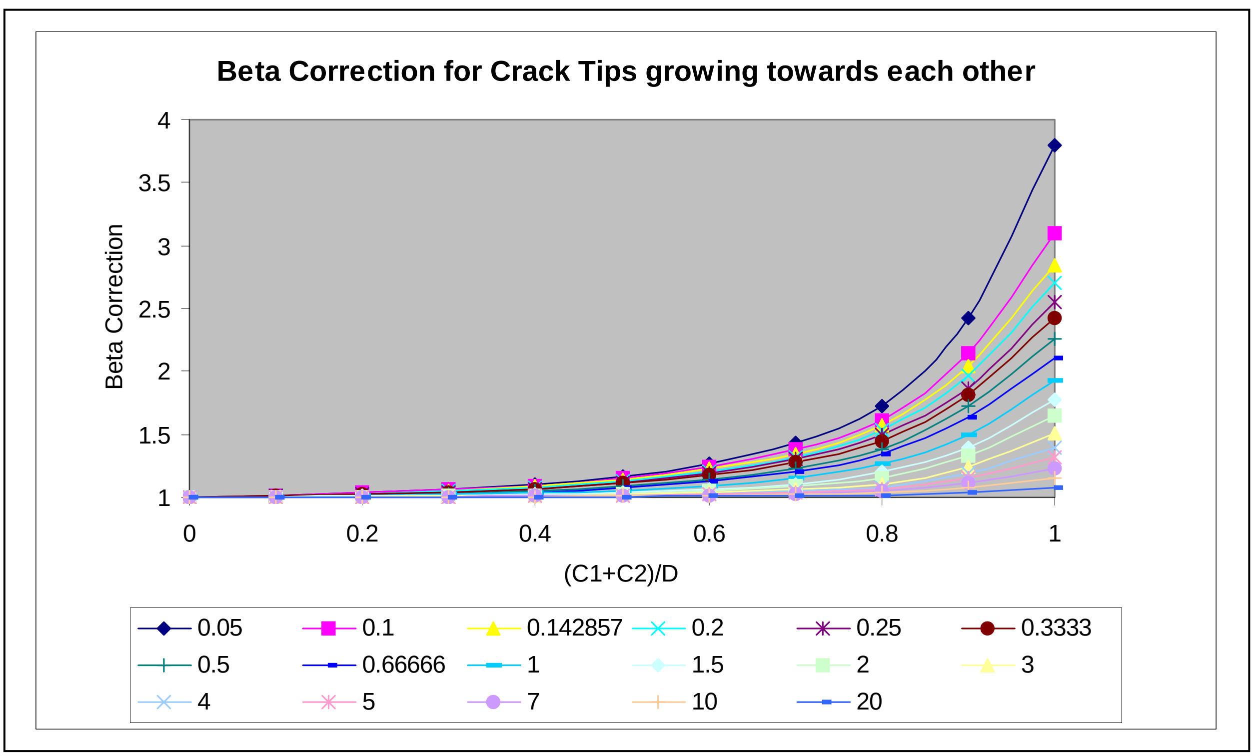
Table 2: Continued

(C1+C2)/D	C1/C2=1	C1/C2=1.5	C1/C2=2	C1/C2=3	C1/C2=4	C1/C2=5	C1/C2=7	C1/C2=10	C1/C2=20
0	1	1	1	1	1	1	1	1	1
0.1	1.004	1.003	1.0025	1.0021	1.0018	1.0016	1.0013	1.001	1.0005
0.2	1.01	1.007	1.006	1.0043	1.0035	1.003	1.0026	1.0023	1.0012
0.3	1.02	1.013	1.01	1.0075	1.0063	1.0055	1.005	1.004	1.0023
0.4	1.0345	1.024	1.019	1.012	1.0093	1.008	1.007	1.006	1.0037
0.5	1.057	1.04	1.031	1.022	1.016	1.013	1.0103	1.009	1.0056
0.6	1.093	1.066	1.051	1.034	1.026	1.021	1.015	1.013	1.008
0.7	1.148	1.105	1.084	1.062	1.045	1.035	1.025086	1.02	1.0115
0.8	1.267	1.2	1.1495	1.1	1.074	1.06	1.045723	1.034	1.0168
0.9	1.5013	1.4	1.325	1.245	1.191	1.153	1.111088	1.077	1.034
1	1.93	1.77	1.65	1.51	1.4	1.32	1.23	1.15	1.07

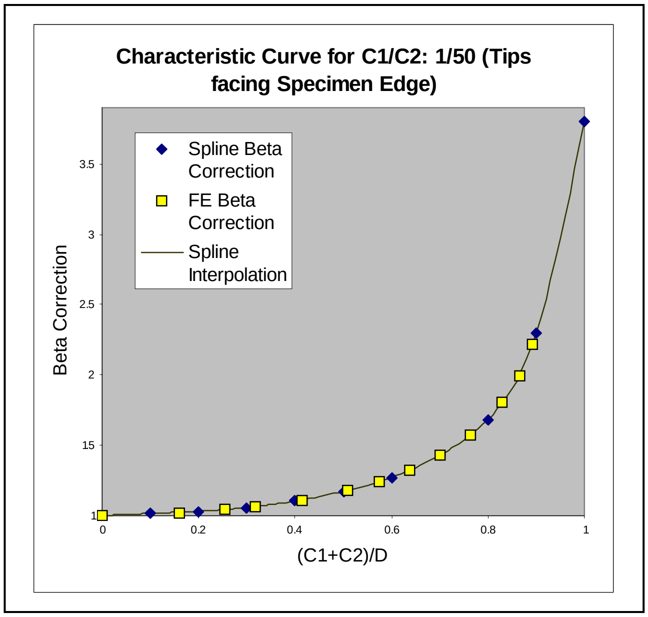
A3. Characteristic Plots for Two Through Cracks

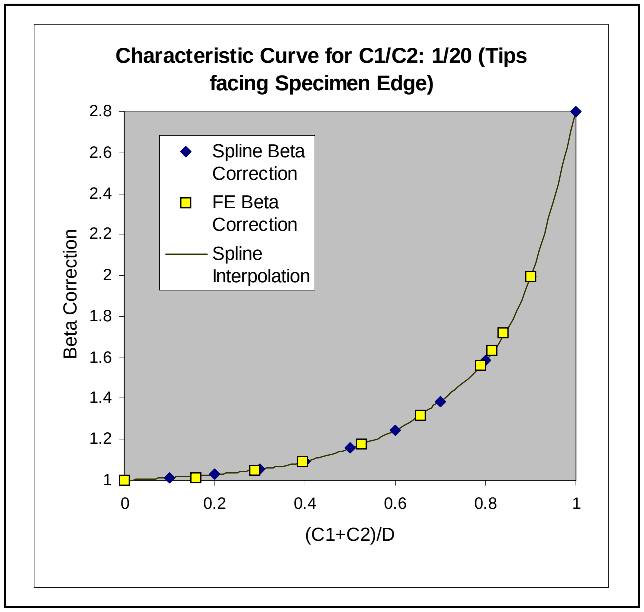
A3.1 (C1+C2)/D vs. Beta Correction for various C1/C2 Ratios

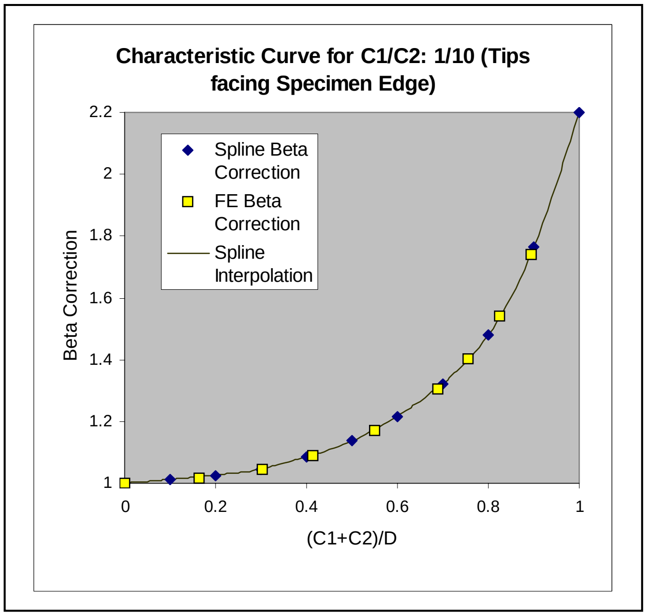


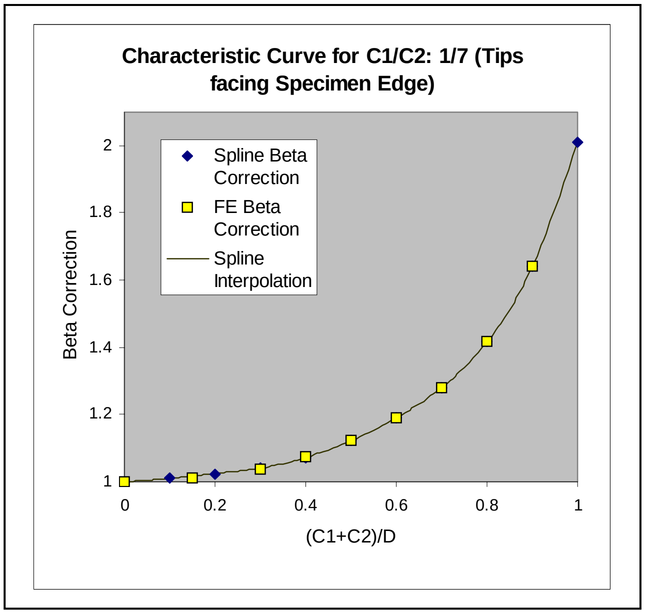


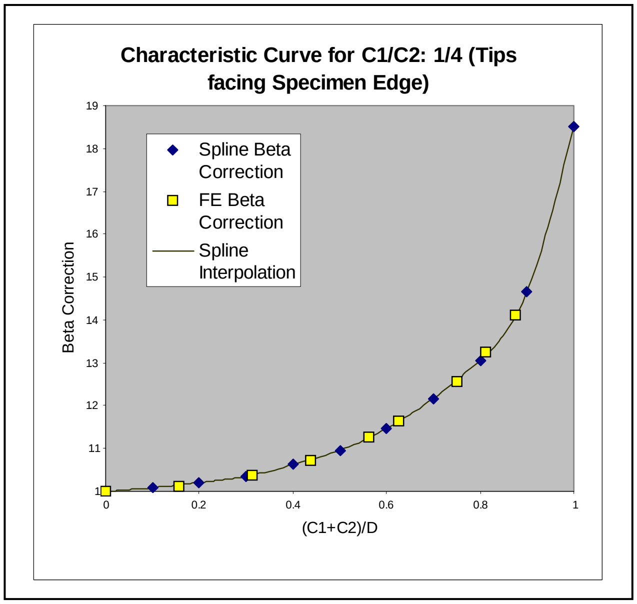
A3.2 Spline Interpolation vs. FEA solution for various C1/C2 ratios

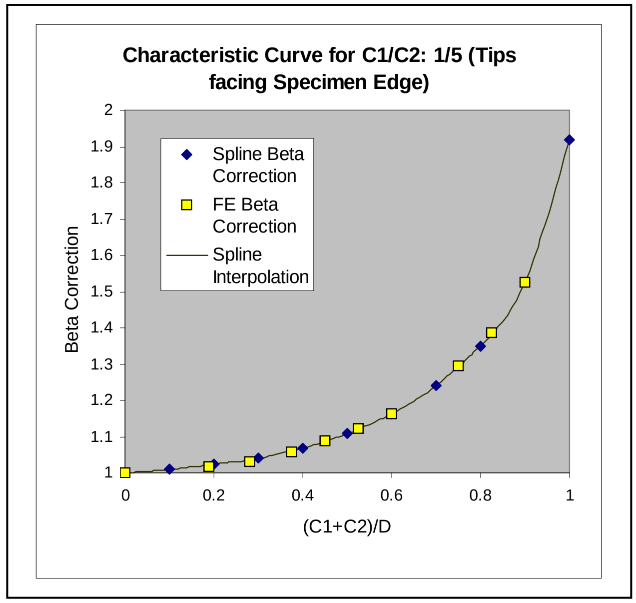


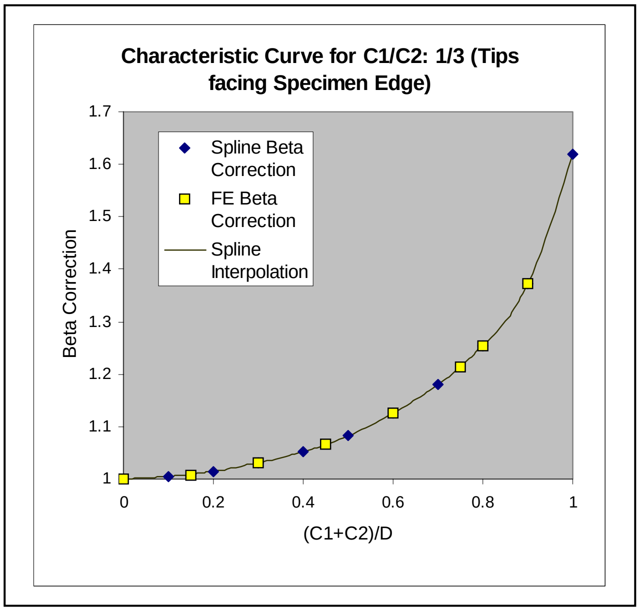


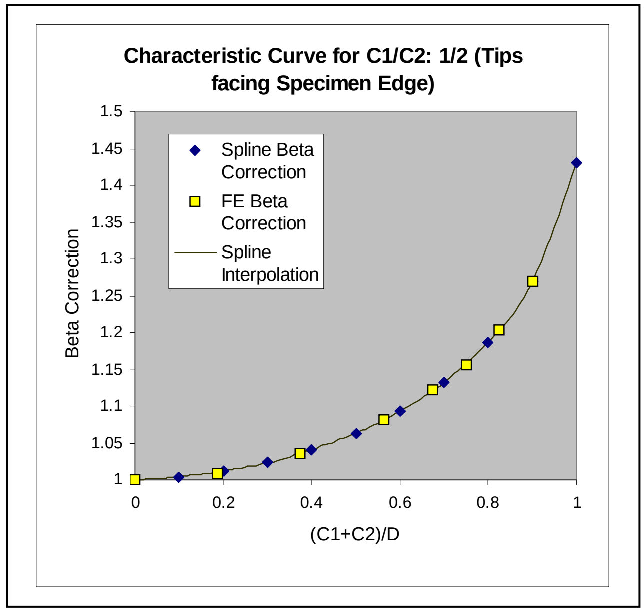


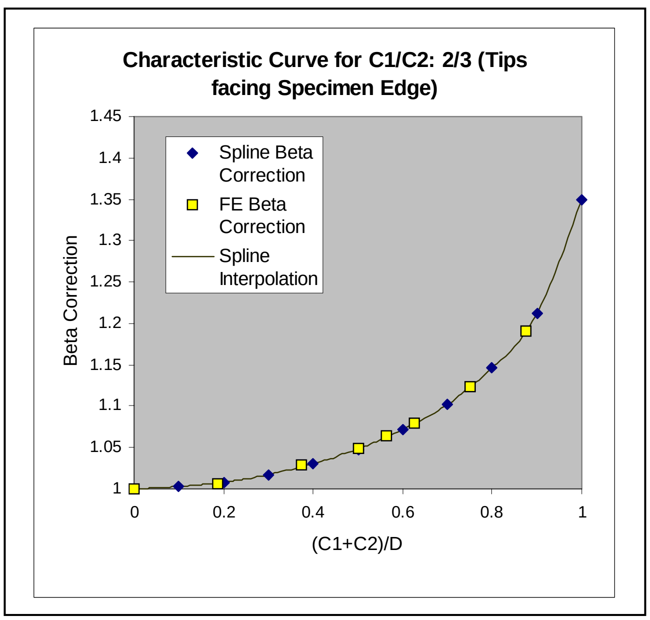


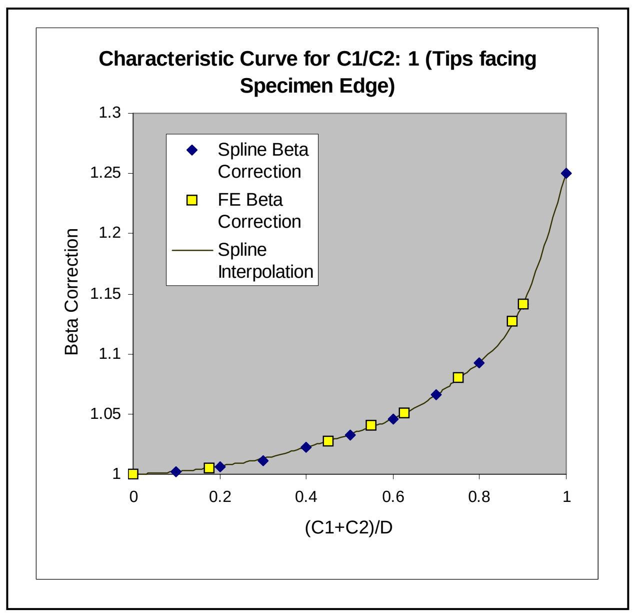

Characteristic Curve for C1/C2: 1/4 (Tips facing Specimen Edge)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
(C1+C2)/D
19
18
17
16
15
14
13
12
11
1
0
0.2
0.4
0.6
0.8
1

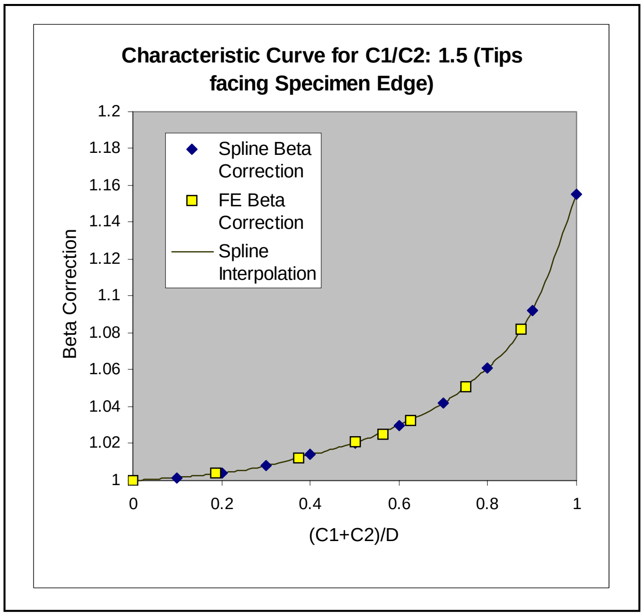

Characteristic Curve for C1/C2: 1/5 (Tips facing Specimen Edge)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
(C1+C2)/D
2
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
0
0.2
0.4
0.6
0.8
1

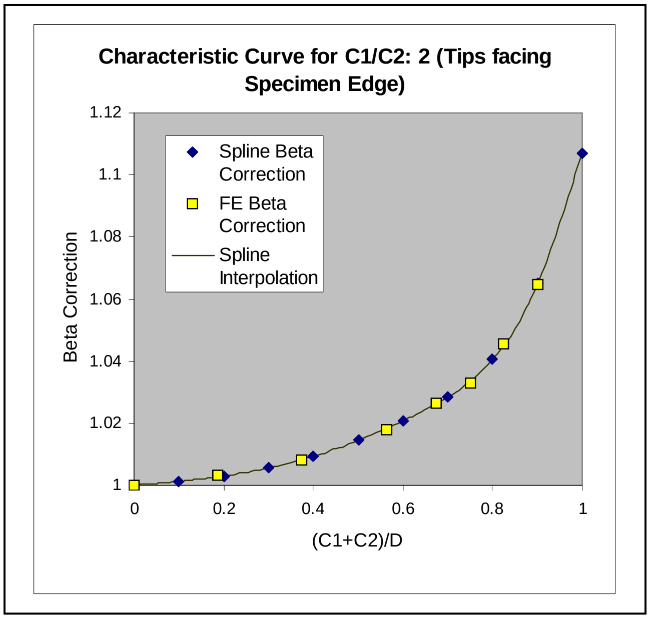

Characteristic Curve for C1/C2: 1/3 (Tips facing Specimen Edge)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
(C1+C2)/D
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
0
0.2
0.4
0.6
0.8
1

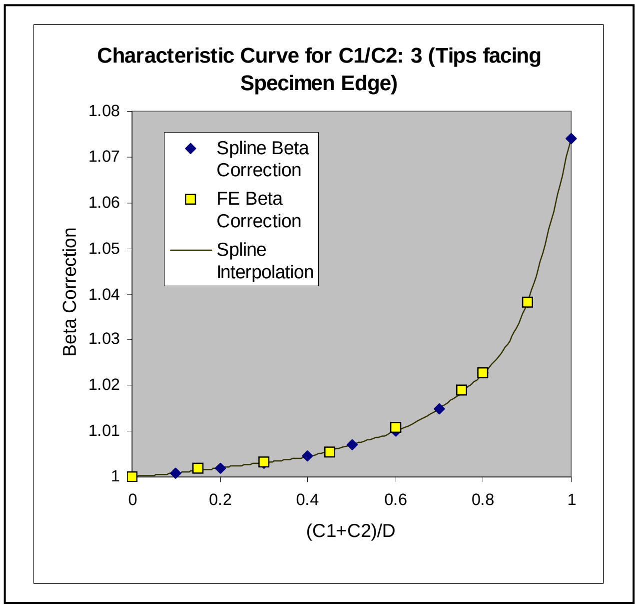

Characteristic Curve for C1/C2: 1/2 (Tips facing Specimen Edge)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
(C1+C2)/D
1.5
1.45
1.4
1.35
1.3
1.25
1.2
1.15
1.1
1.05
1
0
0.2
0.4
0.6
0.8
1

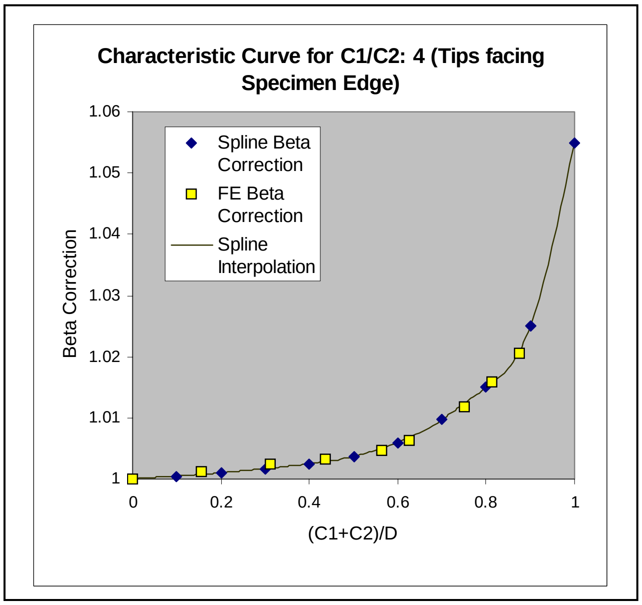

Characteristic Curve for C1/C2: 2/3 (Tips facing Specimen Edge)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
(C1+C2)/D
1.45
1.4
1.35
1.3
1.25
1.2
1.15
1.1
1.05
1
0
0.2
0.4
0.6
0.8
1

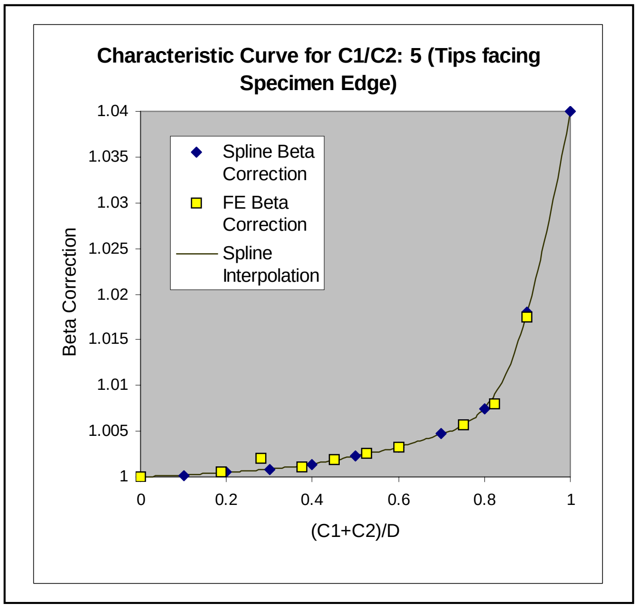

Characteristic Curve for C1/C2: 1 (Tips facing Specimen Edge)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
(C1+C2)/D
1.3
1.25
1.2
1.15
1.1
1.05
1
0
0.2
0.4
0.6
0.8
1

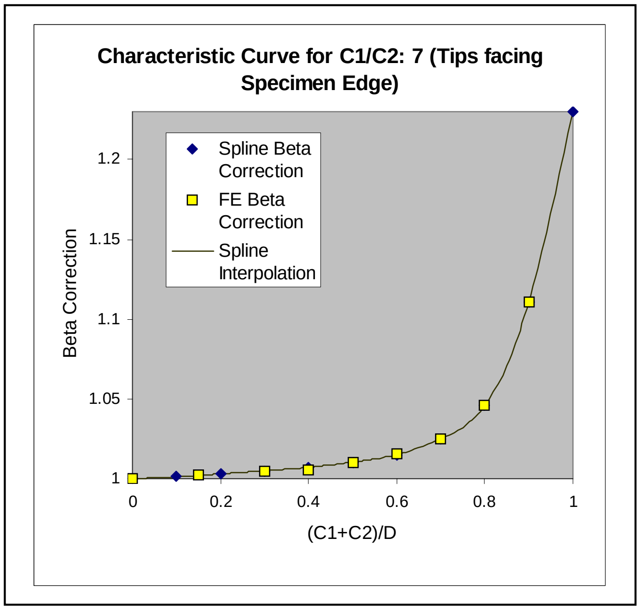

Characteristic Curve for C1/C2: 1.5 (Tips facing Specimen Edge)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
(C1+C2)/D

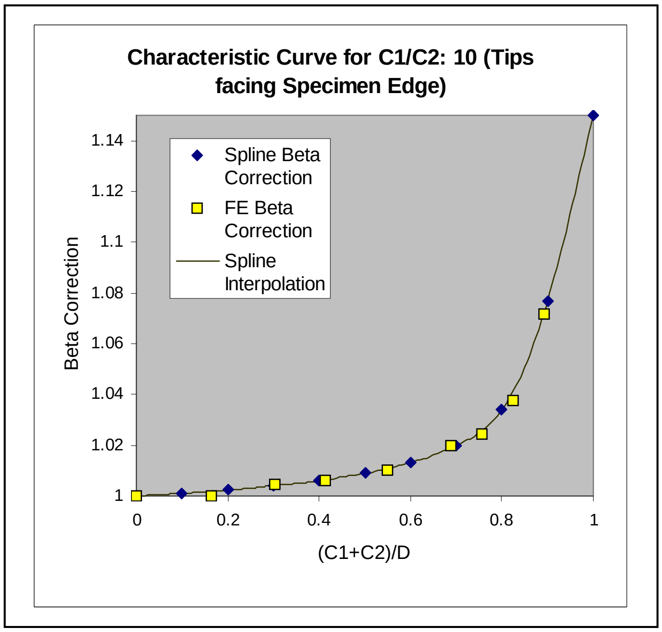

Characteristic Curve for C1/C2: 2 (Tips facing Specimen Edge)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
(C1+C2)/D

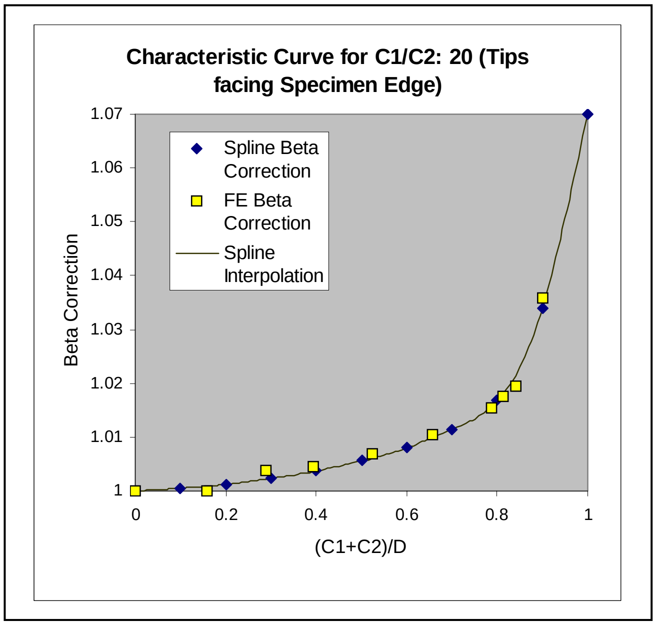

Characteristic Curve for C1/C2: 3 (Tips facing Specimen Edge)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
(C1+C2)/D

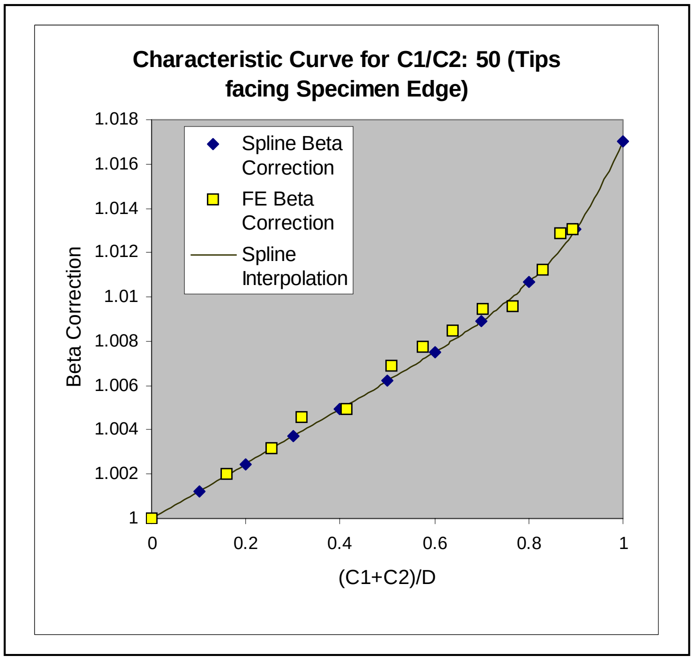

Characteristic Curve for C1/C2: 4 (Tips facing Specimen Edge)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
(C1+C2)/D

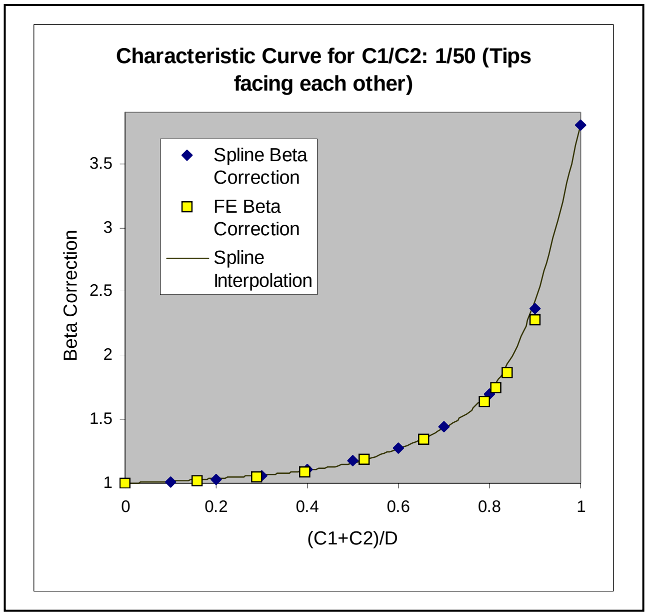

Characteristic Curve for C1/C2: 5 (Tips facing Specimen Edge)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
(C1+C2)/D

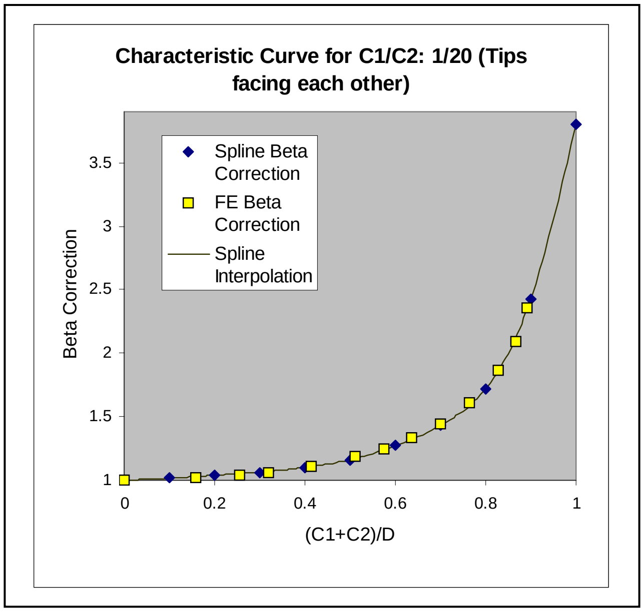

Characteristic Curve for C1/C2: 7 (Tips facing Specimen Edge)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
(C1+C2)/D

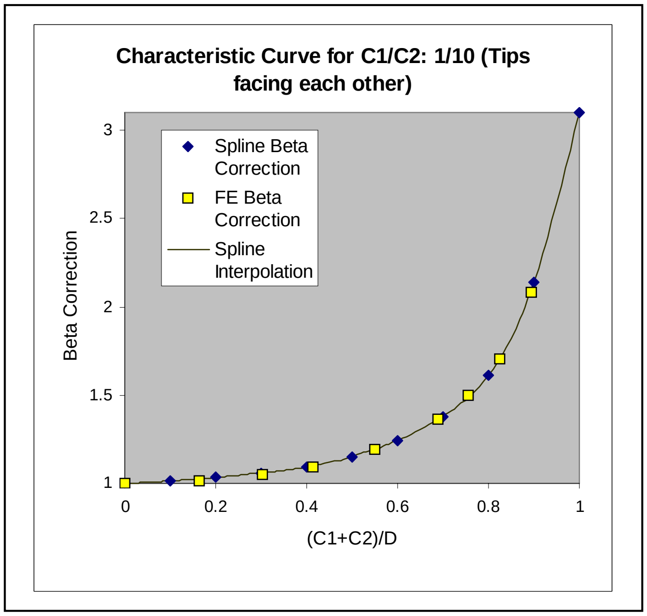

Characteristic Curve for C1/C2: 10 (Tips facing Specimen Edge)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
(C1+C2)/D
1.14
1.12
1.1
1.08
1.06
1.04
1.02
1
0
0.2
0.4
0.6
0.8
1

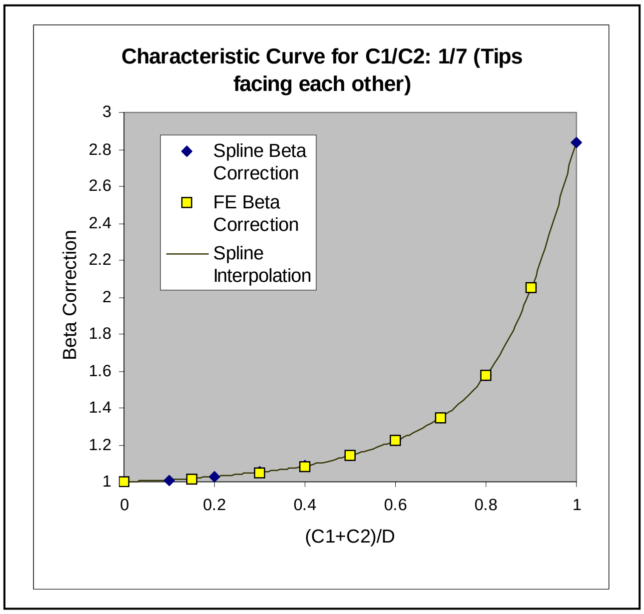

Characteristic Curve for C1/C2: 20 (Tips facing Specimen Edge)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
(C1+C2)/D
1.07
1.06
1.05
1.04
1.03
1.02
1.01
1
0
0.2
0.4
0.6
0.8
1

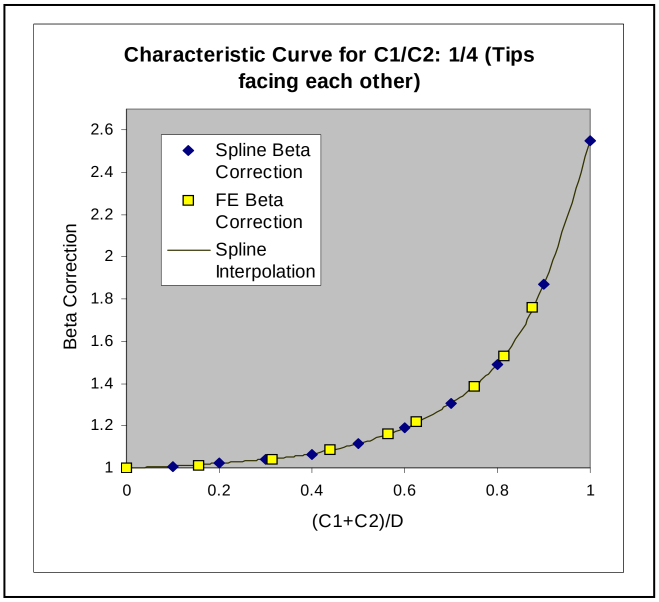

Characteristic Curve for C1/C2: 50 (Tips facing Specimen Edge)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
(C1+C2)/D
1.018
1.016
1.014
1.012
1.01
1.008
1.006
1.004
1.002
1
0
0.2
0.4
0.6
0.8
1

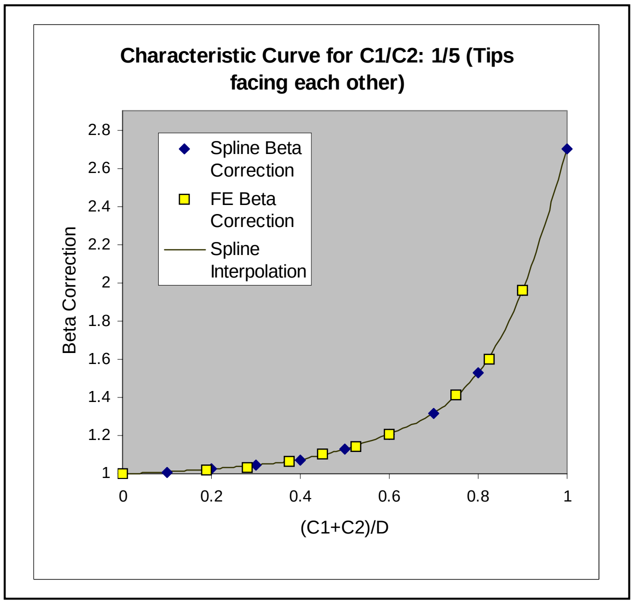

Characteristic Curve for C1/C2: 1/50 (Tips facing each other)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
(C1+C2)/D

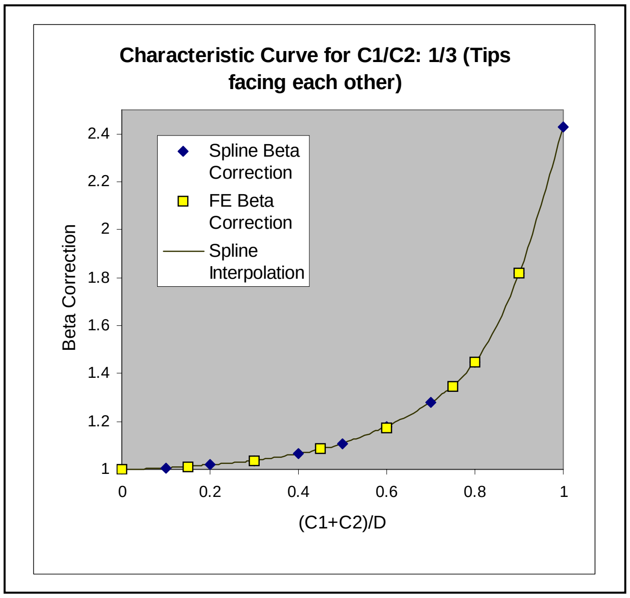

Characteristic Curve for C1/C2: 1/20 (Tips facing each other)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
(C1+C2)/D


Characteristic Curve for C1/C2: 1/10 (Tips facing each other)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
(C1+C2)/D

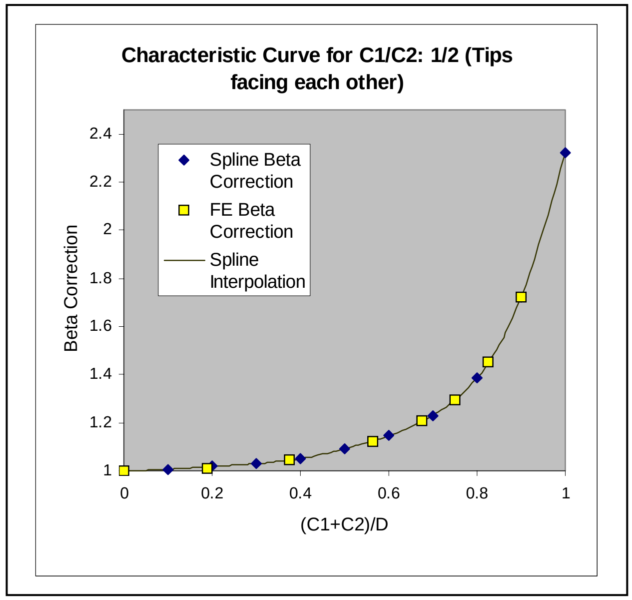

Characteristic Curve for C1/C2: 1/7 (Tips facing each other)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
(C1+C2)/D


Characteristic Curve for C1/C2: 1/4 (Tips facing each other)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
(C1+C2)/D

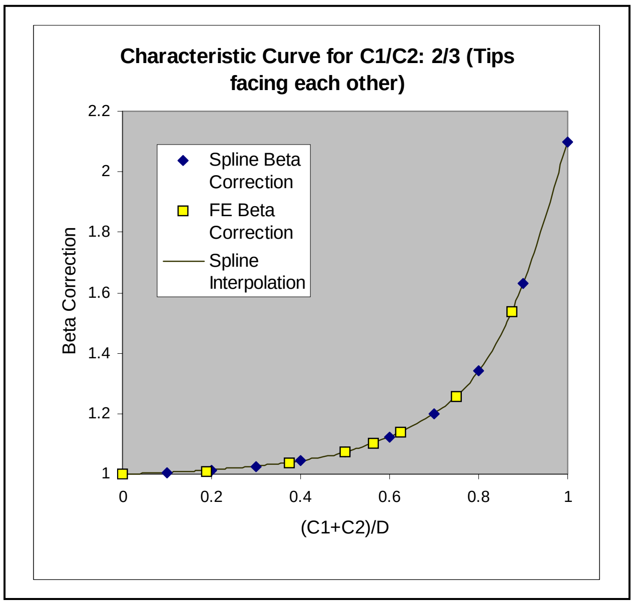

Characteristic Curve for C1/C2: 1/5 (Tips facing each other)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
(C1+C2)/D



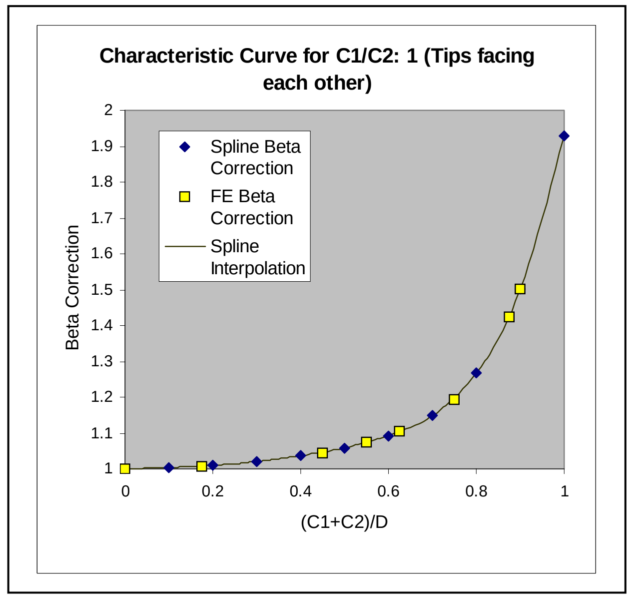
Characteristic Curve for C1/C2: 1/3 (Tips facing each other)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
(C1+C2)/D



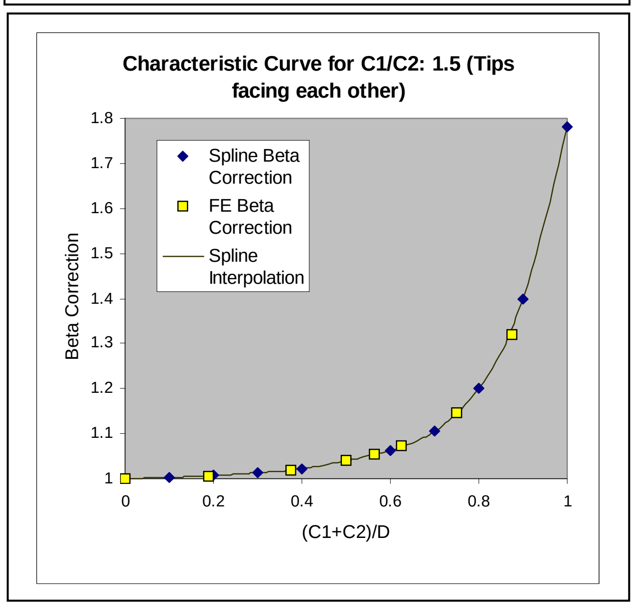
Characteristic Curve for C1/C2: 1/2 (Tips facing each other)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
(C1+C2)/D



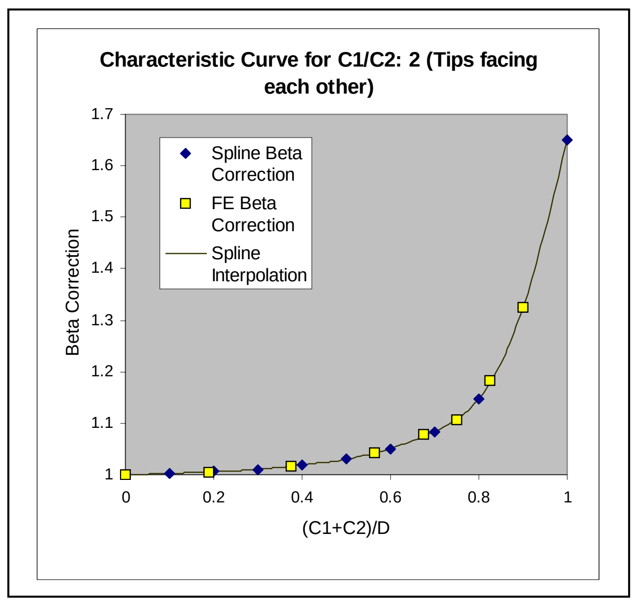
Characteristic Curve for C1/C2: 2/3 (Tips facing each other)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
(C1+C2)/D



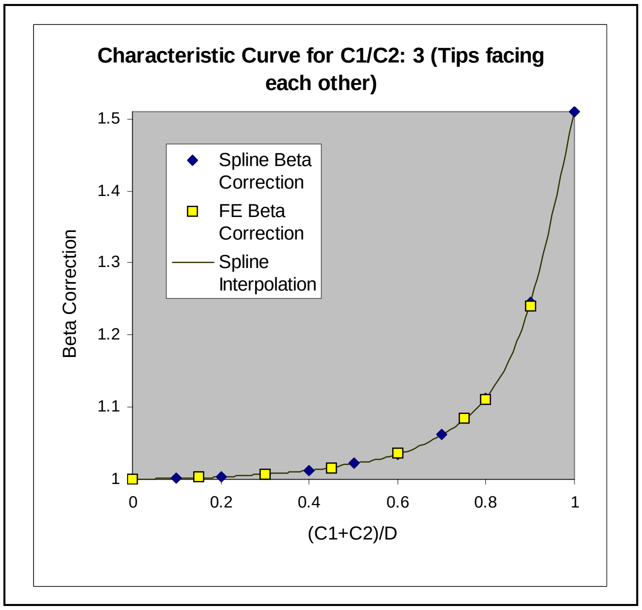
Characteristic Curve for C1/C2: 1 (Tips facing each other)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
(C1+C2)/D

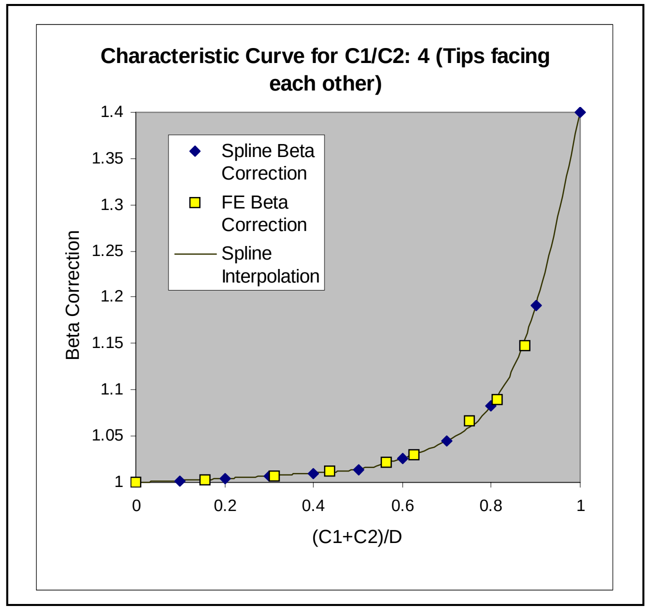


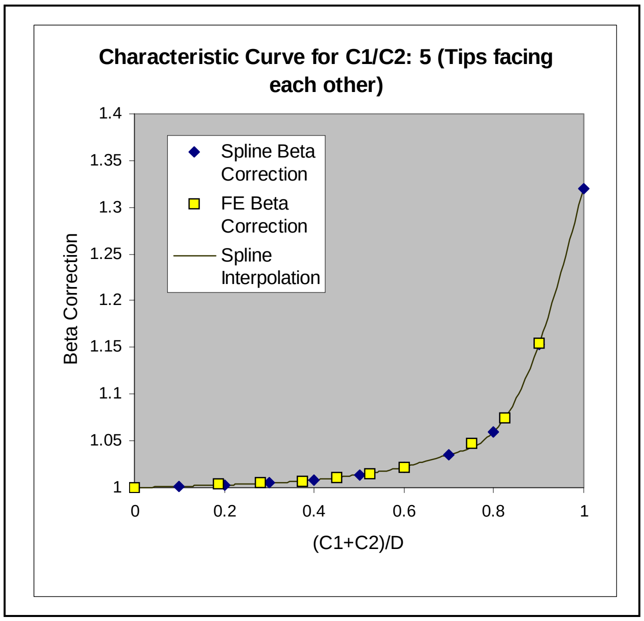
Characteristic Curve for C1/C2: 1.5 (Tips facing each other)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
(C1+C2)/D

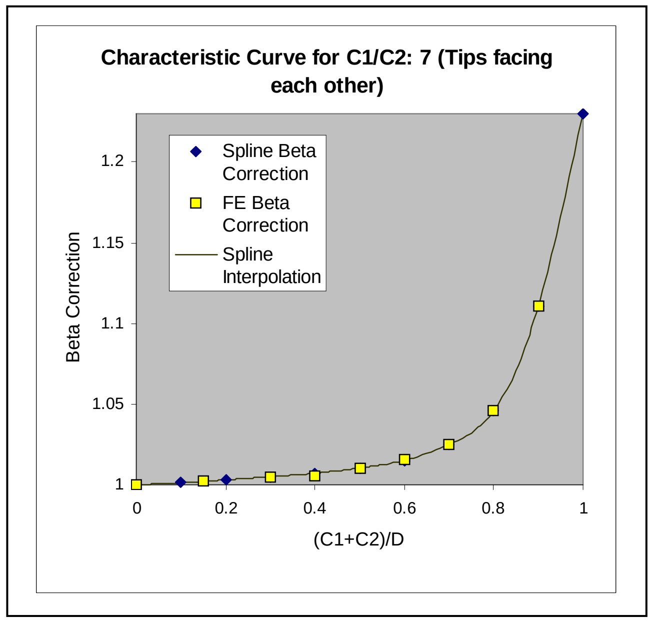


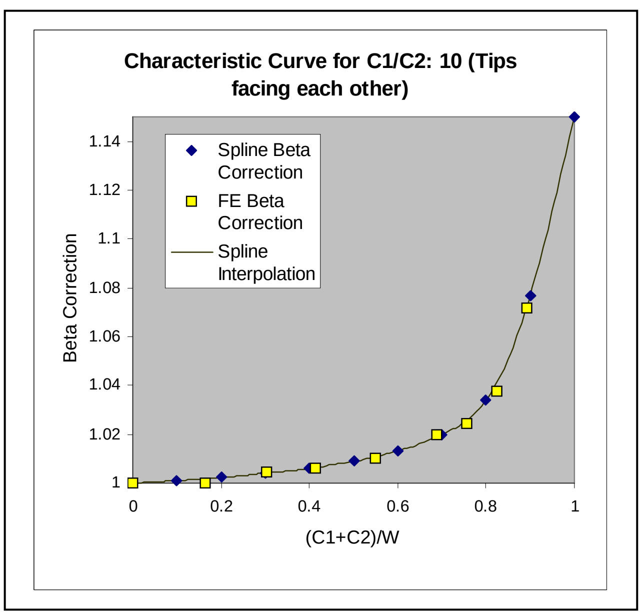
Characteristic Curve for C1/C2: 2 (Tips facing each other)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
(C1+C2)/D

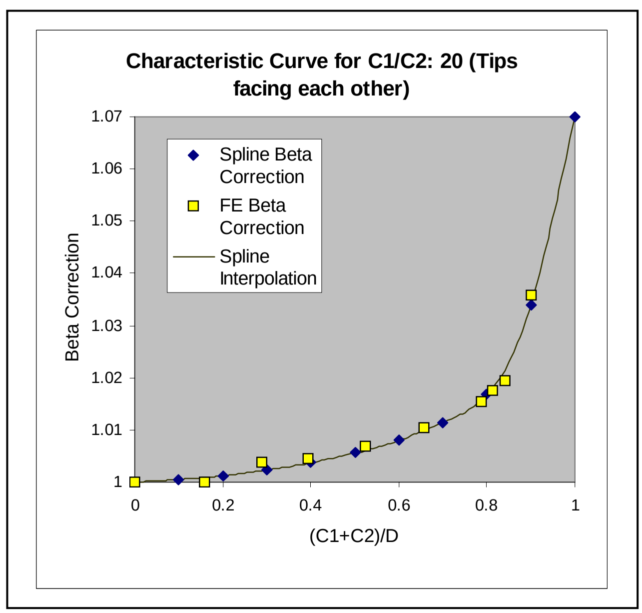

Characteristic Curve for C1/C2: 3 (Tips facing each other)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
1
1.1
1.2
1.3
1.4
1.5
0
0.2
0.4
0.6
0.8
1
(C1+C2)/D

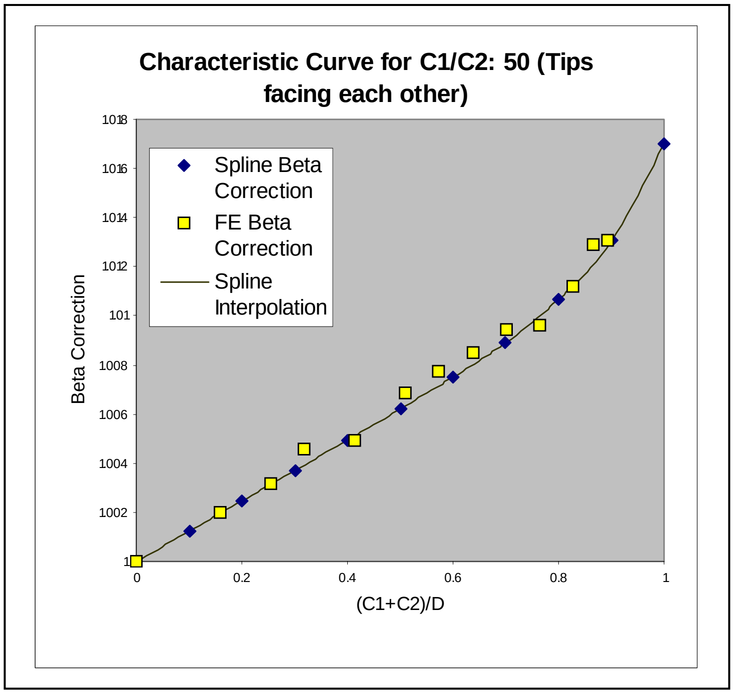

Characteristic Curve for C1/C2: 4 (Tips facing each other)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
1
1.05
1.1
1.15
1.2
1.25
1.3
1.35
1.4
0
0.2
0.4
0.6
0.8
1
(C1+C2)/D


Characteristic Curve for C1/C2: 5 (Tips facing each other)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
1
1.05
1.1
1.15
1.2
1.25
1.3
1.35
1.4
0
0.2
0.4
0.6
0.8
1
(C1+C2)/D


Characteristic Curve for C1/C2: 7 (Tips facing each other)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
1
1.05
1.1
1.15
1.2
0
0.2
0.4
0.6
0.8
1
(C1+C2)/D


Characteristic Curve for C1/C2: 10 (Tips facing each other)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
1
1.02
1.04
1.06
1.08
1.1
1.12
1.14
0
0.2
0.4
0.6
0.8
1
(C1+C2)/W


Characteristic Curve for C1/C2: 20 (Tips facing each other)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
1
1.01
1.02
1.03
1.04
1.05
1.06
1.07
0
0.2
0.4
0.6
0.8
1
(C1+C2)/D


Characteristic Curve for C1/C2: 50 (Tips facing each other)
Spline Beta Correction
FE Beta Correction
Spline Interpolation
Beta Correction
(C1+C2)/D
1018
1016
1014
1012
101
1008
1006
1004
1002
1
0
0.2
0.4
0.6
0.8
1

A4. Comparison Between StressCheck and AFGROW Codes

Case 1: W: 24.0 in., D: 8.0 in. and B: 8.0 in.

C1	C2	(C1+C2)/D	FE-K1	FE-K2	FE-K3	FE-K4	AFG-K1	AFG-K2	AFG-K3	AFG-K4
0.4	4	0.55	1.343	1.372	3.943	4.17	1.352	1.293	3.94	4.178
0.5	5	0.6875	1.712	1.803	4.69	5.27	1.678	1.728	4.719	5.259
0.55	5.5	0.7563	1.973	2.136	5.13	6.014	1.88	2.021	5.177	5.987
0.6	6	0.825	2.326	2.632	5.664	6.999	2.147	2.456	5.743	6.951

Case 2: W: 24.0 in., D: 4.0 in. and B: 4.0 in.

C1	C2	(C1+C2)/D	FE-K1	FE-K2	FE-K3	FE-K4	AFG-K1	AFG-K2	AFG-K3	AFG-K4
2	0.2	0.275	2.863	2.747	0.9765	0.9577	2.871	2.764	0.9164	0.9286

Case 3: W: 24.0 in., D: 8.0 in. and B: 4.0 in.

C1	C2	(C1+C2)/D	FE-K1	FE-K2	FE-K3	FE-K4	AFG-K1	AFG-K2	AFG-K3	AFG-K4
2	0.2	1.1	2.857	2.74	0.8381	0.8358	2.866	2.744	0.7205	0.8226

Case 4: W: 24.0 in., D: 16.0 in. and B: 4.0 in.

C1	C2	(C1+C2)/D	FE-K1	FE-K2	FE-K3	FE-K4	AFG-K1	AFG-K2	AFG-K3	AFG-K4
2	0.2	0.275	2.855	2.738	0.7794	0.7777	2.865	2.744	0.7189	0.8049

Case 5: W: 20.0 in., D: 2.0 in. and B: 9.0 in.

C1	C2	(C1+C2)/D	FE-K1	FE-K2	FE-K3	FE-K4	AFG-K1	AFG-K2	AFG-K3	AFG-K4
0.15	0.1	0.125	0.6847	0.6847	0.5626	0.5625	0.688	0.6892	0.5644	0.5627
0.2	0.15	0.175	0.7959	0.7964	0.691	0.6904	0.7958	0.798	0.6935	0.6907
0.3	0.25	0.275	0.9792	0.982	0.8995	0.8968	0.9793	0.9853	0.9038	0.8959
0.4	0.35	0.375	1.139	1.147	1.078	1.069	1.141	1.152	1.084	1.071
0.5	0.45	0.475	1.287	1.307	1.248	1.228	1.29	1.314	1.253	1.23

Case 6: W: 16.0 in., D: 4.0 in. and B: 4.0 in.

C1	C2	(C1+C2)/D	FE-K1	FE-K2	FE-K3	FE-K4	AFG-K1	AFG-K2	AFG-K3	AFG-K4
0.05	1	0.2625	0.414	0.4147	1.795	1.795	0.413	0.4173	1.792	1.790
0.1	2	0.525	0.67	0.6761	2.613	2.609	0.6586	0.6636	2.62	2.611
0.15	3	0.7875	1.116	1.171	3.393	3.372	1.094	1.193	3.412	3.382
1	0.05	0.2625	1.834	1.824	0.4146	0.4143	1.833	1.817	0.4172	0.413

Case 7: W: 16.0 in., D: 8.0 in. and B: 4.0 in.

C1	C2	(C1+C2)/D	FE-K1	FE-K2	FE-K3	FE-K4	AFG-K1	AFG-K2	AFG-K3	AFG-K4
2	2	0.5	2.917	2.906	2.902	2.92	2.987	2.706	2.706	2.987
3	3	0.75	4.724	4.666	4.672	4.724	4.79	4.684	4.684	4.79

Case 8: W: 8.0 in., D: 2.0 in. and B: 3.0 in.

C1	C2	(C1+C2)/D	FE-K1	FE-K2	FE-K3	FE-K4	AFG-K1	AFG-K2	AFG-K3	AFG-K4
0.15	0.1	0.125	0.6878	0.6879	0.5634	0.5632	0.6889	0.6898	0.5646	0.5631
0.2	0.15	0.175	0.7978	0.7982	0.6924	0.6918	0.7976	0.7991	0.694	0.6916
0.3	0.25	0.275	0.9841	0.9854	0.9024	0.9005	0.9843	0.9883	0.9058	0.8991
0.4	0.35	0.375	1.151	1.158	1.088	1.081	1.152	1.159	1.089	1.079
0.5	0.45	0.475	1.308	1.327	1.267	1.246	1.309	1.325	1.262	1.244
0.6	0.55	0.575	1.463	1.507	1.454	1.406	1.459	1.501	1.449	1.403
0.7	0.65	0.675	1.623	1.716	1.67	1.573	1.616	1.696	1.656	1.565

Case 9: W: 4.0 in., D: 2.0 in. and B: 1.0 in.

C1	C2	(C1+C2)/D	FE-K1	FE-K2	FE-K3	FE-K4	AFG-K1	AFG-K2	AFG-K3	AFG-K4
0.04	0.8	0.42	0.3782	0.3832	2.081	2.717	0.3839	0.3535	2.088	2.696
0.08	0.8	0.44	0.5322	0.5462	2.083	2.716	0.5438	0.5061	2.095	2.697
0.13	0.91	0.52	0.6879	0.7368	2.535	4.126	0.7026	0.695	2.518	4.089
0.56	0.08	0.32	1.621	1.504	0.5199	0.5148	1.616	1.508	0.4595	0.5313
0.6	0.06	0.33	1.742	1.584	0.452	0.4481	1.736	1.59	0.4027	0.4617
0.8	0.8	0.8	2.685	2.66	2.659	2.688	2.725	2.689	2.689	2.725

Appendix B
Edge and Internal Cracks in a Plate

B1. Cases

Case 1: W: 40.0 in. and B: 20.0 in.

C1	C2	(C1+C2)/B	FE-K1(EDGE)	FE-K2(THRU)	FE-K3(THRU)
0.5	3.5	0.2	1.416	3.375	3.375
1	7	0.4	2.117	5.084	5.075
1.25	8.75	0.5	2.492	5.963	5.944
1.5	10.5	0.6	2.99	7.021	6.971
1.75	12.25	0.7	3.739	8.31	8.186
2	14	0.8	5.154	10.2	9.869
2.2	15.4	0.88	7.633	12.704	11.751
2.4	16.8	0.96	16.17	20	14.79
0.5	2.5	0.15	1.409	2.829	2.829
1	5	0.3	2.055	4.132	4.127
1.5	7.5	0.45	2.668	5.333	5.303
2	10	0.6	3.498	6.743	6.654
2.5	12.5	0.75	5.027	8.729	8.404
2.75	13.75	0.825	6.5302	10.345	9.601
3	15	0.9	9.675	13.11	11.18
3.1	15.5	0.93	12.125	15.409	12.014
0.75	2.25	0.15	1.7307	2.678	2.677
1.5	4.5	0.3	2.53	3.899	3.889
2	6	0.4	3.031	4.624	4.597
3	9	0.6	4.315	6.274	6.1
3.5	10.5	0.7	5.351	7.362	6.963
4	12	0.8	7.086	8.995	8.026
4.25	12.75	0.85	8.503	10.371	8.68
4.5	13.5	0.9	10.9	12.38	9.442
4.75	14.25	0.95	16.008	17.352	10.426
1	1.5	0.125	2	2.18	2.179
2	3	0.25	2.901	3.132	3.124
3	4.5	0.375	3.708	3.937	3.891
4	6	0.5	4.612	4.786	4.639
5	7.5	0.625	5.741	5.779	5.382
6	9	0.75	7.49	7.3	6.239
6.6	9.9	0.825	9.178	8.798	6.834
7.333	11	0.91665	13.6	12.96	7.83
7.6	11.4	0.95	17.594	17.151	8.366
1	1	0.1	1.997	1.774	1.774
3	3	0.3	3.624	3.157	3.135
5	5	0.5	5.205	4.381	4.199
6	6	0.6	6.166	5.12	4.715
7	7	0.7	7.449	6.168	5.311
8	8	0.8	9.358	7.833	5.989
9	9	0.9	13.3	11.59	6.958

C1	C2	(C1+C2)/B	FE-K1(EDGE)	FE-K2(THRU)	FE-K3(THRU)
9.5	9.5	0.95	18.472	16.972	7.763
1.5	1	0.125	2.4602	1.778	1.776
3	2	0.25	3.585	2.548	2.536
4.5	3	0.375	4.632	3.219	3.159
6	4	0.5	5.776	3.964	3.769
7.5	5	0.625	7.135	4.901	4.388
9	6	0.75	9.104	6.485	5.16
9.9	6.6	0.825	10.889	8.092	5.729
11	7.333	0.91665	15.36	12.63	6.859
11.4	7.6	0.95	19.329	17.154	7.545
1.5	0.5	0.1	2.457	1.255	1.255
4.5	1.5	0.3	4.564	2.248	2.227
6.75	2.25	0.45	6.083	2.943	2.839
9	3	0.6	7.914	3.921	3.557
10.5	3.5	0.7	9.436	4.938	4.158
12	4	0.8	11.6	6.737	5.039
13.5	4.5	0.9	15.5	10.73	6.492
14.1	4.7	0.94	18.894	14.604	7.48
2.5	0.5	0.15	3.2209	1.2608	1.259
5	1	0.3	4.875	1.846	1.831
7.5	1.5	0.45	6.55	2.446	2.373
10	2	0.6	8.575	3.337	3.071
12.5	2.5	0.75	11.26	5.039	4.177
13.75	2.75	0.825	13.164	6.678	5.066
15	3	0.9	16.24	9.929	6.524
15.75	3.15	0.945	19.841	14.294	7.975
2.8	0.4	0.16	3.429	1.129	1.128
3.5	0.5	0.2	3.896	1.2707	1.269
5.25	0.75	0.3	5.014	1.6	1.59
7	1	0.4	6.175	1.949	1.916
8.75	1.25	0.5	7.452	2.365	2.2809
10.5	1.5	0.6	8.916	2.96	2.761
12.25	1.75	0.7	10.673	3.874	3.432
14	2	0.8	12.95	5.525	4.504
15.05	2.15	0.86	14.791	7.282	5.496
15.75	2.25	0.9	16.525	9.226	6.462
16.8	2.4	0.96	22	15.77	8.918
3	0.3	0.165	3.563	0.9797	0.9787
6	0.6	0.33	5.495	1.455	1.446
10	1	0.55	8.439	2.284	2.195
13	1.3	0.715	11.38	3.59	3.232
15	1.5	0.825	14.13	5.545	4.601
16.5	1.65	0.9075	17.199	8.879	6.532
17	1.7	0.935	19	11.12	7.597
3	0.15	0.1575	3.565	0.6925	0.6922
6	0.3	0.315	5.491	1.029	1.025

C1	C2	(C1+C2)/B	FE-K1(EDGE)	FE-K2(THRU)	FE-K3(THRU)
10	0.5	0.525	8.392	1.595	1.563
14	0.7	0.735	12.42	2.946	2.752
16	0.8	0.84	15.201	4.685	4.143
17	0.85	0.8925	16.984	6.414	5.392
18	0.9	0.945	19.71	9.955	7.552
3	0.5	0.175	3.566	1.265	1.264
6	1	0.35	5.507	1.884	1.862
9	1.5	0.525	7.665	2.646	2.524
12	2	0.7	10.485	4.073	3.57
13.8	2.3	0.805	12.852	5.867	4.675
14.4	2.4	0.84	13.896	6.847	5.209
15	2.5	0.875	15.205	8.199	5.884
15.6	2.6	0.91	16.946	10.227	6.773
15.9	2.65	0.9275	18.172	11.707	7.344
2	0.25	0.1125	2.856	0.8888	0.8885
4	0.5	0.225	4.215	1.278	1.275
6.4	0.8	0.36	5.763	1.701	1.682
9.6	1.2	0.54	8.1203	2.442	2.337
12.8	1.6	0.72	11.236	3.944	3.482
14.4	1.8	0.81	13.395	5.532	4.538
16	2	0.9	16.648	8.937	6.414
16.8	2.1	0.945	19.836	12.949	8.105
0.25	2	0.1125	0.99604	2.521	2.521
0.5	4	0.225	1.422	3.6309	3.63
0.8	6.4	0.36	1.858	4.784	4.7802
1.2	9.6	0.54	2.506	6.438	6.416
1.6	12.8	0.72	3.694	8.795	8.6906
1.8	14.4	0.81	5.0307	10.736	10.355
2	16	0.9	8.327	14.14	12.875
2.1	16.8	0.945	12.719	17.97	14.871
0.5	3	0.175	1.408	3.11	3.11
1	6	0.35	2.079	4.597	4.591
1.5	9	0.525	2.795	6.112	6.08
2	12	0.7	4.025	8.188	7.974
2.3	13.8	0.805	5.602	10.172	9.664
2.4	14.4	0.84	6.5	11.031	10.365
2.5	15	0.875	7.792	12.257	11.155
2.6	15.6	0.91	9.84	14.183	12.143
2.65	15.9	0.9275	11.372	15.506	12.706
0.3	3	0.165	1.0952	3.113	3.112
0.5	5	0.275	1.434	4.118	4.117
0.8	8	0.44	1.913	5.568	5.562
1.1	11	0.605	2.525	7.318	7.253
1.2	12	0.66	2.831	8.042	7.996
1.5	15	0.825	4.822	11.372	11.122
1.65	16.5	0.9075	8.146	14.916	13.932

C1	C2	(C1+C2)/B	FE-K1(EDGE)	FE-K2(THRU)	FE-K3(THRU)
1.7	17	0.935	10.678	17.123	15.286
0.15	3	0.1575	0.7735	3.112	3.111
0.3	6	0.315	1.114	4.591	4.589
0.5	10	0.525	1.55	6.66	6.654
0.6	12	0.63	1.835	8.011	7.997
0.7	14	0.735	2.322	9.892	9.848
0.75	15	0.7875	2.744	11.194	11.154
0.8	16	0.84	3.491	12.937	12.873
0.85	17	0.8925	5.039	15.538	15.321
0.9	18	0.945	9.364	20.298	19.367

Case 2: W: 40.0 in. and B: 10.0 in.

C1	C2	(C1+C2)/B	FE-K1(EDGE)	FE-K2(THRU)	FE-K3(THRU)
0.2	4	0.42	0.8227	3.883	3.772
0.3	6	0.63	1.281	5.508	5.01
0.35	7	0.735	1.651	6.718	5.72
0.42	8.4	0.882	3.164	9.854	7.033
0.4	4	0.44	1.372	3.885	3.784
0.5	5	0.55	1.651	4.621	4.379
0.6	6	0.66	2.094	5.522	5.016
0.8	8	0.88	4.749	8.958	6.624
0.5	3.5	0.4	1.505	3.552	3.49
0.8	5.6	0.64	2.38	5.159	4.754
1	7	0.8	3.643	6.903	5.753
1.1	7.7	0.88	5.417	8.528	6.386
0.6	3	0.36	1.644	3.228	3.192
1	5	0.6	2.526	4.655	4.394
1.25	6.25	0.75	3.618	5.96	5.223
1.5	7.5	0.9	6.706	8.751	6.303
1	3	0.4	2.15	3.242	3.199
1.5	4.5	0.6	3.055	4.336	4.118
2	6	0.8	4.905	6.112	5.158
2.3	6.9	0.92	8.218	9.062	6.049
1	1.5	0.25	2.034	2.207	2.204
2	3	0.5	3.161	3.331	3.248
3	4.5	0.75	4.945	4.941	4.326
3.6	5.4	0.9	7.806	7.604	5.266
2	2	0.4	2.985	2.633	2.602
3	3	0.6	4.038	3.536	3.356
4	4	0.8	5.831	5.163	4.251
4.5	4.5	0.9	7.927	7.216	4.945
1.5	1	0.25	2.484	1.803	1.797
3	2	0.5	3.757	2.764	2.685
4.5	3	0.75	5.477	4.266	3.691
5.4	3.6	0.9	7.925	6.777	4.647
3	1	0.4	3.605	1.898	1.876

C1	C2	(C1+C2)/B	FE-K1(EDGE)	FE-K2(THRU)	FE-K3(THRU)
4.5	1.5	0.6	4.734	2.669	2.527
6	2	0.8	6.261	4.128	3.448
6.9	2.3	0.92	8.377	6.607	4.425
3	0.6	0.36	3.561	1.463	1.452
5	1	0.6	4.964	2.248	2.145
6.25	1.25	0.75	5.964	3.12	2.786
7.5	1.5	0.9	7.692	5.211	3.881
3.5	0.5	0.4	3.915	1.371	1.359
5.6	0.8	0.64	5.331	2.15	2.046
7	1	0.8	6.464	3.209	2.819
7.7	1.1	0.88	7.311	4.295	3.442
4	0.4	0.44	4.235	1.268	1.256
5	0.5	0.55	4.889	1.558	1.524
6	0.6	0.66	5.561	1.959	1.873
8	0.8	0.88	7.299	3.81	3.173
4	0.2	0.42	4.226	0.8933	0.8891
6	0.3	0.63	5.518	1.362	1.332
7	0.35	0.735	6.185	1.765	1.69
8.4	0.42	0.882	7.337	2.969	2.631

Case 3: W: 40.0 in. and B: 5.0 in.

C1	C2	(C1+C2)/B	FE-K1(EDGE)	FE-K2(THRU)	FE-K3(THRU)
0.1	2	0.42	0.4095	2.691	2.639
0.15	3	0.63	0.6508	3.696	3.436
0.2	4	0.84	1.184	5.369	4.368
0.22	4.4	0.924	2.292	6.691	4.861
0.2	2	0.44	0.8073	2.689	2.64
0.25	2.5	0.55	0.8862	3.151	3.033
0.3	3	0.66	1.252	3.689	3.436
0.4	4	0.88	2.732	5.519	4.386
0.3	2.1	0.48	1.074	2.781	2.718
0.4	2.8	0.64	1.515	3.472	3.275
0.5	3.5	0.8	2.048	3.472	3.147
0.55	3.85	0.88	3.357	5.327	4.25
0.4	2	0.48	1.383	2.697	2.642
0.5	2.5	0.6	1.587	3.17	3.038
0.6	3	0.72	2.21	3.76	3.452
0.75	3.75	0.9	4.153	5.495	4.194
0.6	1.8	0.48	1.706	2.537	2.488
0.8	2.4	0.64	2.223	3.133	2.978
1	3	0.8	3.176	4.028	3.507
1.1	3.3	0.88	4.194	4.89	3.828
0.6	0.9	0.3	1.59	1.712	1.706
1	1.5	0.5	2.168	2.315	2.269
1.5	2.25	0.75	3.256	3.32	2.962
1.8	2.7	0.9	4.949	4.885	3.521

C1	C2	(C1+C2)/B	FE-K1(EDGE)	FE-K2(THRU)	FE-K3(THRU)
1	1	0.4	2.07	1.839	1.82
1.5	1.5	0.6	2.743	2.438	2.326
2	2	0.8	3.801	3.43	2.896
2.25	2.25	0.9	5.003	4.638	3.288
0.9	0.6	0.3	1.922	1.402	1.397
1.5	1	0.5	2.572	1.914	1.865
2.25	1.5	0.75	3.583	2.847	2.506
2.7	1.8	0.9	4.992	4.344	3.071
1.8	0.6	0.48	2.758	1.502	1.47
2.4	0.8	0.64	3.304	1.945	1.827
3	1	0.8	4.01	2.7	2.29
3.3	1.1	0.88	4.62	3.451	2.616
2	0.4	0.48	2.886	1.246	1.225
2.5	0.5	0.6	3.283	1.524	1.462
3	0.6	0.72	3.708	1.918	1.758
3.75	0.75	0.9	4.737	3.26	2.466
2.1	0.3	0.48	2.953	1.089	1.074
2.8	0.4	0.64	3.487	1.445	1.382
3.5	0.5	0.8	4.076	2.057	1.815
3.85	0.55	0.88	4.508	2.69	2.181
2	0.2	0.44	2.869	0.8737	0.8658
2.5	0.25	0.55	3.242	1.059	1.038
3	0.3	0.66	3.601	1.303	1.251
4	0.4	0.88	4.472	2.377	1.978
2	0.1	0.42	2.863	0.6173	0.6151
3	0.15	0.63	3.583	0.9074	0.8894
4	0.2	0.84	4.28	1.548	1.42
4.4	0.22	0.924	4.675	2.267	1.858

Case 4: W: 24.0 in. and B: 12.0 in.

C1	C2	(C1+C2)/B	FE-K1(EDGE)	FE-K2(THRU)	FE-K3(THRU)
0.2	4	0.35	0.918	3.811	3.81
0.3	6	0.525	1.207	5.163	5.16
0.4	8	0.7	1.6404	7.115	7.106
0.5	10	0.875	3.386	11.27	11.18
0.4	4	0.3667	1.319	3.813	3.811
0.6	6	0.55	1.777	5.183	5.169
0.8	8	0.7333	2.655	7.165	7.096
1	10	0.9167	6.869	12.06	11.14
2	2	0.3333	3.006	2.599	2.572
3	3	0.5	4.03	3.394	3.253
4.5	4.5	0.75	6.425	5.335	4.361
5.5	5.5	0.9167	11.22	9.914	5.562
4	0.4	0.3667	4.61	1.212	1.2
6	0.6	0.55	6.548	1.771	1.7
8	0.8	0.7333	9.138	2.965	2.641
10	1	0.91667	13.77	7.381	5.311
4	0.2	0.35	4.605	0.8559	0.8517
6	0.3	0.525	6.527	1.234	1.212
8	0.4	0.7	9.021	2.013	1.902
10	0.5	0.875	12.69	4.434	3.804

Case 5: W: 24.0 in. and B: 6.0 in.

C1	C2	(C1+C2)/B	FE-K1(EDGE)	FE-K2(THRU)	FE-K3(THRU)
0.1	2	0.35	0.5021	2.663	2.624
0.15	3	0.525	0.7509	3.571	3.39
0.2	4	0.7	0.9333	4.846	4.234
0.25	5	0.875	2.107	7.426	5.379
0.2	2	0.3667	0.8735	2.664	2.625
0.3	3	0.55	1.297	3.573	3.391
0.4	4	0.7333	1.917	4.884	4.246
0.5	5	0.9167	4.849	7.964	5.432
1	1	0.3333	2.068	1.825	1.814
1.5	1.5	0.5	2.697	2.367	2.303
2.2	2.2	0.7333	3.931	3.452	3.039
2.7	2.7	0.9	6.137	5.595	3.831
2	0.2	0.3667	2.944	0.8573	0.8532
3	0.3	0.55	3.78	1.207	1.18
4	0.4	0.7333	4.685	1.806	1.684
5	0.5	0.9167	6.064	3.56	2.772
2	0.1	0.35	2.94	0.6055	0.6041
3	0.15	0.525	3.77	0.8469	0.8375
4	0.2	0.7	4.626	1.244	1.204
5	0.25	0.875	5.628	2.218	1.97

Case 6: W: 24.0 in. and B: 3.0 in.

C1	C2	(C1+C2)/B	FE-K1(EDGE)	FE-K2(THRU)	FE-K3(THRU)
0.05	1	0.35	0.269	1.857	1.837
0.07	1.4	0.49	0.3321	2.317	2.245
0.1	2	0.7	0.6053	3.188	2.884
0.13	2.6	0.91	1.407	4.933	3.698
0.1	1	0.3667	0.4632	1.858	1.838
0.15	1.5	0.55	0.6992	2.44	2.347
0.2	2	0.7333	1.205	3.2	2.887
0.24	2.4	0.88	2.068	4.223	3.395
0.5	0.5	0.3333	1.444	1.283	1.276
0.8	0.8	0.5333	1.939	1.719	1.669
1.1	1.1	0.7333	2.599	2.326	2.081
1.3	1.3	0.8667	3.469	3.177	2.431
1	0.1	0.3667	2.017	0.5961	0.5936
1.5	0.15	0.55	2.512	0.8161	0.7978
2	0.2	0.7333	2.984	1.178	1.101
2.4	0.24	0.88	3.465	1.841	1.543
1	0.05	0.35	2.016	0.4211	0.4203
1.4	0.07	0.49	2.412	0.5401	0.536
2	0.1	0.7	2.95	0.8151	0.7883
2.6	0.13	0.91	3.556	1.6	1.398

Case 7: W: 16.0 in. and B: 8.0 in.

C1	C2	(C1+C2)/B	FE-K1(EDGE)	FE-K2(THRU)	FE-K3(THRU)
0.15	3	0.39375	0.8039	3.363	3.362
0.2	4	0.525	0.9774	4.225	4.222
0.3	6	0.7875	1.746	7.09	7.069
0.35	7	0.9188	4.165	11.01	10.8
0.25	2.5	0.3438	1.037	2.988	2.989
0.4	4	0.55	1.449	4.234	4.222
0.6	6	0.825	3.058	7.218	7.056
0.66	6.6	0.9075	5.154	9.452	8.836
1.5	1.5	0.375	2.645	2.272	2.236
2	2	0.5	3.29	2.771	2.657
3	3	0.75	5.246	4.355	3.56
3.6	3.6	0.9	8.376	7.32	4.396
2.5	0.25	0.3438	3.59	0.9471	0.9399
4	0.4	0.55	5.349	1.446	1.388
6	0.6	0.825	8.929	3.509	2.906
6.6	0.66	0.9075	10.92	5.629	4.135
3	0.15	0.3938	4.12	0.7632	0.7574
4	0.2	0.525	5.328	1.009	0.9892
6	0.3	0.7875	8.69	2.304	2.102
7	0.35	0.9188	11.49	4.919	3.983

Case 8: W: 16.0 in. and B: 4.0 in.

C1	C2	(C1+C2)/B	FE-K1(EDGE)	FE-K2(THRU)	FE-K3(THRU)
0.07	1.4	0.3675	0.4353	2.244	2.204
0.1	2	0.525	0.6175	2.917	2.768
0.15	3	0.7875	0.9215	4.75	3.862
0.17	3.4	0.8925	2.146	6.447	4.514
0.12	1.2	0.33	0.6106	2.039	2.017
0.2	2	0.55	1.065	2.922	2.769
0.28	2.8	0.77	1.821	4.29	3.625
0.33	3.3	0.9075	3.589	6.241	4.369
0.6	0.6	0.3	1.59	1.406	1.4
1	1	0.5	2.202	1.932	1.88
1.4	1.4	0.7	3.022	2.648	2.386
1.8	1.8	0.9	5.008	4.569	3.128
1.2	0.12	0.33	2.261	0.6519	0.6497
2	0.2	0.55	3.089	0.9855	0.9639
2.8	0.28	0.77	3.985	1.634	1.496
3.3	0.33	0.9075	4.856	2.764	2.195
1.4	0.07	0.3675	2.467	0.5112	0.5096
2	0.1	0.525	3.079	0.6918	0.6842
3	0.15	0.7875	4.158	1.303	1.227
3.4	0.17	0.8925	4.702	1.977	1.732

B2. Beta Interaction Tables for Crack Tips in an Infinite Plate

Table 1: Beta Interaction values for Edge Crack Tip growing to Internal Crack

(C1+C2)/B	C1/C2=1/20	C1/C2=1/10	C1/C2=1/5	C1/C2=1/3	C1/C2=2/3	C1/C2=1
0	0.993	0.993	0.993	0.993	0.993	0.993
0.1	0.99951	0.99831	0.99707	0.996301	0.99573	0.9954
0.2	1.00613	1.00613	1.0043	1.00135	0.99998	0.9994
0.3	1.01743	1.02115	1.02442	1.017	1.01362	1.01019
0.4	1.04253	1.05098	1.0691	1.05389	1.04237	1.03155
0.5	1.08501	1.10345	1.13315	1.10889	1.08808	1.067
0.6	1.15242	1.19313	1.23	1.20281	1.15678	1.11987
0.7	1.30341	1.36916	1.40901	1.3661	1.26689	1.20884
0.75	1.43076	1.54351	1.5601	1.49016	1.36032	1.28401
0.8	1.65691	1.79524	1.77338	1.67176	1.48861	1.36593
0.85	2.03257	2.14075	2.08176	1.93382	1.62864	1.485
0.9	2.96316	2.9351	2.6969	2.39286	1.93329	1.71
0.95	5.22	4.9	4.02	3.39656	2.67752	2.31711
1	9.9	9.2	7	5.5	4.3	3.5

Table 1: Continued

(C1+C2)/B	C1/C2=1.5	C1/C2=3	C1/C2=5	C1/C2=6	C1/C2=8	C1/C2=10	C1/C2=20
0	0.993	0.993	0.993	0.993	0.993	0.993	0.993
0.1	0.995	0.9945	0.993929	0.99366	0.99354	0.993388	0.992983
0.2	0.9985	0.997	0.996	0.9955	0.994996	0.994532	0.994142
0.3	1.00628	1.00193	0.999	0.998164	0.997419	0.996976	0.996
0.4	1.02199	1.00871	1.00374	1.00236	1.001	1	0.998
0.5	1.04903	1.02307	1.01221	1.00866	1.0051	1.0035	1
0.6	1.08553	1.04334	1.024	1.01729	1.01084	1.00827	1.0023
0.7	1.1465	1.07313	1.04039	1.03175	1.02	1.01635	1.007
0.75	1.20022	1.10466	1.05629	1.04321	1.03	1.024	1.00993
0.8	1.2743	1.14147	1.08047	1.06153	1.04	1.034	1.013
0.85	1.3539	1.18942	1.1131	1.09438	1.05819	1.04682	1.01758
0.9	1.54024	1.32089	1.19764	1.15788	1.11358	1.08379	1.03511
0.95	2.01491	1.58	1.385	1.33	1.252	1.19	1.09
1	2.7	1.95	1.63	1.54	1.43	1.35	1.25

Table 2: Beta Interaction values for Through Crack Tip growing to Edge Crack

(C1+C2)/B	C1/C2=1/20	C1/C2=1/10	C1/C2=1/5	C1/C2=1/3	C1/C2=2/3	C1/C2=1
0	1	1	1	1	1	1
0.1	1.00014	1.00019	1.00028	1.0004	1.00098	1.0012
0.2	1.00023	1.00039	1.0008	1.00113	1.00276	1.0033
0.3	1.00029	1.0007	1.0014	1.003	1.0073	1.0115
0.4	1.00041	1.00137	1.00236	1.00719	1.01925	1.03086
0.5	1.0007	1.00261	1.00731	1.01525	1.04239	1.06389
0.6	1.00124	1.00395	1.01416	1.03158	1.07682	1.11523
0.7	1.00234	1.00627	1.02309	1.05852	1.13788	1.21715
0.75	1.00307	1.01048	1.03889	1.07807	1.20028	1.29
0.8	1.00441	1.01662	1.06567	1.12455	1.28	1.405
0.85	1.00631	1.02415	1.09225	1.20355	1.39	1.56
0.9	1.01689	1.05747	1.17563	1.3274	1.61177	1.90564
0.95	1.052	1.149	1.4	1.70787	2.27015	2.66897
1	1.1	1.27	1.9	2.4	3.5	4.5

Table 2: Continued

(C1+C2)/B	C1/C2=1.5	C1/C2=3	C1/C2=5	C1/C2=8	C1/C2=20
0	1	1	1	1	1
0.1	1.00158	1.0021	1.00255	1.00271	1.00294
0.2	1.00414	1.007	1.011	1.01415	1.01648
0.3	1.01533	1.023	1.033	1.0416	1.0519
0.4	1.04318	1.06501	1.08578	1.10017	1.12112
0.5	1.0915	1.14191	1.1743	1.20175	1.23516
0.6	1.16547	1.26011	1.32316	1.35439	1.41585
0.7	1.29426	1.46181	1.57065	1.65818	1.78199
0.75	1.41243	1.64214	1.78119	1.91816	2.08741
0.8	1.57	1.85506	2.07403	2.24524	2.50819
0.85	1.75	2.13025	2.44465	2.65539	3.08189
0.9	2.17	2.76724	3.19055	3.54362	4.06
0.95	3.20037	4.2	4.85	5.3115	6.25
1	5.9	7.5	8.8	10	12.5

Table 3: Beta Interaction values for Through Crack Tip growing to Specimen Edge

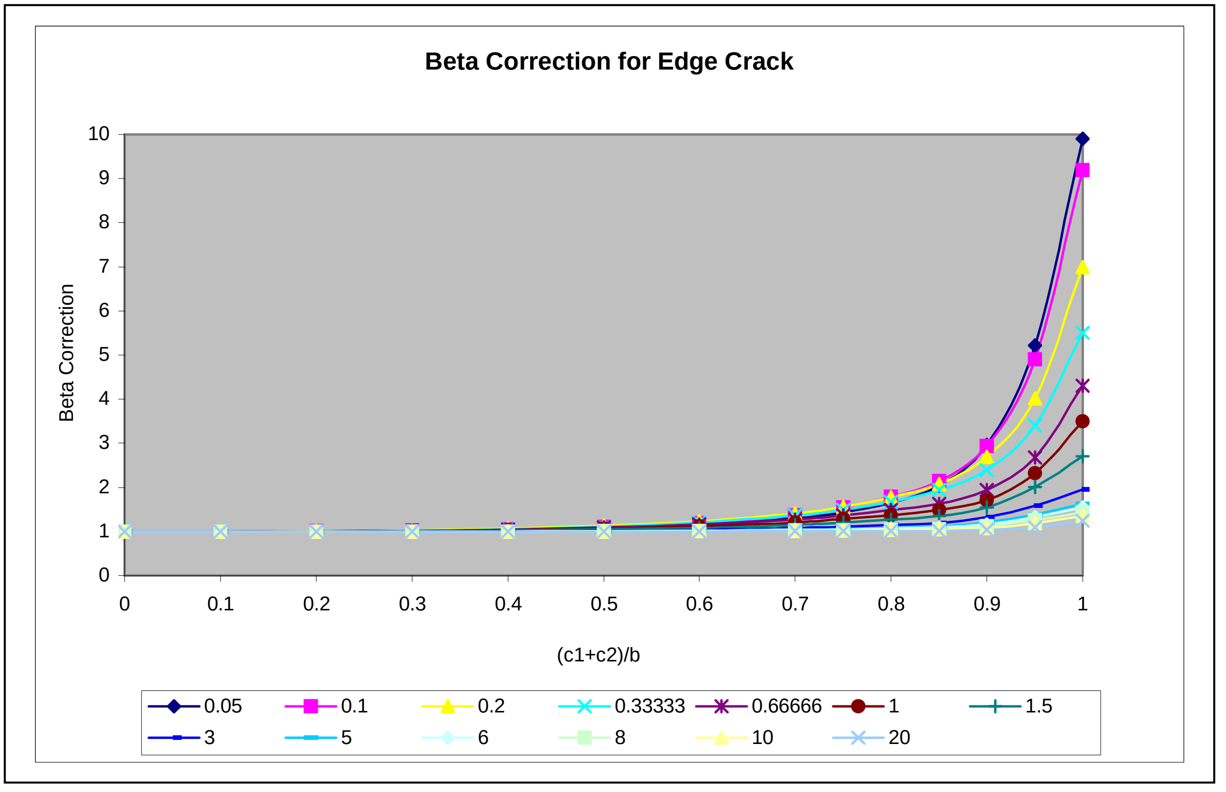
(C1+C2)/B	C1/C2=1/5	C1/C2=1/3	C1/C2=2/3	C1/C2=1	C1/C2=1.5
0	1	1	1	1	1
0.1	1.0000002	1.000015	1.00009	1.00016	1.000559
0.2	1.0000008	1.00005	1.000274	1.001303	1.003409
0.3	1.0000025	1.0001	1.000897	1.004	1.008531
0.4	1.000008	1.00018	1.002549	1.008726	1.018278
0.5	1.00002	1.000294	1.006	1.016	1.035
0.6	1.00004	1.0005	1.011709	1.027009	1.057657
0.7	1.00011	1.00115	1.019373	1.046	1.097119
0.75	1.00021	1.001966	1.025825	1.060217	1.123845
0.8	1.0005	1.003402	1.03505	1.077158	1.155521
0.85	1.0012	1.006	1.045118	1.099473	1.198498
0.9	1.00256	1.012386	1.064104	1.144044	1.273664
0.95	1.01	1.026181	1.107346	1.220789	1.407649
1	1.023	1.048	1.22	1.45	1.7

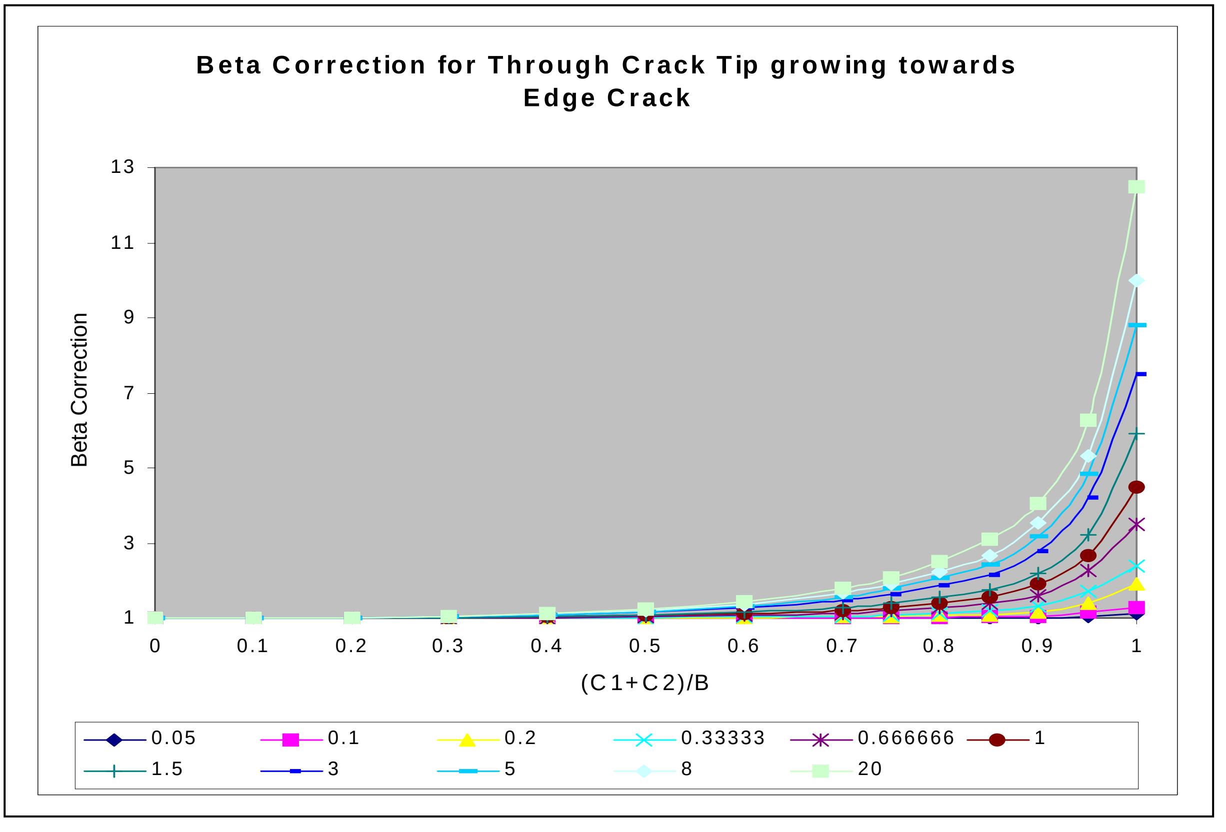
Table 3: Continued

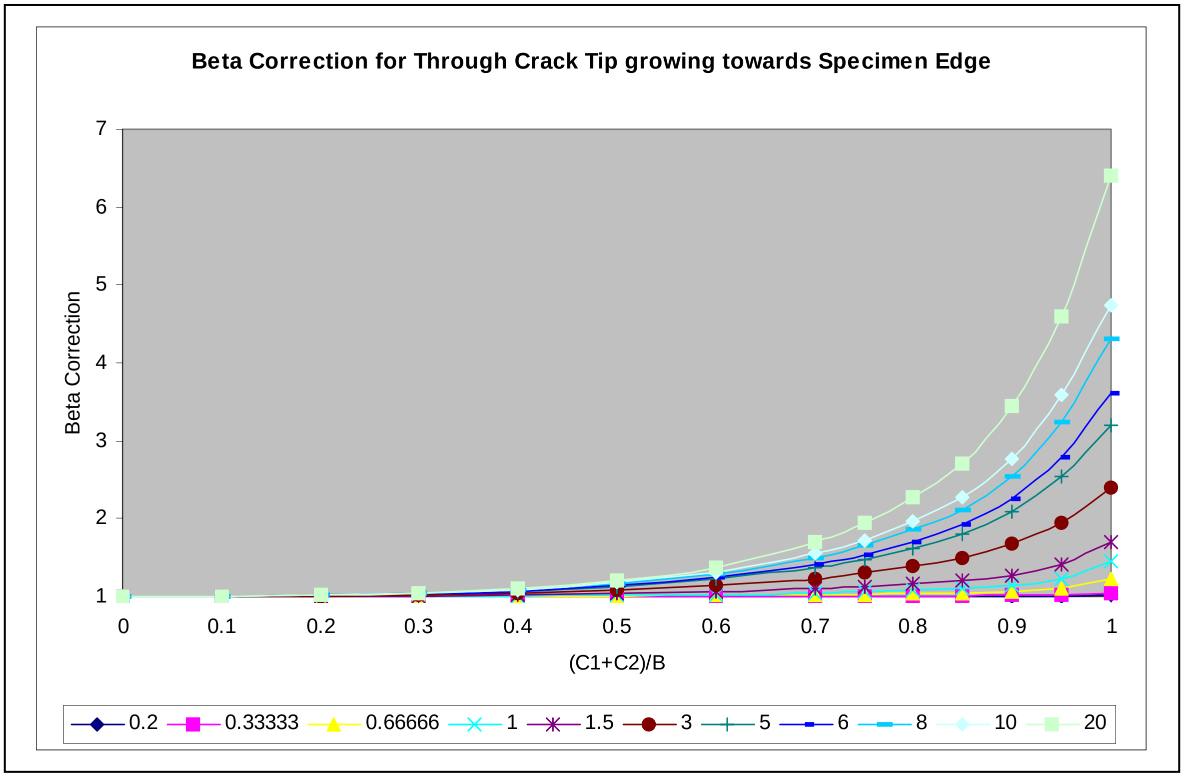
(C1+C2)/B	C1/C2=3	C1/C2=5	C1/C2=6	C1/C2=8	C1/C2=10	C1/C2=20
0	1	1	1	1	1	1
0.1	1.00085	1.000777	1.002395	1.001784	1.002528	1.002675
0.2	1.008522	1.010327	1.011077	1.012259	1.013208	1.015834
0.3	1.022467	1.031549	1.032108	1.035893	1.04004	1.048637
0.4	1.0432	1.064684	1.071232	1.081467	1.09105	1.109513
0.5	1.082366	1.122442	1.136954	1.159119	1.176863	1.212295
0.6	1.143129	1.217684	1.242317	1.279111	1.311285	1.379544
0.7	1.230906	1.3674	1.415543	1.487918	1.545934	1.691094
0.75	1.300456	1.476493	1.54056	1.648502	1.728547	1.935226
0.8	1.387508	1.617956	1.706339	1.8533	1.968324	2.264025
0.85	1.497517	1.804192	1.932414	2.11507	2.278173	2.711743
0.9	1.674273	2.096401	2.259496	2.54322	2.758954	3.438565
0.95	1.95	2.55	2.78	3.25	3.58	4.6
1	2.4	3.2	3.6	4.3	4.75	6.4

B3. Characteristic Plots for the Edge-Through Crack Case

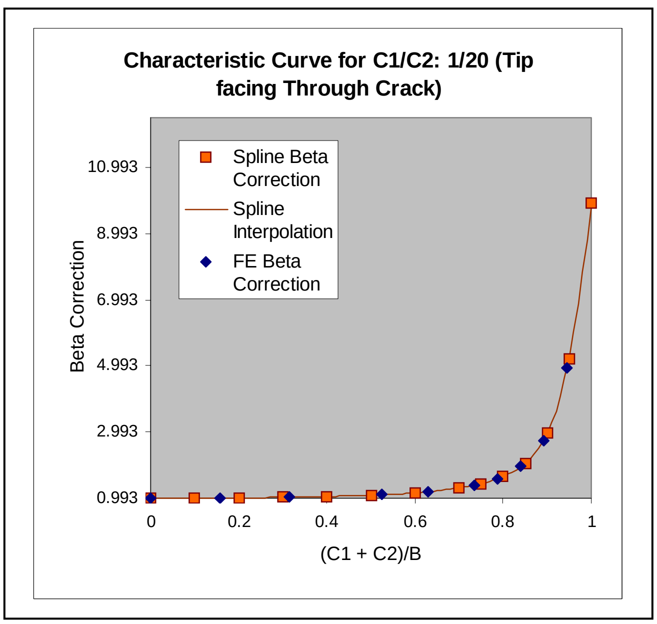
B3.1 (C1+C2)/B vs. Beta Correction for various C1/C2 Ratios

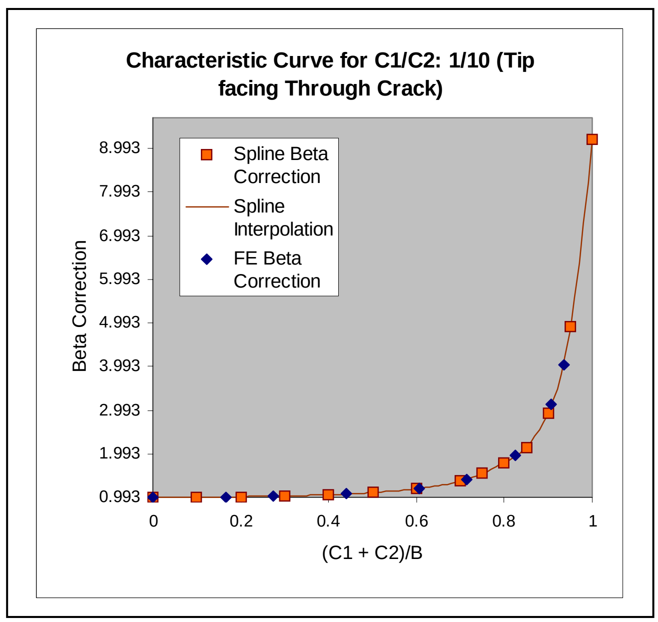


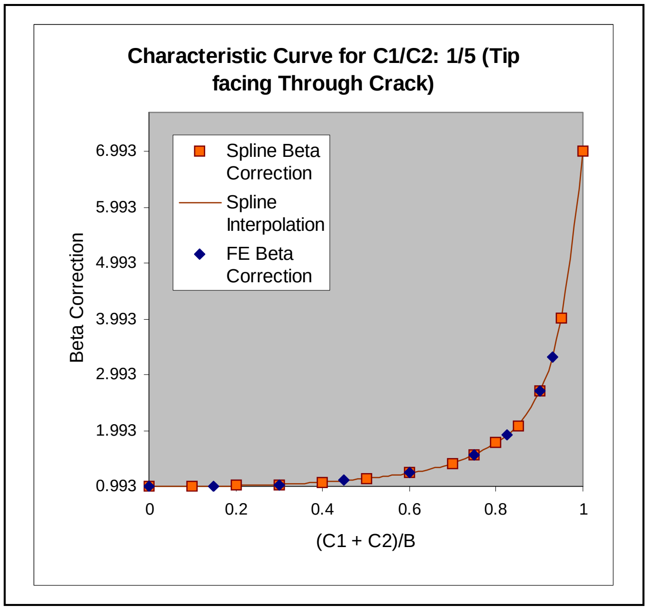


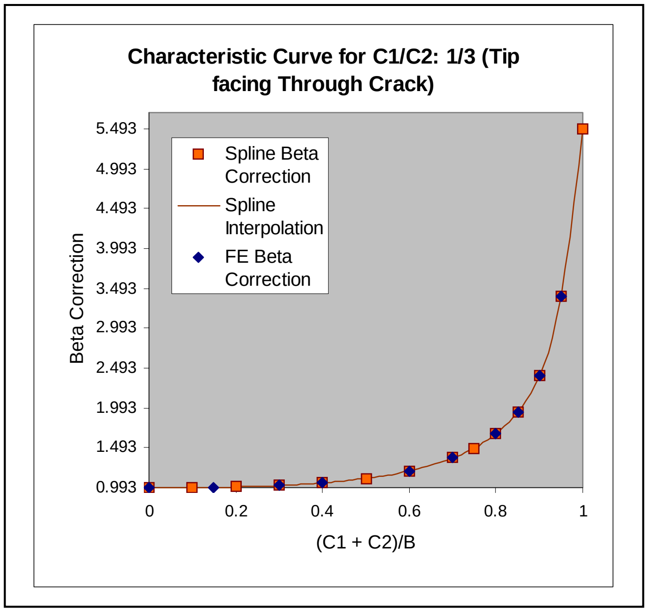


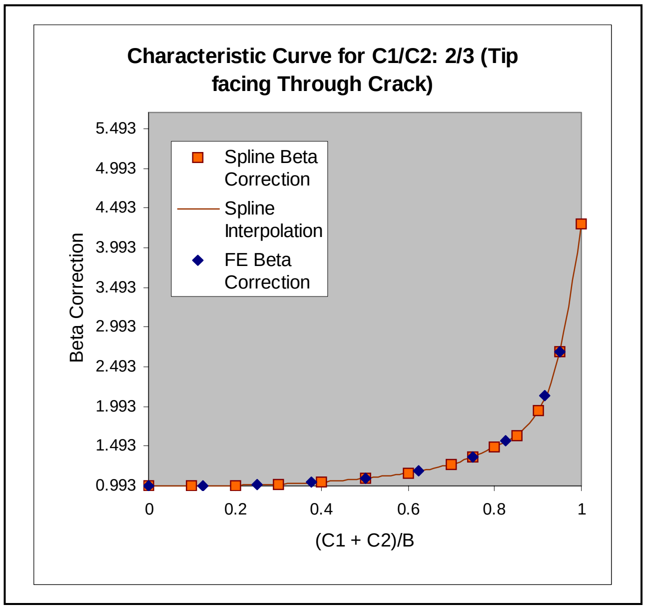
B3.2 Spline Interpolation vs. FEA solution for various C1/C2 ratios

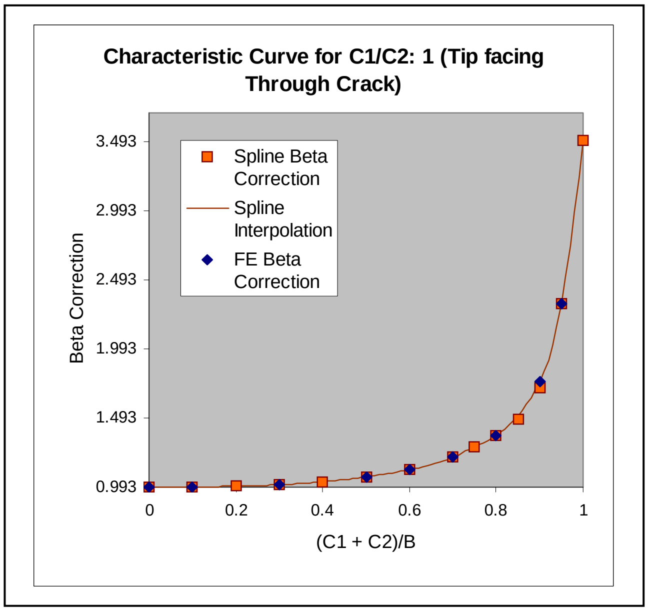


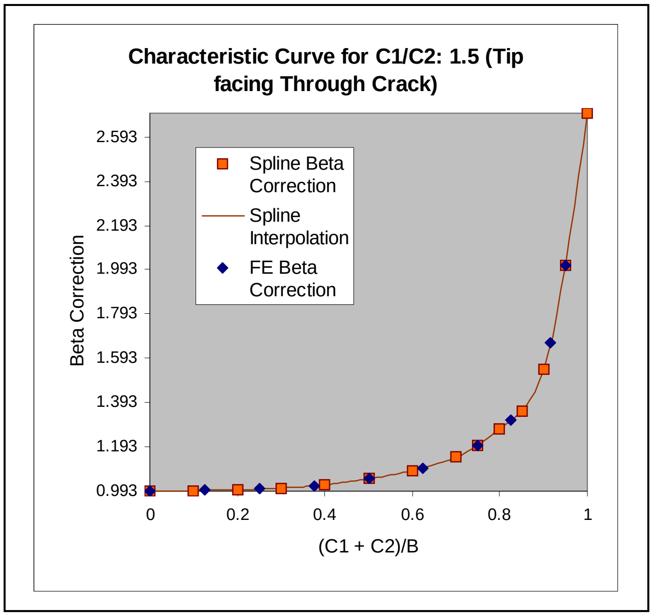


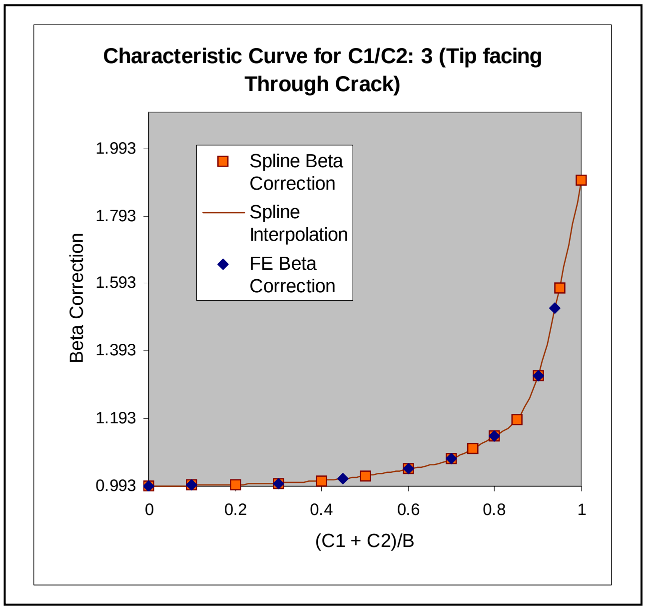

Characteristic Curve for C1/C2: 1/5 (Tip facing Through Crack)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B
6.993
5.993
4.993
3.993
2.993
1.993
0.993
0
0.2
0.4
0.6
0.8
1

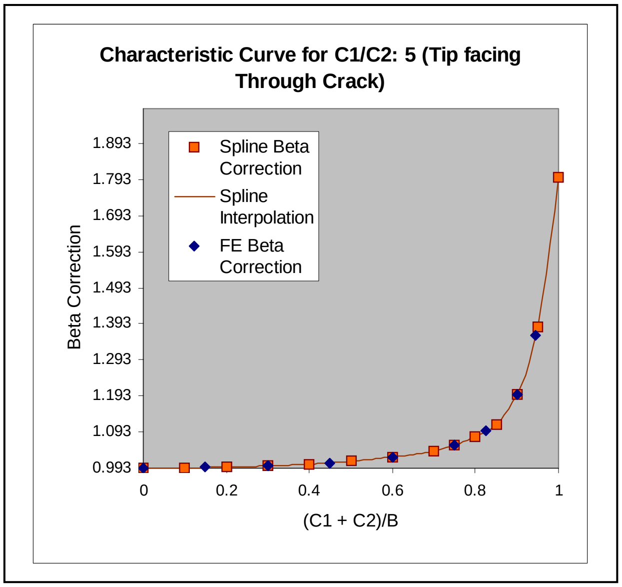

Characteristic Curve for C1/C2: 1/3 (Tip facing Through Crack)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B
5.493
4.993
4.493
3.993
3.493
2.993
2.493
1.993
1.493
0.993
0
0.2
0.4
0.6
0.8
1

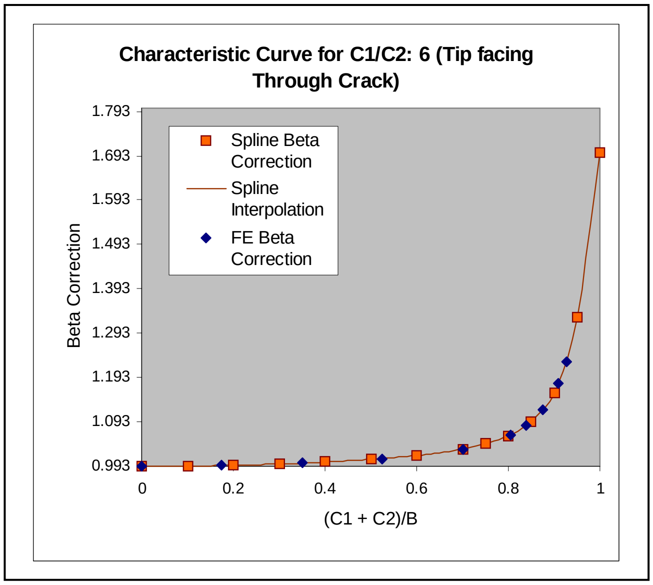

Characteristic Curve for C1/C2: 2/3 (Tip facing Through Crack)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B
5.493
4.993
4.493
3.993
3.493
2.993
2.493
1.993
1.493
0.993
0
0.2
0.4
0.6
0.8
1

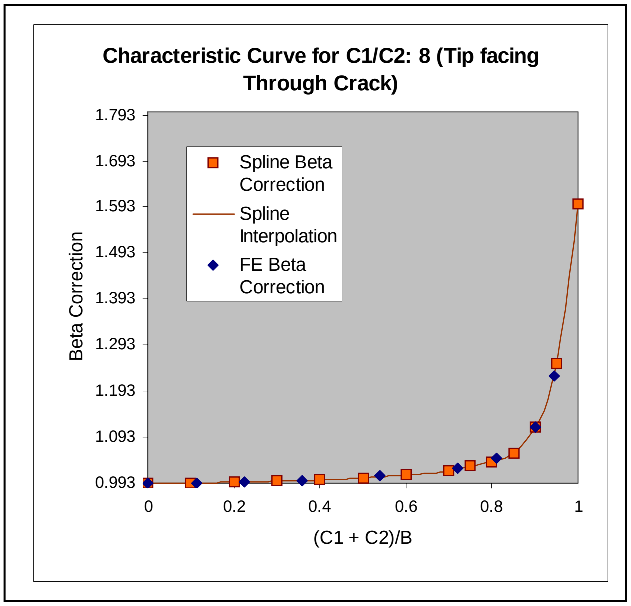

Characteristic Curve for C1/C2: 1 (Tip facing Through Crack)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B
3.493
2.993
2.493
1.993
1.493
0.993
0
0.2
0.4
0.6
0.8
1

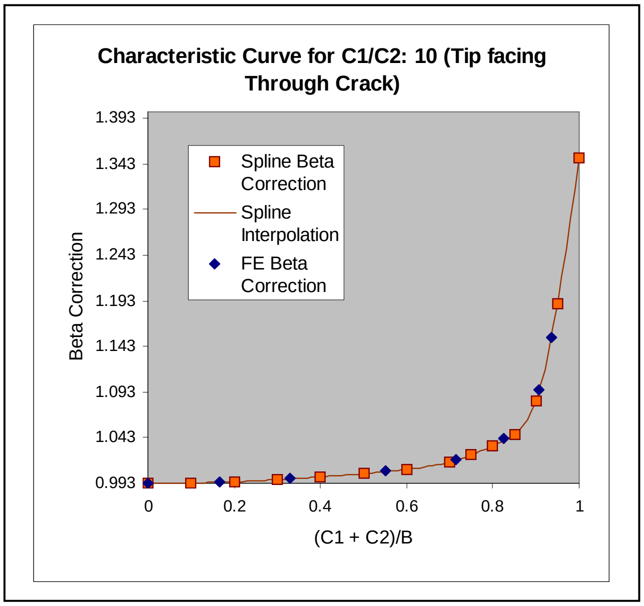

Characteristic Curve for C1/C2: 1.5 (Tip facing Through Crack)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B
2.593
2.393
2.193
1.993
1.793
1.593
1.393
1.193
0.993
0
0.2
0.4
0.6
0.8
1

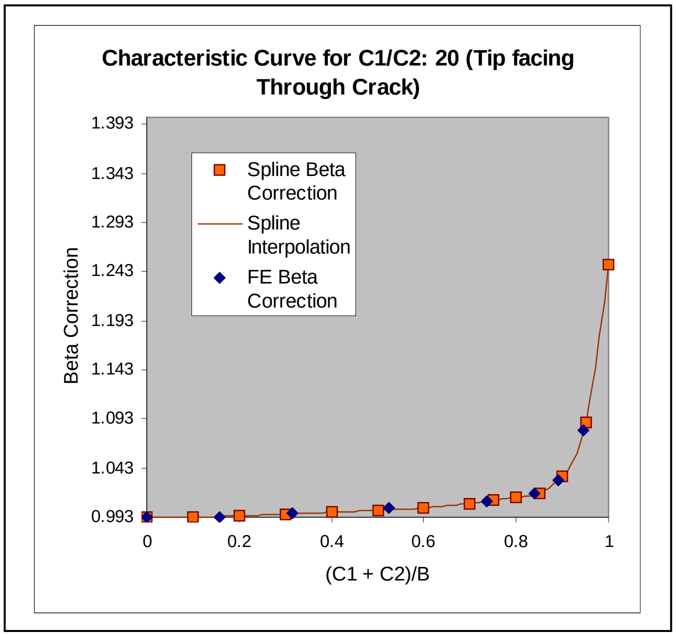

Characteristic Curve for C1/C2: 3 (Tip facing Through Crack)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B
1.993
1.793
1.593
1.393
1.193
0.993
0
0.2
0.4
0.6
0.8
1

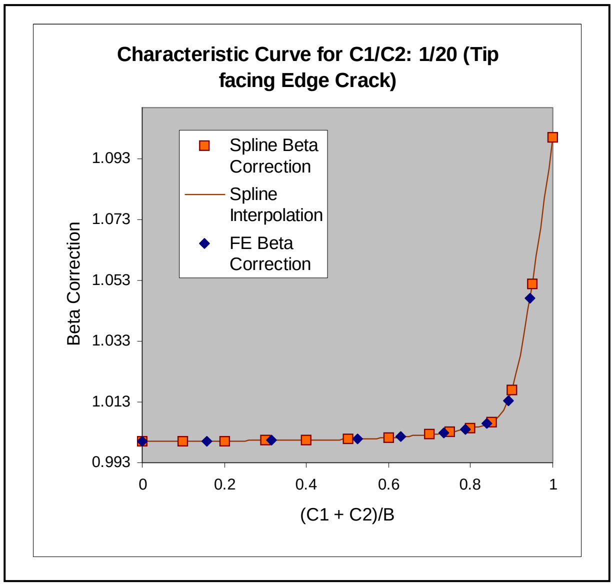

Characteristic Curve for C1/C2: 5 (Tip facing Through Crack)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B
1.893
1.793
1.693
1.593
1.493
1.393
1.293
1.193
1.093
0.993
0
0.2
0.4
0.6
0.8
1

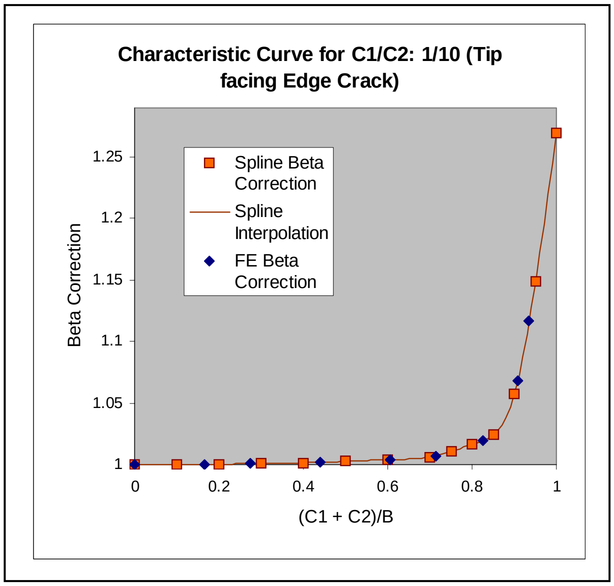

Characteristic Curve for C1/C2: 6 (Tip facing Through Crack)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B
1.793
1.693
1.593
1.493
1.393
1.293
1.193
1.093
0.993
0
0.2
0.4
0.6
0.8
1

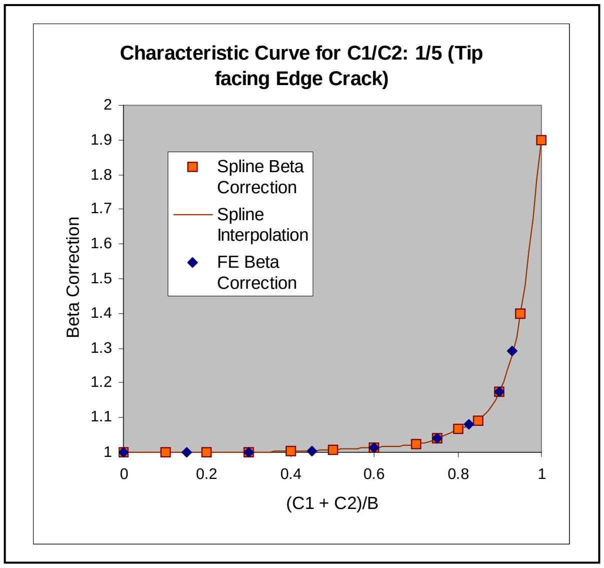

Characteristic Curve for C1/C2: 8 (Tip facing Through Crack)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B
1.793
1.693
1.593
1.493
1.393
1.293
1.193
1.093
0.993
0
0.2
0.4
0.6
0.8
1

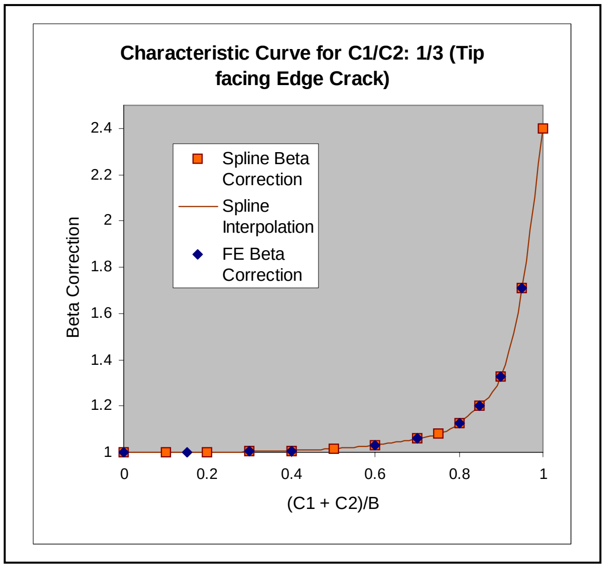

Characteristic Curve for C1/C2: 10 (Tip facing Through Crack)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B
1.393
1.343
1.293
1.243
1.193
1.143
1.093
1.043
0.993
0
0.2
0.4
0.6
0.8
1

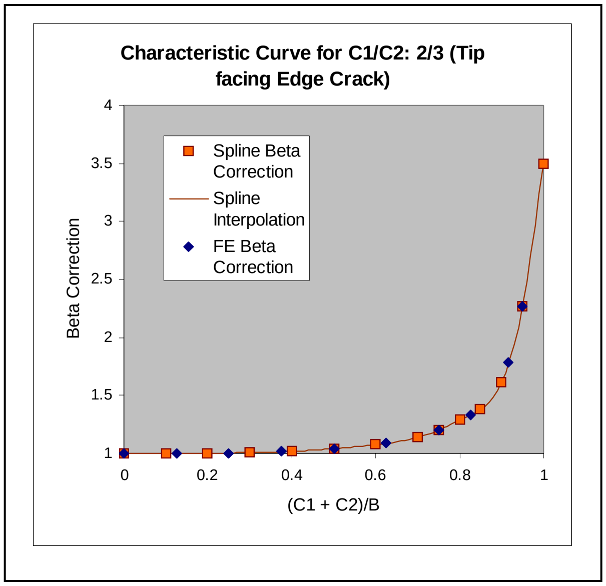

Characteristic Curve for C1/C2: 20 (Tip facing Through Crack)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B
1.393
1.343
1.293
1.243
1.193
1.143
1.093
1.043
0.993
0
0.2
0.4
0.6
0.8
1

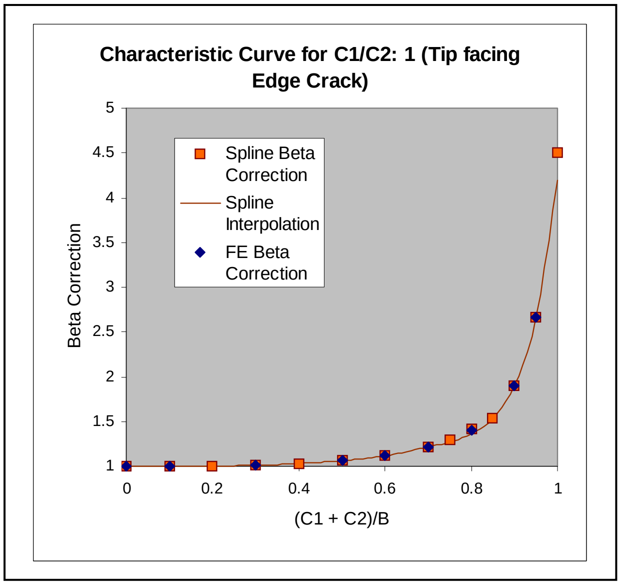

Characteristic Curve for C1/C2: 1/20 (Tip facing Edge Crack)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B

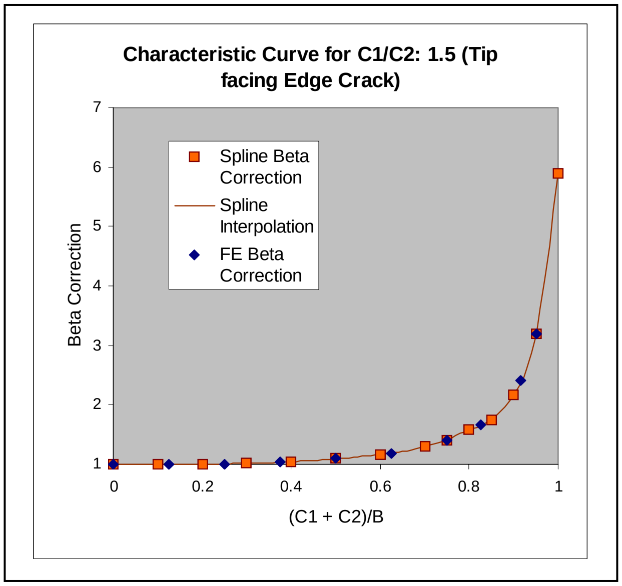

Characteristic Curve for C1/C2: 1/10 (Tip facing Edge Crack)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B

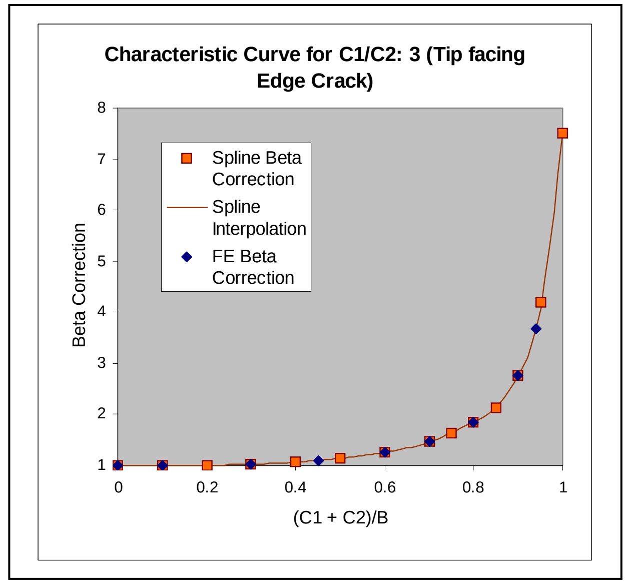

Characteristic Curve for C1/C2: 1/5 (Tip facing Edge Crack)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B

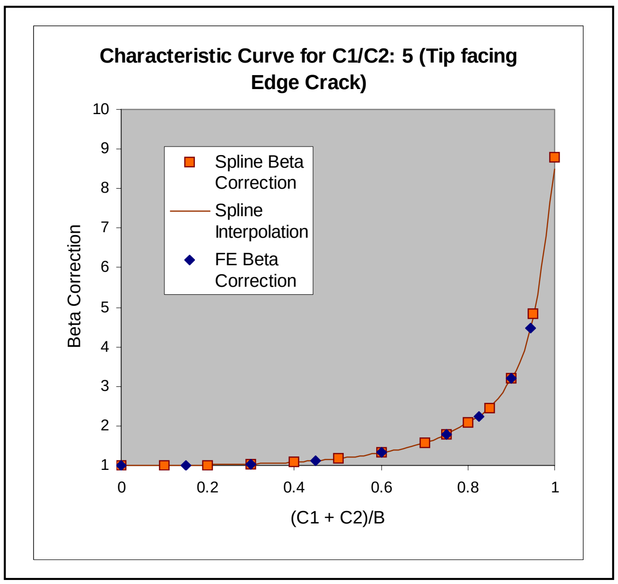

Characteristic Curve for C1/C2: 1/3 (Tip facing Edge Crack)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B

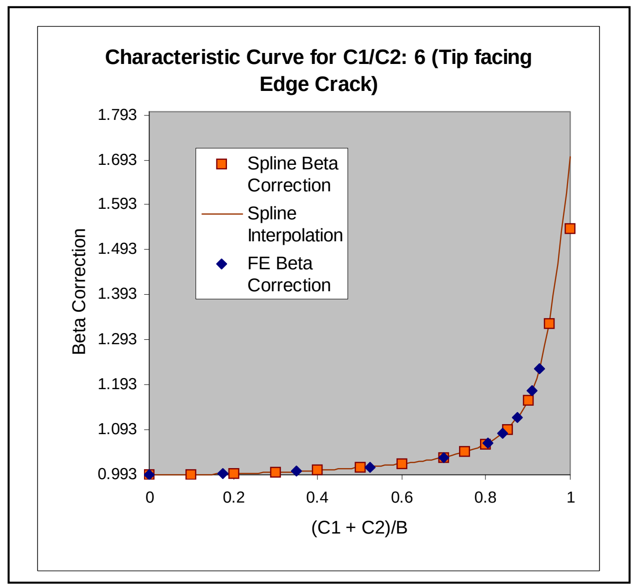

Characteristic Curve for C1/C2: 2/3 (Tip facing Edge Crack)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B

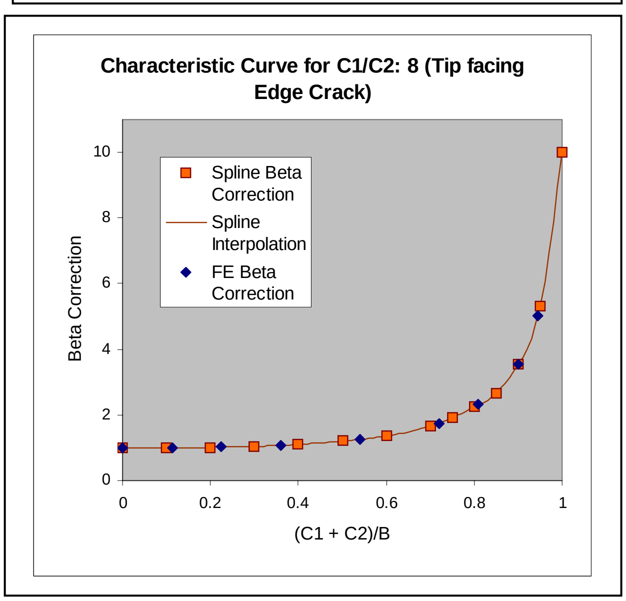

Characteristic Curve for C1/C2: 1 (Tip facing Edge Crack)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B

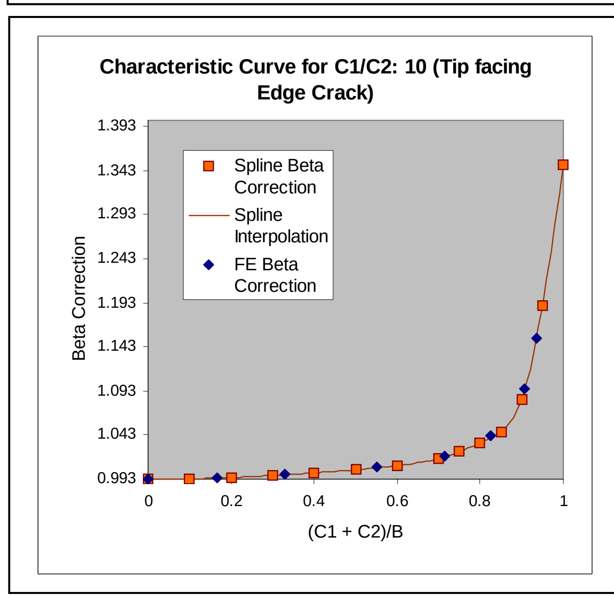

Characteristic Curve for C1/C2: 1.5 (Tip facing Edge Crack)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B
1
2
3
4
5
6
7
0
0.2
0.4
0.6
0.8
1

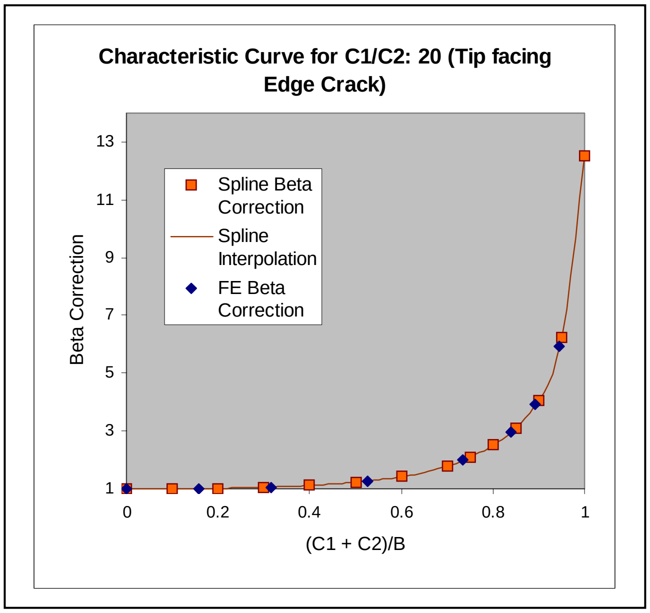

Characteristic Curve for C1/C2: 3 (Tip facing Edge Crack)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B
1
2
3
4
5
6
7
8
0
0.2
0.4
0.6
0.8
1

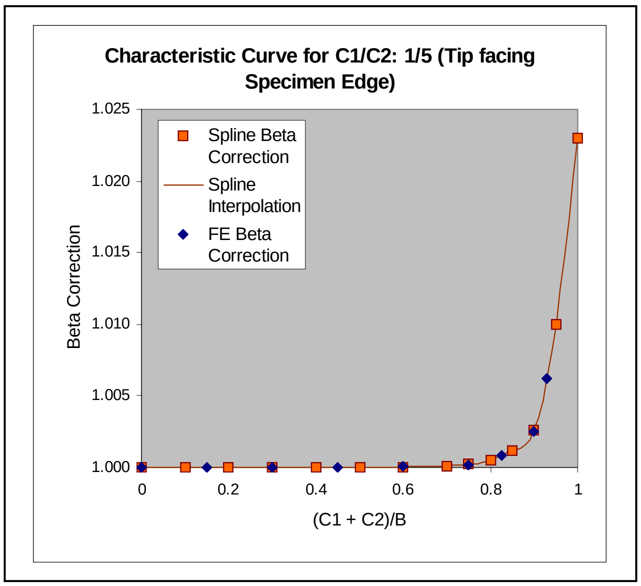

Characteristic Curve for C1/C2: 5 (Tip facing Edge Crack)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B
1
2
3
4
5
6
7
8
9
10
0
0.2
0.4
0.6
0.8
1

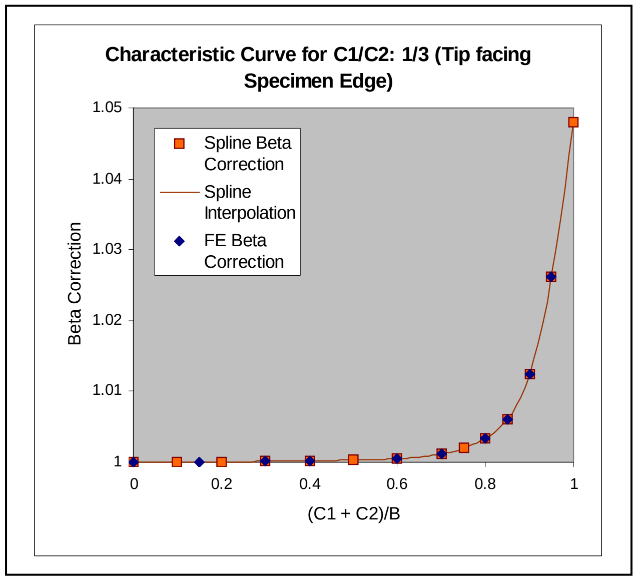

Characteristic Curve for C1/C2: 6 (Tip facing Edge Crack)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B
0.993
1.093
1.193
1.293
1.393
1.493
1.593
1.693
1.793
0
0.2
0.4
0.6
0.8
1

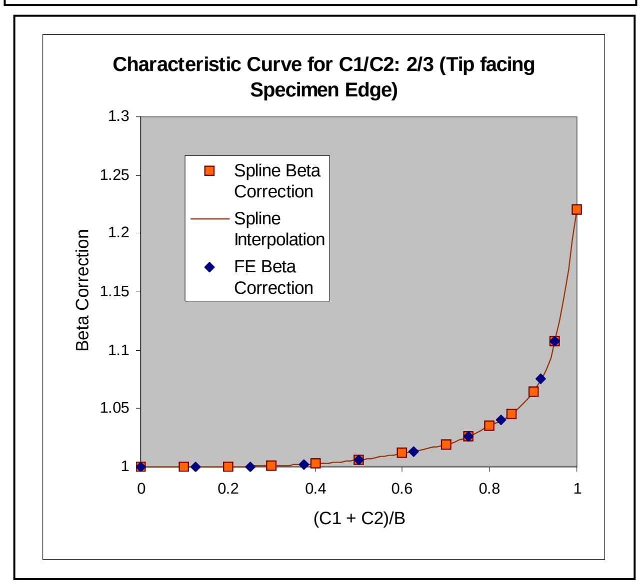

Characteristic Curve for C1/C2: 8 (Tip facing Edge Crack)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B
0
2
4
6
8
10
0
0.2
0.4
0.6
0.8
1

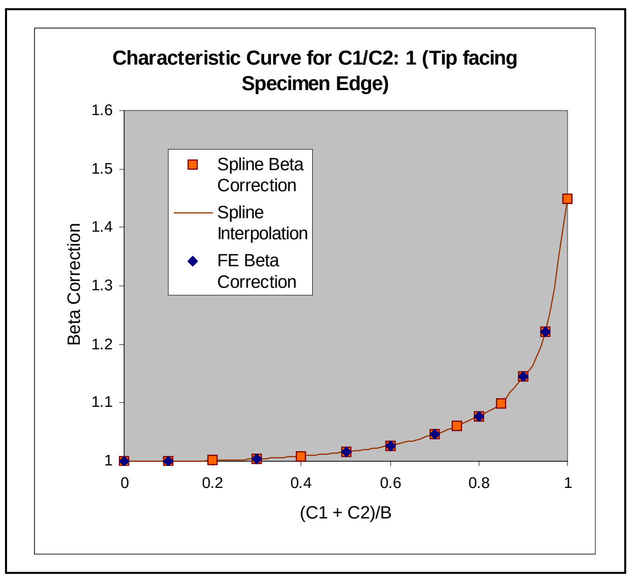

Characteristic Curve for C1/C2: 10 (Tip facing Edge Crack)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B
0.993
1.043
1.093
1.143
1.193
1.243
1.293
1.343
1.393
0
0.2
0.4
0.6
0.8
1

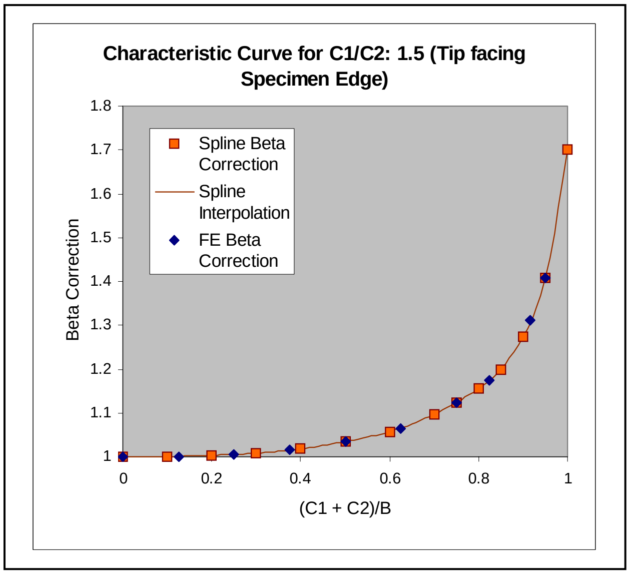

Characteristic Curve for C1/C2: 20 (Tip facing Edge Crack)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B

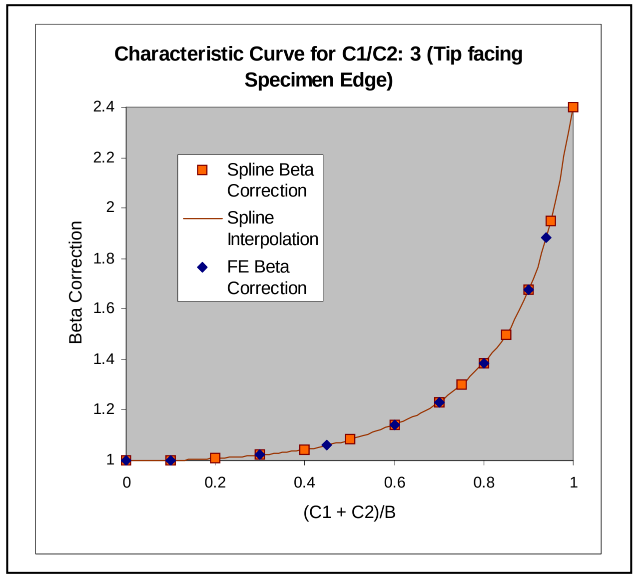

Characteristic Curve for C1/C2: 1/5 (Tip facing Specimen Edge)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B

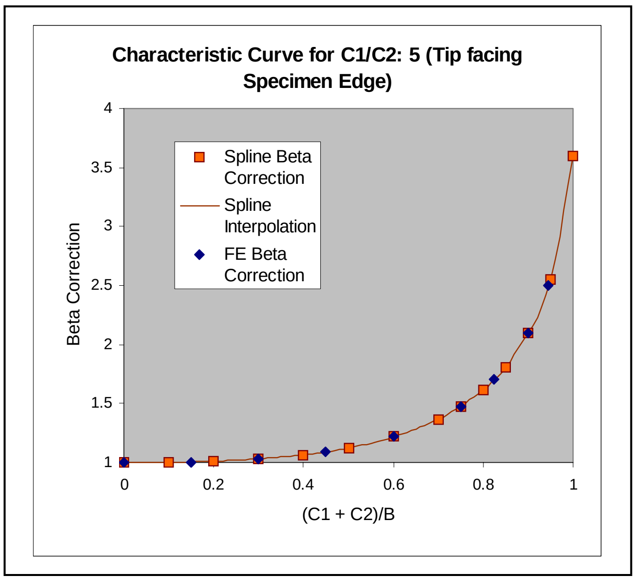

Characteristic Curve for C1/C2: 1/3 (Tip facing Specimen Edge)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B

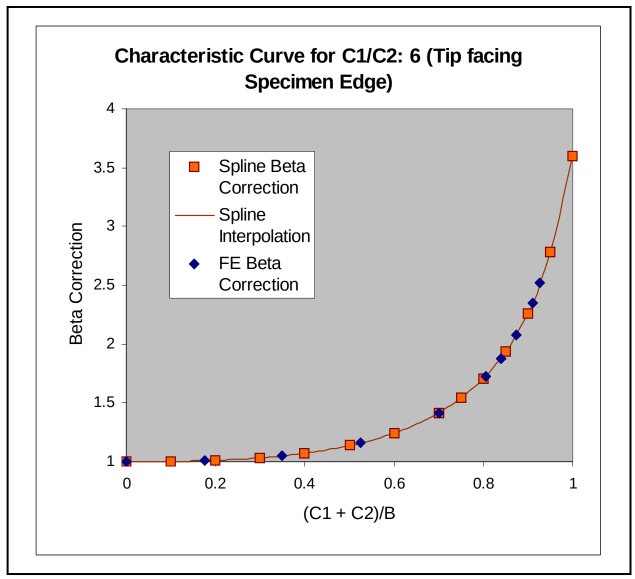

Characteristic Curve for C1/C2: 2/3 (Tip facing Specimen Edge)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B

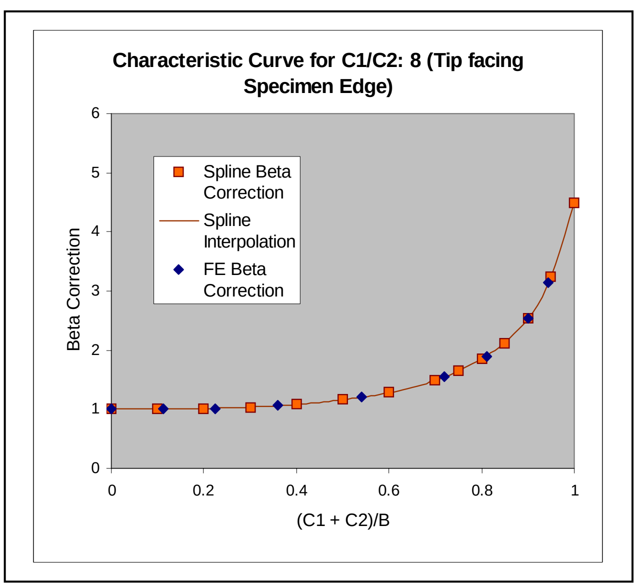

Characteristic Curve for C1/C2: 1 (Tip facing Specimen Edge)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B

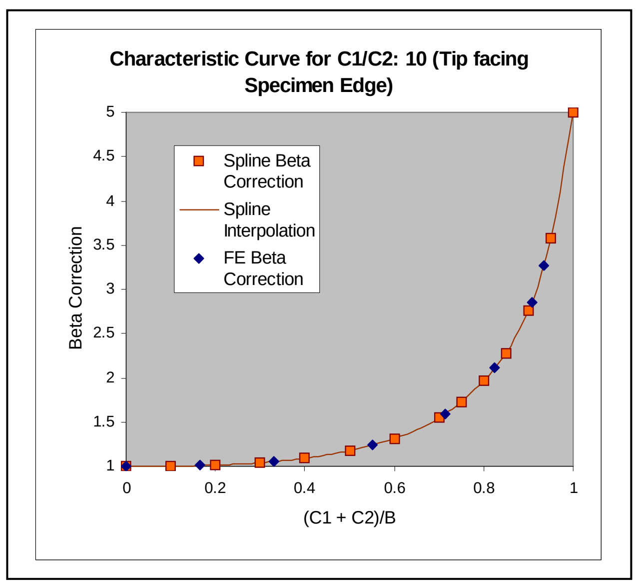

Characteristic Curve for C1/C2: 1.5 (Tip facing Specimen Edge)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B

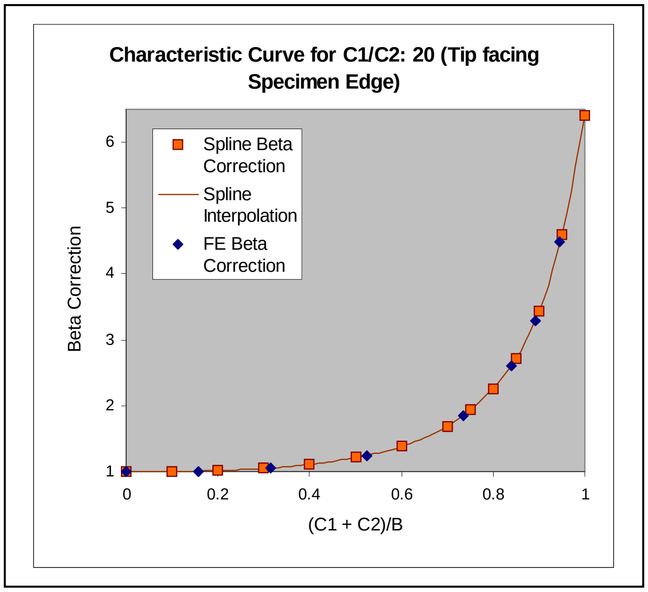

Characteristic Curve for C1/C2: 3 (Tip facing Specimen Edge)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B


Characteristic Curve for C1/C2: 5 (Tip facing Specimen Edge)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B


Characteristic Curve for C1/C2: 6 (Tip facing Specimen Edge)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B


Characteristic Curve for C1/C2: 8 (Tip facing Specimen Edge)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B


Characteristic Curve for C1/C2: 10 (Tip facing Specimen Edge)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B



Characteristic Curve for C1/C2: 20 (Tip facing Specimen Edge)
Spline Beta Correction
Spline Interpolation
FE Beta Correction
Beta Correction
(C1 + C2)/B
0
0.2
0.4
0.6
0.8
1
1
2
3
4
5
6

B4. Comparison of StressCheck and AFGROW Codes

Case 1: W: 80.0 in. and B: 40.0 in.

C1	C2	(C1+C2)/B	FE-K1	FE-K2	FE-K3	AFG-K1	AFG-K2	AFG-K3
32	4	0.9	23.6	12.65	9.073	23.54	12.64	9.069
20	10	0.75	13.65	8.73	6.906	13.6	8.806	6.914

Case 2: W: 12.0 in. and B: 6.0 in.

C1	C2	(C1+C2)/B	FE-K1	FE-K2	FE-K3	AFG-K1	AFG-K2	AFG-K3
0.5	3.5	0.6666667	1.89	4.291	4.248	1.888	4.283	4.244
3.5	0.5	0.6666667	5.501	1.923	1.74	5.497	1.906	1.735

Case 3: W: 12.0 in. and B: 4.0 in.

C1	C2	(C1+C2)/B	FE-K1	FE-K2	FE-K3	AFG-K1	AFG-K2	AFG-K3
0.5	2	0.625	1.875	3.005	2.796	1.768	3.014	2.852
2	0.5	0.625	3.372	1.658	1.55	3.369	1.659	1.534

Case 4: W: 12.0 in. and B: 3.0 in.

C1	C2	(C1+C2)/B	FE-K1	FE-K2	FE-K3	AFG-K1	AFG-K2	AFG-K3
0.45	1.35	0.6	1.676	2.377	2.255	1.621	2.404	2.299
1.35	0.45	0.6	2.595	1.461	1.382	2.603	1.47	1.368

Case 5: W: 8.0 in. and B: 4.0 in.

C1	C2	(C1+C2)/B	FE-K1	FE-K2	FE-K3	AFG-K1	AFG-K2	AFG-K3
0.3	3	0.825	2.168	5.101	4.992	2.143	5.084	4.991
3	0.3	0.825	6.317	2.477	2.054	6.303	2.417	2.056
3.2	0.4	0.9	7.465	4	2.866	7.443	3.996	2.868
2	1	0.75	4.318	2.759	2.183	4.299	2.785	2.186

Case 6: W: 8.0 in. and B: 3.0 in.

C1	C2	(C1+C2)/B	FE-K1	FE-K2	FE-K3	AFG-K1	AFG-K2	AFG-K3
0.2	1	0.4	0.9807	1.896	1.861	0.9683	1.901	1.847
1	0.2	0.4	2.186	0.8677	0.857	2.186	0.8629	0.8453

Case 7: W: 8.0 in. and B: 2.0 in.

C1	C2	(C1+C2)/B	FE-K1	FE-K2	FE-K3	AFG-K1	AFG-K2	AFG-K3
0.25	0.5	0.375	1.058	1.307	1.294	1.047	1.31	1.284
0.5	0.25	0.375	1.47	0.9326	0.9227	1.423	0.9301	0.9116

Case 8: W: 4.0 in. and B: 2.0 in.

C1	C2	(C1+C2)/B	FE-K1	FE-K2	FE-K3	AFG-K1	AFG-K2	AFG-K3
0.5	0.5	0.5	1.645	1.385	1.328	1.643	1.385	1.323
0.75	0.75	0.75	2.622	2.177	1.783	2.64	2.167	1.781
1.6	0.2	0.9	5.288	2.824	2.026	5.263	2.826	2.028
1	0.5	0.75	3.053	1.95	1.54	3.04	1.969	1.546

Case 9: W: 4.0 in. and B: 1.5 in.

C1	C2	(C1+C2)/B	FE-K1	FE-K2	FE-K3	AFG-K1	AFG-K2	AFG-K3
0.3	0.45	0.5	1.257	1.302	1.262	1.202	1.308	1.279
0.45	0.3	0.5	1.52	1.085	1.044	1.511	1.092	1.053
0.5	0.6	0.73333	2.03	1.839	1.581	1.983	1.869	1.564
1	0.25	0.83333	3.015	1.819	1.431	2.981	1.876	1.419

Case 10: W: 4.0 in. and B: 1.0 in.

C1	C2	(C1+C2)/B	FE-K1	FE-K2	FE-K3	AFG-K1	AFG-K2	AFG-K3
0.25	0.25	0.5	1.099	0.9648	0.9362	1.06	0.9734	0.9455
0.45	0.3	0.75	1.734	1.347	1.165	1.708	1.411	1.138
0.5	0.25	0.75	1.775	1.267	1.096	1.768	1.346	1.077

Appendix C
Unequal Edge Cracks in a Plate

C1. Cases

Case 1: W: 40.0 in.

C1	C2	(C1+C2)/W	FE-K1	FE-K2
0.3	6	0.1575	0.8713	5.439
0.7	14	0.3675	-0.3731	12.34
0.9	18	0.4725	-2.692	18.35
1.2	24	0.63	-12.48	35.72
1.5	30	0.7875	-53.86	89.03
1.6	32	0.84	-97.659	138.31
1.7	34	0.8925	-205.6	253.4
1.81	36.2	0.95025	-750.804	810.454
0.64	6.4	0.176	1.258	5.76
1	10	0.275	0.9149	8.399
1.5	15	0.4125	-0.9309	13.66
1.7	17	0.4675	-2.425	16.66
2.5	25	0.6875	-20.92	42.86
3	30	0.825	-77.69	107.3
3.15	31.5	0.86625	-129.104	161.976
3.2	32	0.88	-158.354	192.77
3.3	33	0.9075	-247.1	285
3.46	34.6	0.9515	-710.346	757.544
1	7	0.2	1.489	6.151
1.5	10.5	0.3	1.029	8.812
2	14	0.4	-0.3146	12.36
2.4	16.8	0.48	-2.424	16.42
3	21	0.6	-8.954	26.25
3.5	24.5	0.7	-21.535	42.081
4	28	0.8	-54.93	80.15
4.25	29.75	0.85	-96.232	124.383
4.5	31.5	0.9	-198.2	232.3
4.75	33.25	0.95	-620.443	663.61
1	5	0.15	1.739	4.846
2	10	0.3	1.424	8.358
2.5	12.5	0.375	0.6206	10.69
3	15	0.45	-0.8417	13.73
4	20	0.6	-7.568	23.97
5	25	0.75	-29.62	50.97
5.5	27.5	0.825	-62.96	88.539
5.7	28.5	0.855	-90.111	117.635
6	30	0.9	-175.1	208
6.35	31.75	0.9525	-593.625	643.596
1.5	4.5	0.15	2.211	4.5
3	9	0.3	2.247	7.474
4	12	0.4	1.469	10.09

C1	C2	(C1+C2)/W	FE-K1	FE-K2
4.5	13.5	0.45	0.7257	11.76
6	18	0.6	-4.096	19.73
7	21	0.7	-12.26	30.69
8.25	24.75	0.825	-44.549	68.79
9	27	0.9	-127.5	159.6
9.5	28.5	0.95	-404.902	449.361
2	3	0.125	2.722	3.495
6	9	0.375	3.955	7.021
8	12	0.5	3.74	9.434
10.66	16	0.6665	1.492	15.5
12	18	0.75	-2.171	22.08
12.8	19.2	0.8	-6.985	29.132
13.4	20.1	0.8375	-13.375	38.175
14	21	0.875	-25.66	54.2
15	22.5	0.9375	-99.652	140.368
2	4	0.15	2.628	4.143
4	8	0.3	3.098	6.573
6	12	0.45	2.465	9.82
8	16	0.6	-0.3136	15.531
10	20	0.75	-10.233	30.048
11	22	0.825	-25.871	49.791
11.5	23	0.8625	-43.356	70.53
12	24	0.9	-79.759	111.471
12.5	25	0.9375	-180.996	221.469
1	4	0.125	1.818	4.176
3	12	0.375	1.0475	10.169
4	16	0.5	-1.586	15.185
5	20	0.625	-8.03	24.625
6	24	0.75	-25.862	46.565
6.5	26	0.8125	-48.539	72.472
6.8	27.2	0.85	-75.126	101.843
7.2	28.8	0.9	-156.042	188.426
7.5	30	0.9375	-343.536	383.216
3	3	0.15	3.437	3.437
7	7	0.35	5.27	5.27
10	10	0.5	6.517	6.517
15	15	0.75	9.853	9.853
17	17	0.85	12.91	12.91
18	18	0.9	15.89	15.89
19	19	0.95	22.54	22.54
9	6	0.375	7.01	3.954
12	8	0.5	9.427	3.739
16	10.66	0.6665	15.5	1.493
18	12	0.75	22.08	-2.171
21	14	0.875	54.22	-25.69
4.5	1.5	0.15	4.498	2.213

C1	C2	(C1+C2)/W	FE-K1	FE-K2
9	3	0.3	7.473	2.249
12	4	0.4	10.08	1.467
13.5	4.5	0.45	11.76	0.7258
18	6	0.6	19.72	-4.096
21	7	0.7	30.69	-12.25
27	9	0.9	159.3	-127.3
5	1	0.15	4.843	1.74
10	2	0.3	8.36	1.424
12.5	2.5	0.375	10.69	0.6205
15	3	0.45	13.73	0.8427
20	4	0.6	23.95	-7.567
25	5	0.75	50.94	-29.6
30	6	0.9	207.9	-175.1
7	1	0.2	6.152	1.487
10.5	1.5	0.3	8.808	1.031
14	2	0.4	12.37	-0.3146
16.8	2.4	0.48	16.43	-2.428
21	3	0.6	26.24	-8.945
28	4	0.8	80.12	-54.86
31.5	4.5	0.9	232.2	-198.2
6.4	0.64	0.176	5.76	1.263
10	1	0.275	8.398	0.915
17	1.7	0.4675	16.66	-2.425
25	2.5	0.6875	42.84	-20.91
30	3	0.825	107.3	-77.79
33	3.3	0.9075	285.1	-247.2
14	0.7	0.3675	12.35	-0.3736
18	0.9	0.4725	18.34	-2.687
24	1.2	0.63	35.72	-12.52
30	1.5	0.7875	89.16	-53.87
34	1.7	0.8925	253.8	-205.9

Case 2: W: 24.0 in.

C1	C2	(C1+C2)/W	FE-K1	FE-K2
0.1	2	0.0875	0.587	2.916
0.2	4	0.175	0.6722	4.579
0.45	9	0.39375	-0.613	10.56
0.6	12	0.525	-3.686	17.49
0.9	18	0.7875	-41.75	68.97
1	20	0.875	-121.1	156.1
0.2	2	0.09167	0.829	2.916
0.4	4	0.18333	0.9596	4.582
0.8	8	0.36667	-0.05632	8.995
1.2	12	0.55	-4.746	17.75
1.7	17	0.77917	-37.33	57.81
2	20	0.91667	-227.4	257.8
0.5	3.5	0.16667	1.155	4.146
1	7	0.33333	0.544	7.617
1.5	10.5	0.5	-2.45	13.66
2.2	15.4	0.73333	-22.43	39.5
2.7	18.9	0.9	-153.3	179.5
1	5	0.25	1.308	5.449
1.5	7.5	0.375	0.4806	8.277
2	10	0.5	-1.826	12.66
3	15	0.75	-22.88	39.42
3.6	18	0.9	-135.2	160.6
2	2	0.16667	2.765	2.765
4	4	0.33333	3.974	3.974
6	6	0.5	5.09	5.09
9	9	0.75	7.691	7.693
11	11	0.91667	13.56	13.52
7.5	1.5	0.375	8.277	0.4804
10	2	0.5	12.66	-1.826
15	3	0.75	39.49	-22.93
18	3.6	0.9	160.7	-135.3
7	1	0.33333	7.62	0.5445
10.5	1.5	0.5	13.66	-2.45
15.4	2.2	0.73333	39.4	-22.44
18.9	2.7	0.9	179.8	-153
8	0.8	0.36667	8.987	-0.05619
12	1.2	0.55	17.74	-4.746
17	1.7	0.77917	57.84	-37.34
20	2	0.91667	258.1	-227.4
9	0.45	0.39375	10.55	-0.6169
12	0.6	0.525	17.48	-3.682
18	0.9	0.7875	68.98	-41.75

C1	C2	(C1+C2)/W	FE-K1	FE-K2
20	1	0.875	156	-121

Case 3: W: 16.0 in.

C1	C2	(C1+C2)/W	FE-K1	FE-K2
0.1	2	0.13125	0.5408	3.058
0.2	4	0.2625	0.3904	5.283
0.3	6	0.39375	-0.5023	8.617
0.4	8	0.525	-3.006	14.28
0.6	12	0.7875	-34.05	56.43
0.7	14	0.91875	-209.6	243.4
0.2	2	0.1375	0.7677	3.052
0.5	5	0.34375	0.1607	6.768
0.8	8	0.55	-3.875	14.49
1.1	11	0.75625	-24.46	40.42
1.3	13	0.89375	-122.8	145.6
0.5	3.5	0.25	0.8509	4.661
0.7	4.9	0.35	0.3134	6.606
1	7	0.5	-2.002	11.16
1.5	10.5	0.75	-2.131	35.71
1.8	12.6	0.9	-124.8	146.6
0.5	2.5	0.1875	1.127	3.55
1	5	0.375	0.3928	6.754
1.5	7.5	0.5625	-3.255	13.05
2	10	0.75	-18.69	32.18
2.4	12	0.9	-110.3	131.5
1	1	0.125	1.965	1.965
3	3	0.375	3.462	3.46
4	4	0.5	4.157	4.157
6	6	0.75	6.282	6.284
7.2	7.2	0.9	10.09	10.08
5	1	0.375	6.751	0.3928
7.5	1.5	0.5625	13.04	-3.258
10	2	0.75	32.19	-18.68
12	2.4	0.9	131.3	-110.3
4.9	0.7	0.35	6.607	0.3136
7	1	0.5	11.16	-2.001
10.5	1.5	0.75	35.69	-21.33
12.6	1.8	0.9	146.8	-125.1
5	0.5	0.34375	6.766	0.1608
8	0.8	0.55	14.49	-3.873
11	1.1	0.75625	40.41	-24.47
13	1.3	0.89375	145.6	-122.9
6	0.3	0.39375	8.618	-0.5017

C1	C2	(C1+C2)/W	FE-K1	FE-K2
8	0.4	0.525	14.28	-3.009
12	0.6	0.7875	56.3	-34.02
14	0.7	0.91875	244.1	-210.7

Case 4: W: 4.0 in.

C1	C2	(C1+C2)/W	FE-K1	FE-K2
0.03	0.6	0.1575	0.2756	1.731
0.05	1	0.2625	0.1948	2.647
0.1	2	0.525	-1.493	7.111
0.15	3	0.7875	-16.997	28.109
0.16	3.2	0.84	-30.816	43.691
0.08	0.8	0.22	0.3685	2.161
0.1	1	0.275	0.289	2.647
0.2	2	0.55	-1.933	7.217
0.3	3	0.825	-24.66	33.854
0.05	0.35	0.1	0.4119	1.224
0.1	0.7	0.2	0.466	1.938
0.2	1.4	0.4	-0.1019	3.891
0.3	2.1	0.6	-2.843	8.245
0.4	2.8	0.8	-17.42	25.265
0.15	0.75	0.225	0.5559	2.033
0.25	1.25	0.375	0.1937	3.353
0.4	2	0.6	-2.392	7.536
0.55	2.75	0.825	-19.873	27.914
0.5	0.5	0.25	1.39	1.39
1	1	0.5	2.046	2.046
1.3	1.3	0.65	2.571	2.571
1.65	1.65	0.825	3.754	3.754

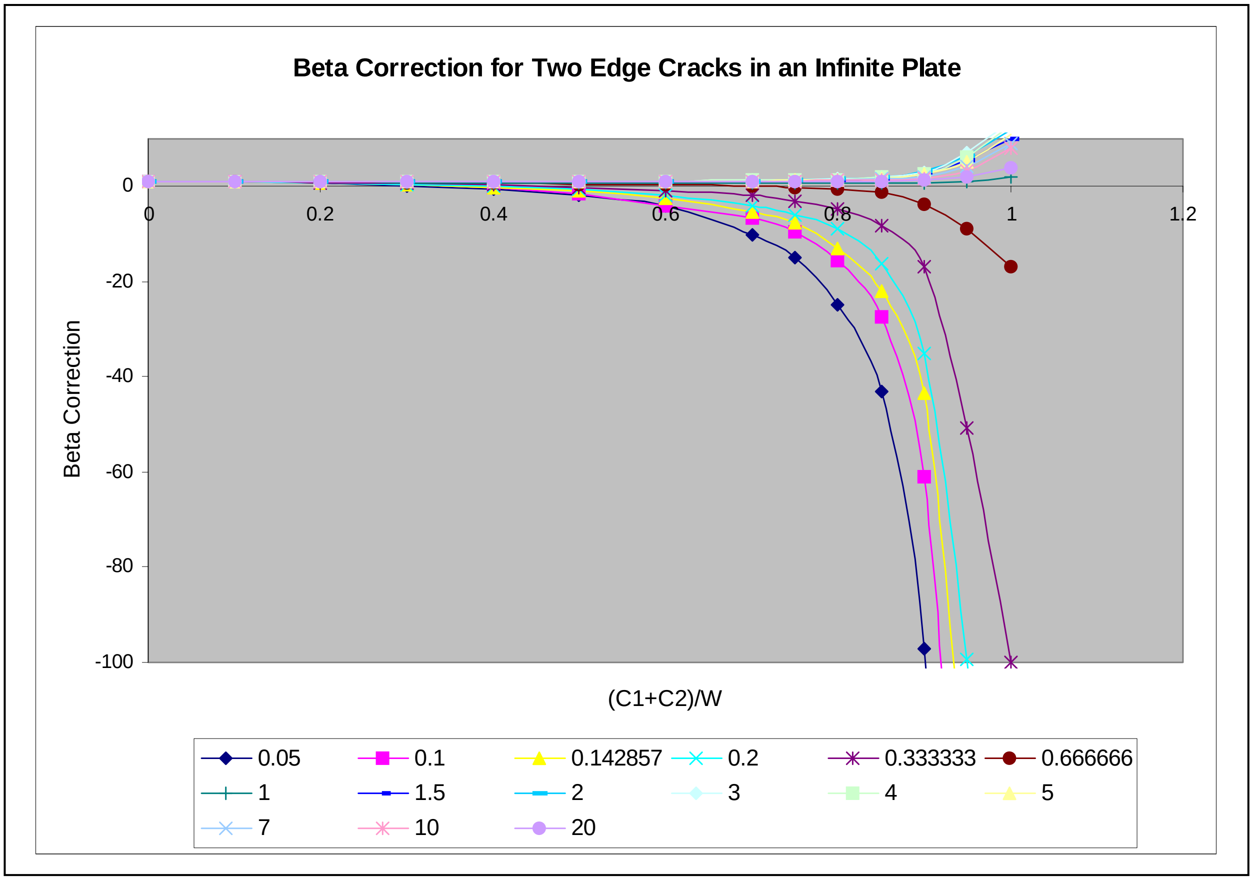
C2. Beta Interaction Tables for Crack Tips in an Infinite Plate

(C1+C2)/W	C1/C2=1/20	C1/C2=1/10	C1/C2=1/7	C1/C2=1/5	C1/C2=1/3	C1/C2=2/3	C1/C2=1
0	1	1	1	1	1	1	1
0.1	0.9	0.909	0.915	0.922	0.933	0.952	0.963
0.2	0.675	0.72567	0.742267	0.774537	0.826304	0.89568	0.93639
0.3	0.23	0.33688	0.416241	0.495167	0.626351	0.80347	0.88323
0.4	-0.525	-0.28382	-0.1094	0.061	0.346573	0.68719	0.82821
0.5	-1.85	-1.47492	-0.95599	-0.58693	-0.085	0.54587	0.7782
0.6	-4.2	-4.04357	-2.49593	-1.78547	-0.74391	0.35589	0.736925
0.7	-10.2	-6.75457	-5.4978	-4	-1.98958	0.04161	0.72008
0.75	-15	-9.5	-7.61255	-6.08171	-3.1	-0.21363	0.72668
0.8	-25	-15.5453	-12.9593	-8.9	-4.8	-0.63674	0.74658
0.85	-43	-27.5	-21.885	-16.2251	-8.2	-1.34167	0.78289
0.9	-97.0246	-61	-43.5105	-35	-16.8089	-3.6	0.87168
0.95	-274.234	-177	-131.645	-99.4144	-50.7905	-9	1.11639
1	-500	-340	-260	-200	-100	-17	2

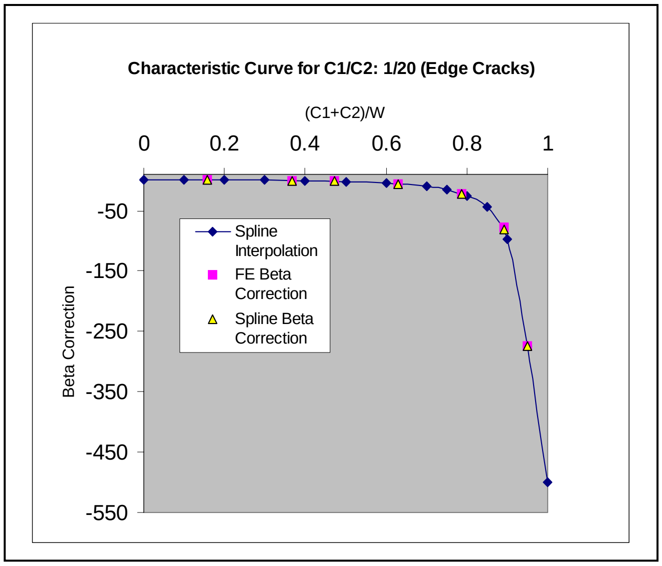
(C1+C2)/W	C1/C2=1.5	C1/C2=2	C1/C2=3	C1/C2=4	C1/C2=5	C1/C2=7	C1/C2=10	C1/C2=20
0	1	1	1	1	1	1	1	1
0.1	0.9735	0.98	0.988	0.993	0.99538	0.99786	0.99854	0.98987
0.2	0.959	0.971	0.98535	0.9915	0.99479	0.99836	0.99983	0.98919
0.3	0.934709	0.95942	0.9852	0.99472	0.99828	1.0017	1.00377	0.99785
0.4	0.92245	0.96039	0.9919	1.00211	1.0053	1.00472	1.00727	1.00428
0.5	0.927642	0.97797	1.02492	1.01572	1.023	1.01993	1.01205	1.0075
0.6	0.966556	1.03886	1.08178	1.0742	1.06894	1.05233	1.03023	1.01526
0.7	1.09342	1.19865	1.23079	1.2079	1.18312	1.1278	1.08946	1.02603
0.75	1.21125	1.34083	1.37	1.32626	1.28277	1.19476	1.14704	1.04866
0.8	1.4134	1.57642	1.62	1.52495	1.44639	1.3399	1.23943	1.09841
0.85	1.7904	2.08872	2.03	1.9129	1.78704	1.5766	1.435	1.16241
0.9	2.85	3.17491	3.07735	2.78819	2.5251	2.13788	1.8	1.37706
0.95	5.5	6.5	6.9668	6.1	5.3	4.1657	3.2	2.02092
1	10	12	13.5	13	11	9	8	4

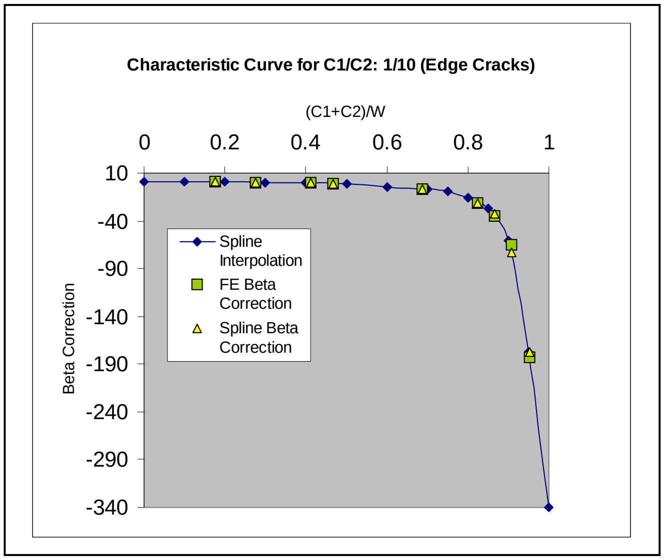
C3. Characteristic Plots for the Two Edge Crack case

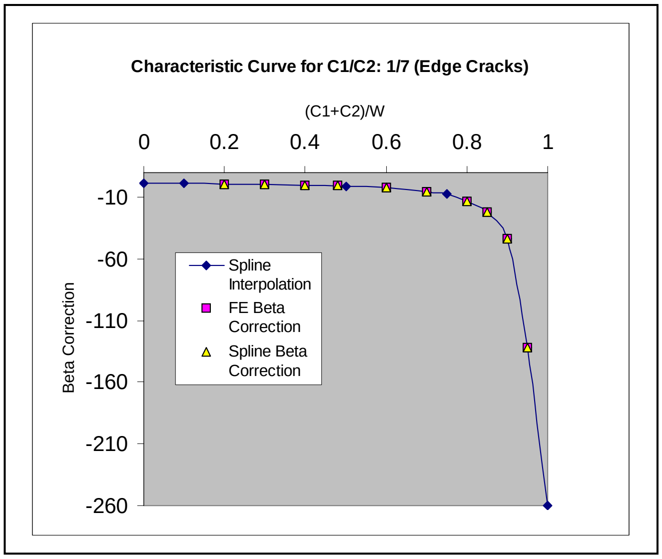
C3.1 (C1+C2)/W vs. Beta Correction for various C1/C2 Ratios

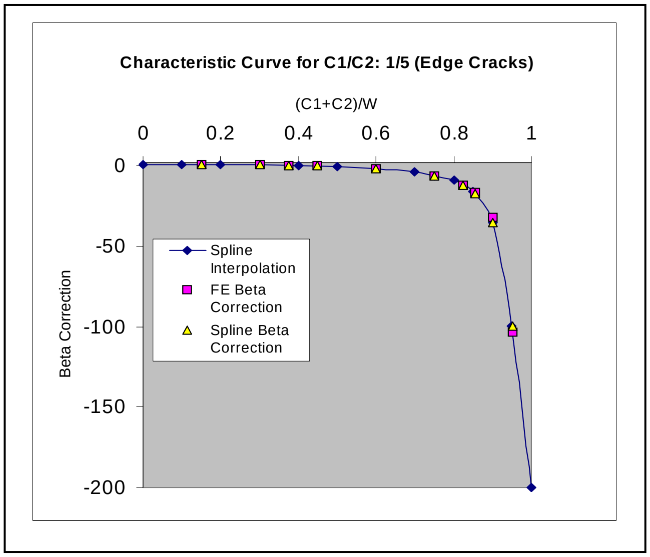


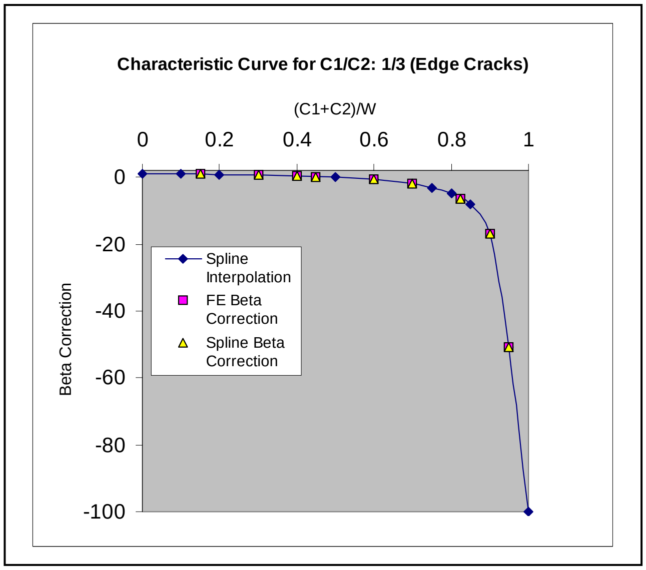
C3.2 Spline Interpolation vs. FEA solution for various C1/C2 ratios

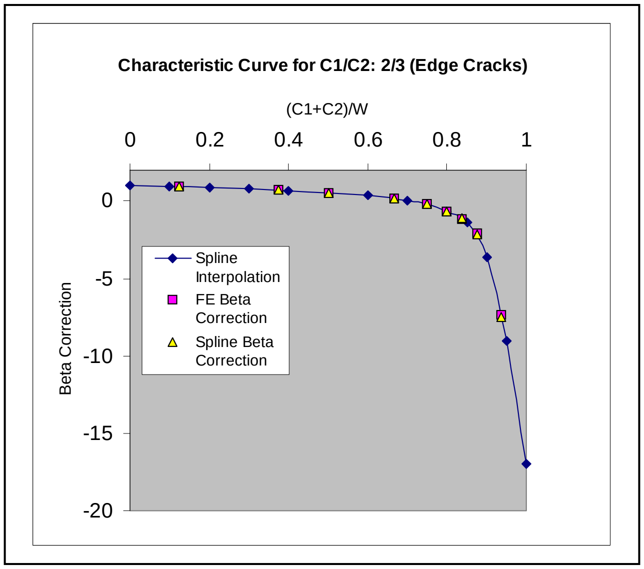


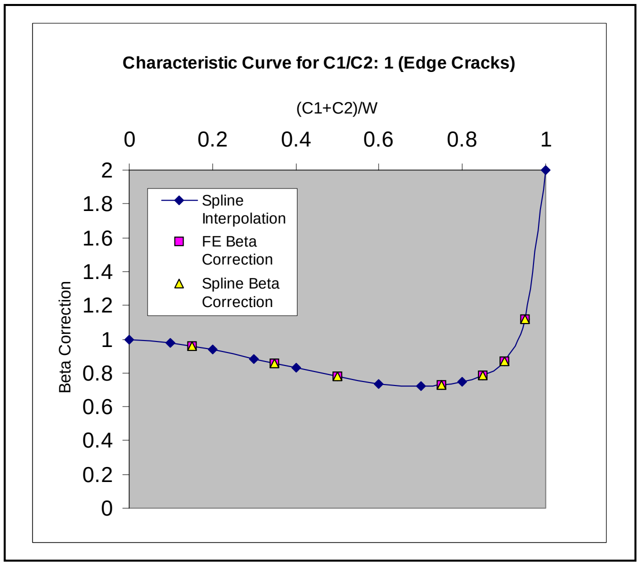


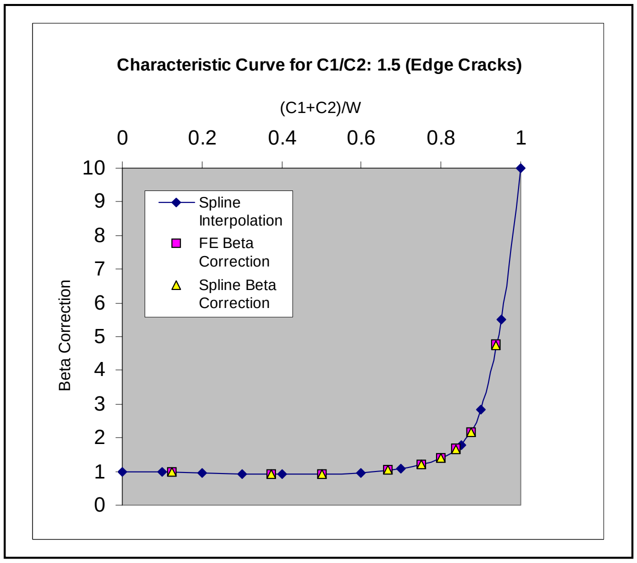


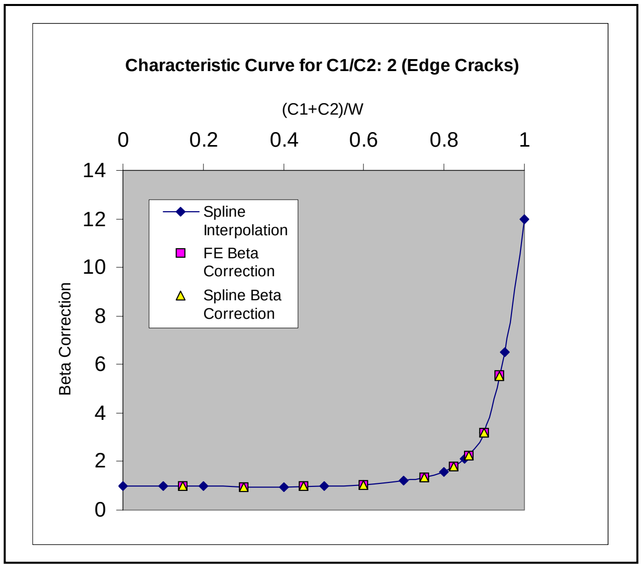


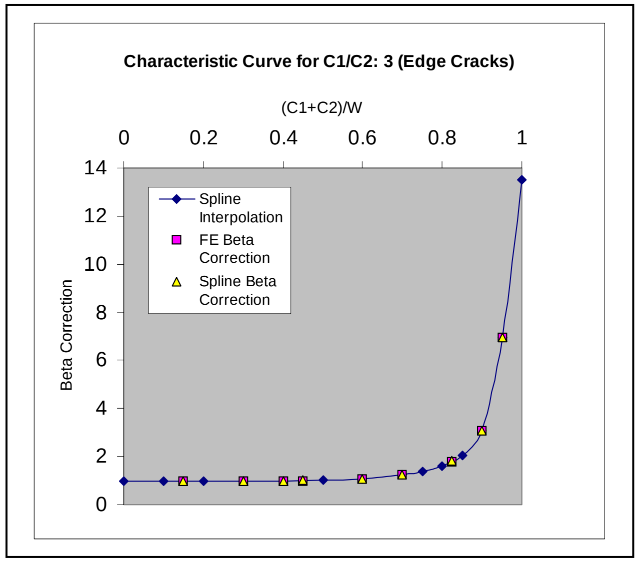

Characteristic Curve for C1/C2: 1/3 (Edge Cracks)
(C1+C2)/W
Beta Correction
Spline Interpolation
FE Beta Correction
Spline Beta Correction

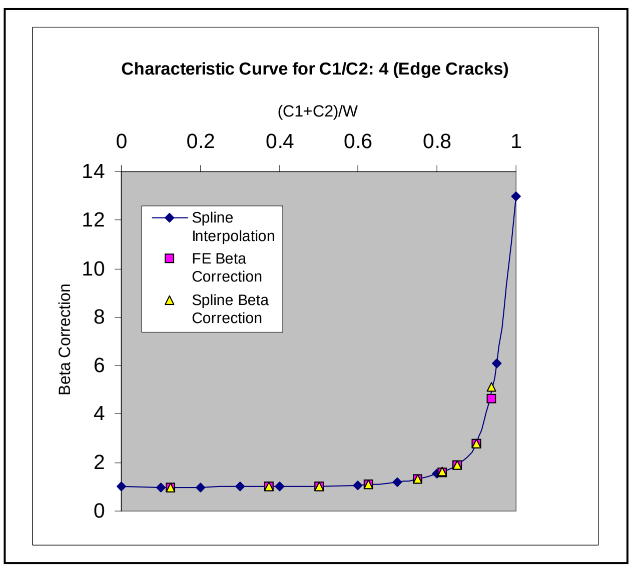

Characteristic Curve for C1/C2: 2/3 (Edge Cracks)
(C1+C2)/W
Beta Correction
Spline Interpolation
FE Beta Correction
Spline Beta Correction

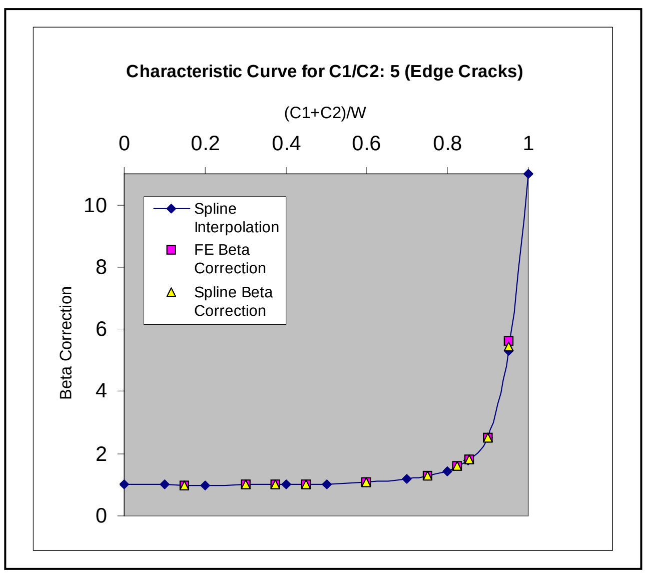

Characteristic Curve for C1/C2: 1 (Edge Cracks)
(C1+C2)/W
Beta Correction
Spline Interpolation
FE Beta Correction
Spline Beta Correction

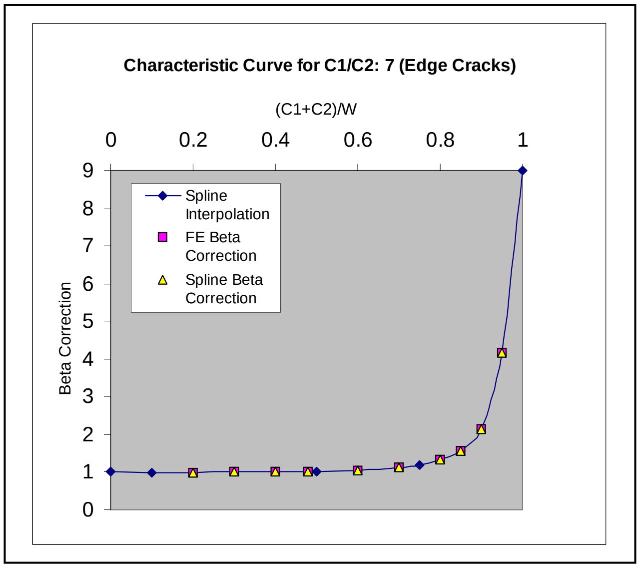

Characteristic Curve for C1/C2: 1.5 (Edge Cracks)
(C1+C2)/W
Beta Correction
Spline Interpolation
FE Beta Correction
Spline Beta Correction

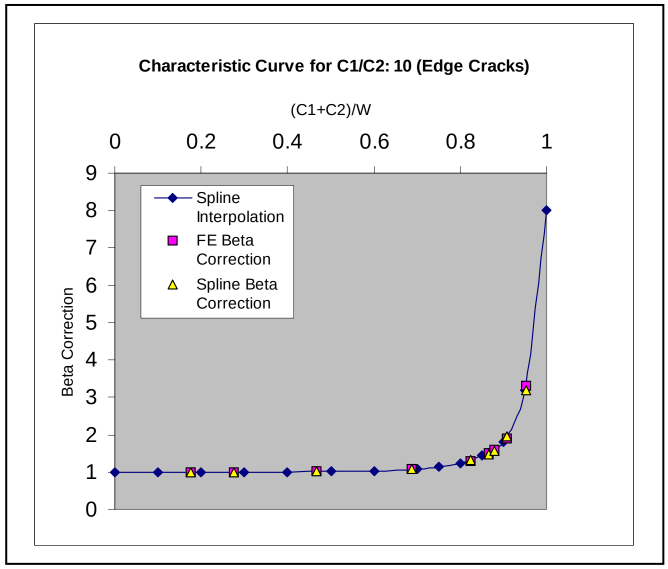

Characteristic Curve for C1/C2: 2 (Edge Cracks)
(C1+C2)/W
Beta Correction
Spline Interpolation
FE Beta Correction
Spline Beta Correction

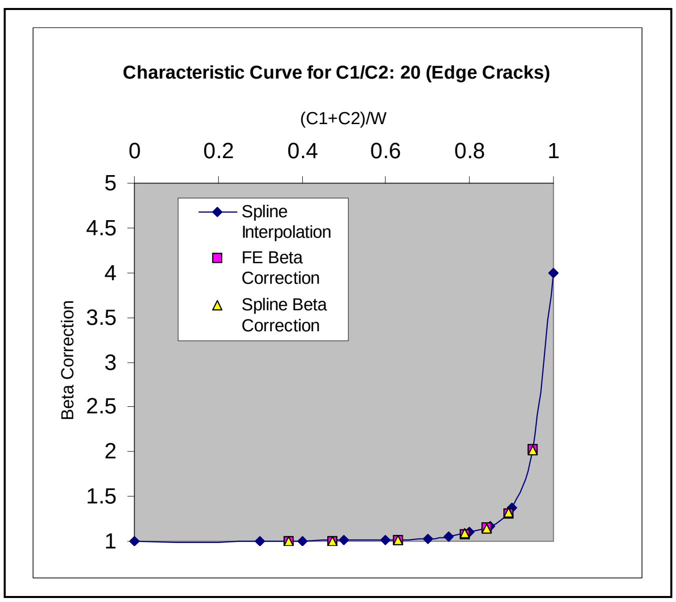

Characteristic Curve for C1/C2: 3 (Edge Cracks)
(C1+C2)/W
Beta Correction
Spline Interpolation
FE Beta Correction
Spline Beta Correction


Characteristic Curve for C1/C2: 4 (Edge Cracks)
(C1+C2)/W
Beta Correction
Spline Interpolation
FE Beta Correction
Spline Beta Correction


Characteristic Curve for C1/C2: 5 (Edge Cracks)
(C1+C2)/W
Beta Correction
Spline Interpolation
FE Beta Correction
Spline Beta Correction


Characteristic Curve for C1/C2: 7 (Edge Cracks)
(C1+C2)/W
Beta Correction
Spline Interpolation
FE Beta Correction
Spline Beta Correction
0
0.2
0.4
0.6
0.8
1
9
8
7
6
5
4
3
2
1
0


Characteristic Curve for C1/C2: 10 (Edge Cracks)
(C1+C2)/W
Beta Correction
Spline Interpolation
FE Beta Correction
Spline Beta Correction
0
0.2
0.4
0.6
0.8
1
9
8
7
6
5
4
3
2
1
0


Characteristic Curve for C1/C2: 20 (Edge Cracks)
(C1+C2)/W
Beta Correction
Spline Interpolation
FE Beta Correction
Spline Beta Correction
0
0.2
0.4
0.6
0.8
1
5
4.5
4
3.5
3
2.5
2
1.5
1

C4. Comparison of FE and AFGROW Solutions

Case 1: W: 24.0 in.

C1	C2	(C1+C2)/W	FE-K1	FE-K2	AFG-K1	AFG-K2
0.4	4	0.18333	0.9596	4.582	0.9711	4.598
0.8	8	0.36667	-0.05632	8.995	0	9.001
0.5	3.5	0.16667	1.155	4.146	1.153	4.173
1	7	0.33333	0.544	7.617	0.5432	7.64
1	5	0.25	1.308	5.449	1.324	5.471
1.5	7.5	0.375	0.4806	8.277	0.453	8.283

Case 2: W: 16.0 in.

C1	C2	(C1+C2)/W	FE-K1	FE-K2	AFG-K1	AFG-K2
0.5	3.5	0.25	0.8509	4.661	0.8541	4.679
0.7	4.9	0.35	0.3134	6.606	0.3143	6.6
1	7	0.5	-2.002	11.16	0	11.18
0.5	2.5	0.1875	1.127	3.55	1.137	3.565
1	5	0.375	0.3928	6.754	0.3699	6.763
1.5	7.5	0.5625	-3.255	13.05	0	13.02
0.4	8	0.525	-3.006	14.28	0	14.3
0.6	12	0.7875	-34.05	56.43	0	56.57
0.8	8	0.55	-3.875	14.49	0	14.43
1.1	11	0.75625	-24.46	40.42	0	40.61

Case 3: W: 4.0 in.

C1	C2	(C1+C2)/W	FE-K1	FE-K2	AFG-K1	AFG-K2
0.4	2	0.6	-2.392	7.536	0	7.574
0.5	0.5	0.25	1.39	1.39	1.405	1.405
0.05	1	0.2625	0.1948	2.647	0.189	2.633
0.16	3.2	0.84	-30.816	43.691	0	43.38
0.1	1	0.275	0.289	2.647	0.2894	2.655
0.2	2	0.55	-1.933	7.217	0	7.214
0.1	0.7	0.2	0.466	1.938	0.4709	1.945
0.3	2.1	0.6	-2.843	8.245	0	8.298
0.15	0.75	0.225	0.5559	2.033	0.5622	2.047
0.25	1.25	0.375	0.1937	3.353	0.1849	3.381
0.55	2.75	0.825	-19.873	27.914	0	28.12

Appendix D
Unequal Edge Cracks in a Plate with Constrained Bending

D1. Cases

Case 1: W: 40.0 in.

C1	C2	(C1+C2)/W	FE-K1	FE-K2
0.64	12.8	0.336	1.041	8.367
0.7	14	0.3675	1.003	8.934
0.9	18	0.4725	0.7885	10.99
1.2	24	0.63	0.3773	14.64
1.5	30	0.7875	0.6	19.8
1.7	34	0.8925	3.554	25.79
1.77	35.4	0.92925	7.194	29.52
1.81	36.2	0.95025	11.4	32.88
0.64	6.4	0.176	1.436	5.319
1	10	0.275	1.552	7.024
1.5	15	0.4125	1.457	9.426
1.7	17	0.4675	1.371	10.45
2.5	25	0.6875	1.291	15.36
3	30	0.825	3.102	19.86
3.15	31.5	0.86625	4.796	21.81
3.2	32	0.88	5.63	22.61
3.3	33	0.9075	7.951	24.51
3.38	33.8	0.9295	10.85	26.6
3.46	34.6	0.9515	15.54	29.88
1	7	0.2	1.754	5.603
1.5	10.5	0.3	1.891	7.252
2	14	0.4	1.872	8.913
2.4	16.8	0.48	1.81	10.33
3	21	0.6	1.795	12.69
3.5	24.5	0.7	2.182	15
4	28	0.8	3.681	17.89
4.25	29.75	0.85	5.444	19.8
4.5	31.5	0.9	8.828	22.53
4.62	32.34	0.924	11.68	24.55
4.75	33.25	0.95	16.79	28.09
1	5	0.15	1.862	4.609
2	10	0.3	2.274	6.999
2.5	12.5	0.375	2.332	8.166
3	15	0.45	2.349	9.397
4	20	0.6	2.545	12.07
5	25	0.75	3.83	15.42
5.5	27.5	0.825	5.765	17.64
5.7	28.5	0.855	7.093	18.81

C1	C2	(C1+C2)/W	FE-K1	FE-K2
6	30	0.9	10.31	21.25
6.2	31	0.93	14	23.91
6.35	31.75	0.9525	18.8	27.35
1.5	4.5	0.15	2.318	4.327
3	9	0.3	2.972	6.493
4	12	0.4	3.24	7.873
4.5	13.5	0.45	3.361	8.59
6	18	0.6	3.962	10.92
7	21	0.7	4.9	12.75
8.25	24.75	0.825	7.842	15.88
9	27	0.9	12.34	19.44
9.25	27.75	0.925	15.17	21.58
9.5	28.5	0.95	19.78	25.3
2	3	0.125	2.768	3.459
6	9	0.375	4.597	6.32
8	12	0.5	5.4	7.685
10.66	16	0.6665	6.992	9.889
12	18	0.75	8.403	11.42
12.8	19.2	0.8	9.673	12.65
13.4	20.1	0.8375	11.01	13.86
14	21	0.875	12.89	15.56
15	22.5	0.9375	19.15	21.28
3	3	0.15	3.416	3.416
7	7	0.35	5.271	5.271
10	10	0.5	6.567	6.567
15	15	0.75	9.94	9.94
17	17	0.85	12.98	12.98
18	18	0.9	15.94	15.94
18.5	18.5	0.925	18.45	18.45
19	19	0.95	22.6	22.6
2	4	0.15	2.721	4.046
4	8	0.3	3.638	5.964
6	12	0.45	4.324	7.78
8	16	0.6	5.263	9.798
10	20	0.75	7.298	12.43
11	22	0.825	9.418	14.41
11.5	23	0.8625	11.15	15.88
12	24	0.9	13.76	18.07
12.5	25	0.9375	18.37	21.98
1	4	0.125	1.905	4.074
3	12	0.375	2.67	7.913
4	16	0.5	2.836	9.871
5	20	0.625	3.255	12.07

C1	C2	(C1+C2)/W	FE-K1	FE-K2
6	24	0.75	4.636	14.75
6.5	26	0.8125	6.227	16.5
6.8	27.2	0.85	7.77	17.81
7.2	28.8	0.9	11.22	20.42
7.5	30	0.9375	16.15	24.05
9	6	0.375	6.323	4.596
12	8	0.5	7.685	5.4
18	12	0.75	11.42	8.404
21	14	0.875	15.56	12.9
13.5	4.5	0.45	8.592	3.36
18	6	0.6	10.92	3.961
21	7	0.7	12.75	4.899
27	9	0.9	19.41	12.33
15	3	0.45	9.393	2.351
20	4	0.6	12.08	2.546
25	5	0.75	15.41	3.831
30	6	0.9	21.23	10.3
16.8	2.4	0.48	10.34	1.807
21	3	0.6	12.69	1.796
28	4	0.8	17.89	3.682
31.5	4.5	0.9	22.53	8.838
17	1.7	0.4675	10.45	1.37
25	2.5	0.6875	15.34	1.292
30	3	0.825	19.85	3.105
33	3.3	0.9075	24.5	7.953
18	0.9	0.4725	10.99	0.7895
24	1.2	0.63	14.64	0.3769
30	1.5	0.7875	19.79	0.5988
34	1.7	0.8925	25.79	3.553

Case 2: W: 24.0 in.

C1	C2	(C1+C2)/W	FE-K1	FE-K2
0.45	9	0.39375	0.7074	7.391
0.6	12	0.525	0.4287	9.523
0.9	18	0.7875	0.2562	15.6
1	20	0.875	1.771	19.23
0.8	8	0.366667	1.14	6.731
1.2	12	0.55	0.8554	9.517
1.7	17	0.779167	1.394	14.28
2	20	0.916667	6.749	19.81
1	7	0.333333	1.441	6.084
1.5	10.5	0.5	1.307	8.405
2.2	15.4	0.733333	1.735	12.51
2.7	18.9	0.9	6.621	17.68
1.5	7.5	0.375	1.767	6.384
2	10	0.5	1.762	8.035
3	15	0.75	2.773	12.15
3.6	18	0.9	7.787	16.67
4	4	0.333333	3.976	3.977
6	6	0.5	5.083	5.085
9	9	0.75	7.696	7.697
11	11	0.916667	13.55	13.54
7.5	1.5	0.375	6.377	1.767
10	2	0.5	8.036	1.762
15	3	0.75	12.13	2.772
18	3.6	0.9	16.65	7.785
7	1	0.333333	6.08	1.442
10.5	1.5	0.5	8.404	1.307
15.4	2.2	0.733333	12.51	1.736
18.9	2.7	0.9	17.67	6.619
8	0.8	0.366667	6.731	1.139
12	1.2	0.55	9.516	0.8544
17	1.7	0.779167	14.28	1.394
20	2	0.916667	19.81	6.756
9	0.45	0.39375	7.39	0.7066
12	0.6	0.525	9.522	0.429
18	0.9	0.7875	15.6	0.2563
20	1	0.875	19.22	1.771

Case 3: W: 16.0 in.

C1	C2	(C1+C2)/W	FE-K1	FE-K2
0.3	6	0.39375	0.606	5.968
0.4	8	0.525	0.4115	7.657
0.6	12	0.7875	0.3793	12.54
0.7	14	0.91875	3.683	17.9
0.5	5	0.34375	0.971	5.192
0.8	8	0.55	0.7816	7.652
1.1	11	0.75625	1.12	10.98
1.3	13	0.89375	4.209	14.84
0.7	4.9	0.35	1.198	5.107
1	7	0.5	1.135	6.775
1.5	10.5	0.75	1.713	10.34
1.8	12.6	0.9	5.587	14.25
1	5	0.375	1.476	5.167
1.5	7.5	0.5625	1.549	7.175
2	10	0.75	2.423	9.757
2.4	12	0.9	6.52	13.44
3	3	0.375	3.465	3.464
4	4	0.5	4.154	4.154
6	6	0.75	6.292	6.292
7.2	7.2	0.9	10.09	10.09
5	1	0.375	5.163	1.476
7.5	1.5	0.5625	7.18	1.55
10	2	0.75	9.749	2.423
12	2.4	0.9	13.44	6.514
4.9	0.7	0.35	5.105	1.199
7	1	0.5	6.775	1.135
10.5	1.5	0.75	10.34	1.715
12.6	1.8	0.9	14.25	5.591
5	0.5	0.34375	5.182	0.9707
8	0.8	0.55	7.652	0.782
11	1.1	0.75625	10.98	1.122
13	1.3	0.89375	14.85	4.206
6	0.3	0.39375	5.969	0.6052
8	0.4	0.525	7.657	0.4114
12	0.6	0.7875	12.53	0.3789
14	0.7	0.91875	17.9	3.686

D2. Beta Interaction Tables for Crack Tips in an Infinite Plate

Table 1: Beta Corrections for both Crack Tips

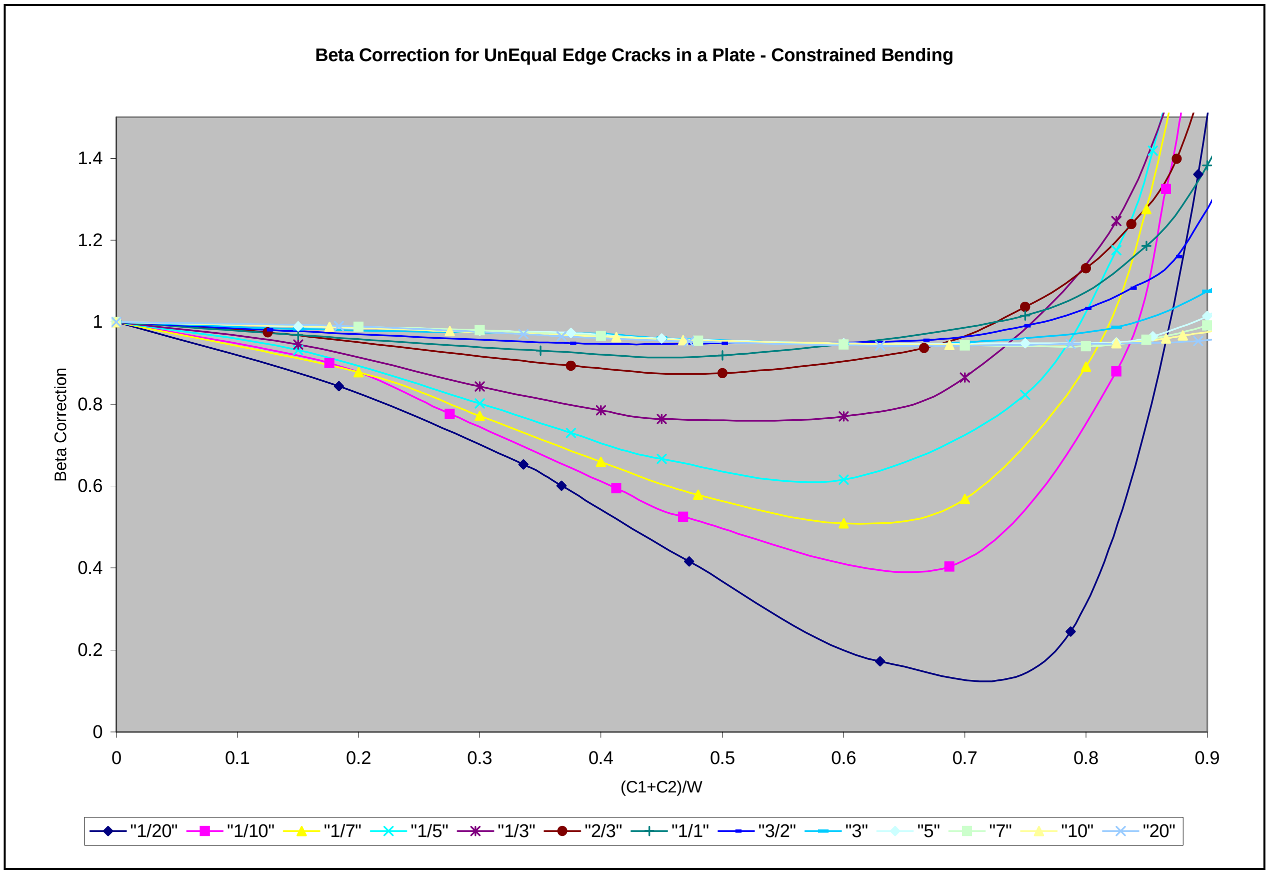
(C1+C2)/W	C1/C2=1/20	C1/C2=1/10	C1/C2=1/7	C1/C2=1/5	C1/C2=1/3	C1/C2=2/3	C1/C2=1/1
0	1	1	1	1	1	1	1
0.1	0.876	0.9	0.911	0.924	0.946	0.972	0.98
0.2	0.752	0.795	0.827	0.853	0.894	0.943	0.96
0.3	0.633	0.7	0.736	0.775	0.84	0.914	0.939
0.4	0.504	0.594	0.652	0.704	0.791	0.889	0.92
0.5	0.376	0.484	0.562	0.633	0.743	0.876	0.918
0.6	0.2	0.433	0.51	0.614	0.77	0.939	0.939
0.7	0.116	0.407	0.704	0.749	0.863	1.005	0.98
0.8	0.265	0.51	0.891	0.904	1.354	1.074	1.058
0.9	1.459	2.022	2.009	2	1.855	1.513	1.381
0.92	1.688	2.371	2.248	2.2	1.964	1.605	1.464

Table 1: Continued

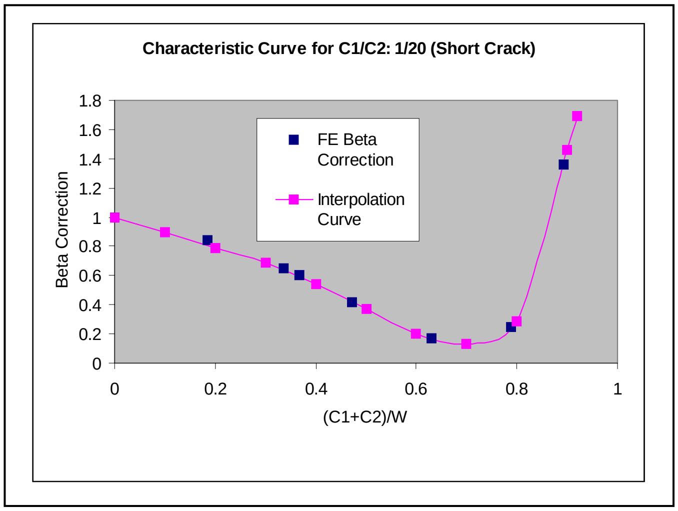
(C1+C2)/W	C1/C2=3/2	C1/C2=3	C1/C2=5	C1/C2=7	C1/C2=10	C1/C2=20
0	1	1	1	1	1	1
0.1	0.984	0.9917	0.9913	0.9908	0.9905	0.9904
0.2	0.97	0.9827	0.9824	0.9817	0.9811	0.9804
0.3	0.955	0.9744	0.9735	0.9724	0.9715	0.9703
0.4	0.948	0.9657	0.9646	0.9633	0.9619	0.9604
0.5	0.948	0.957	0.956	0.954	0.9525	0.9509
0.6	0.966	0.9474	0.949	0.946	0.9474	0.944
0.7	0.98	0.9505	0.9457	0.9432	0.9426	0.9393
0.8	1.006	1.012	0.9496	0.9407	0.941	0.9465
0.9	1.223	1.0735	1.0144	0.9916	0.9736	0.955
0.92	1.284	1.0872	1.0279	1.0017	0.9875	0.9577

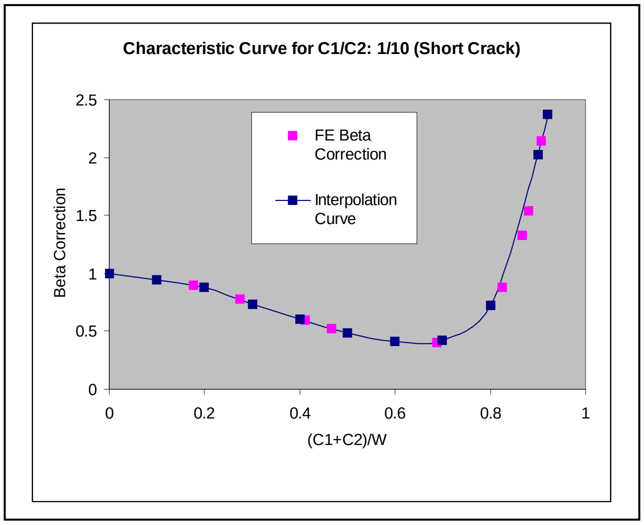
D3. Characteristic Plots for Unequal Edge Cracks with Constrained Bending

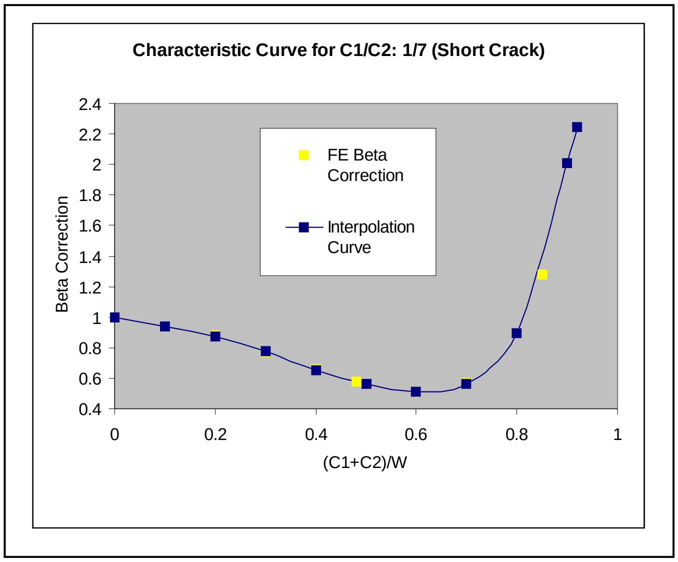
D3.1 (C1+C2)/W vs. Beta Correction for various C1/C2 Ratios

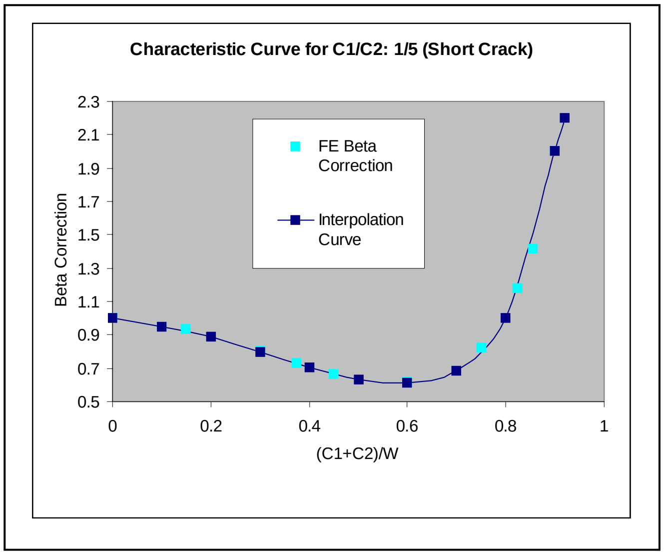


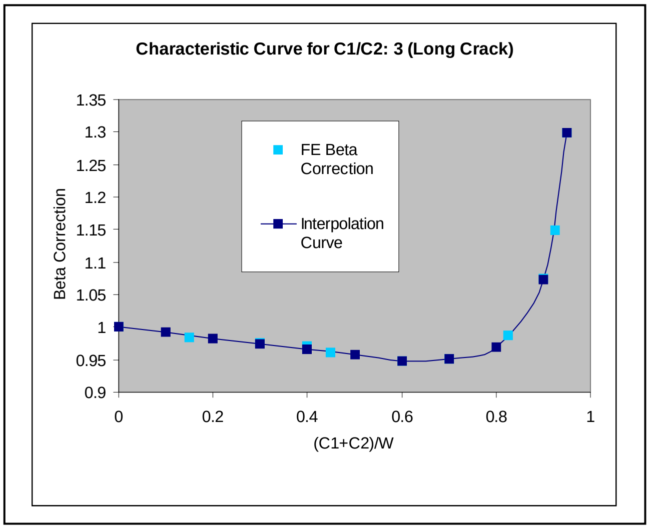
D3.2 Spline Interpolation vs. FEA solution for various C1/C2 ratios

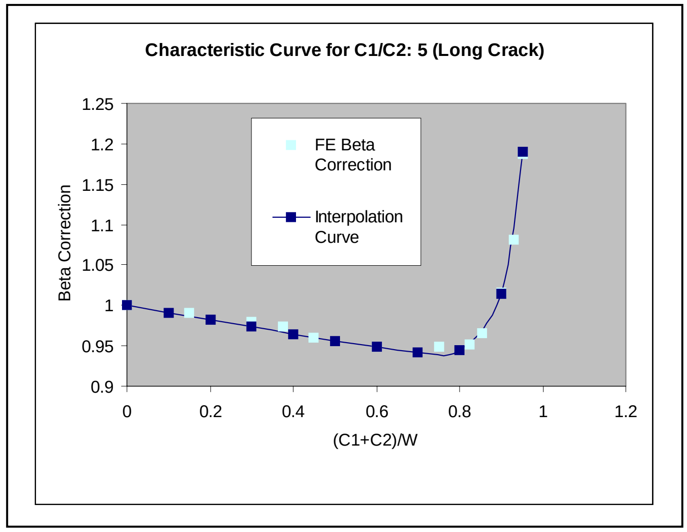


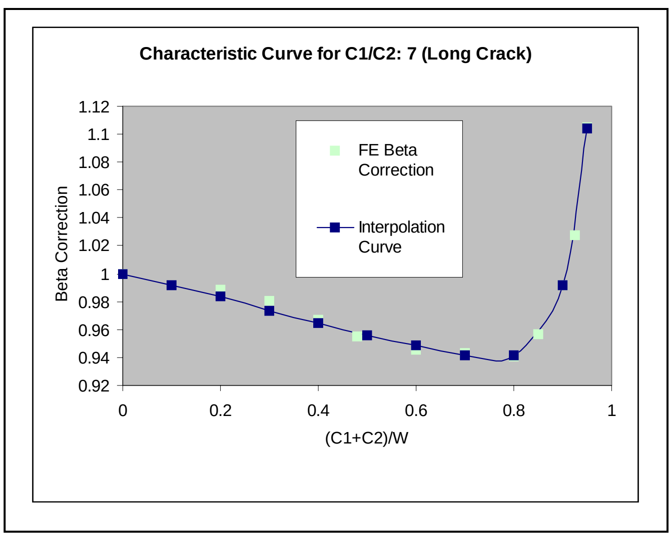


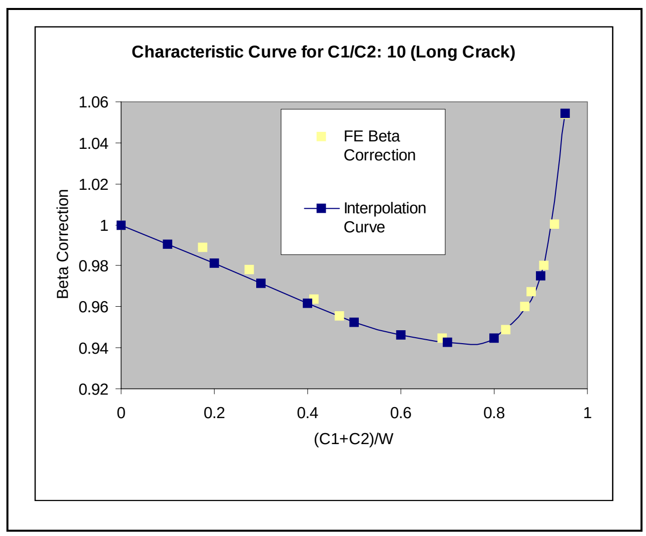


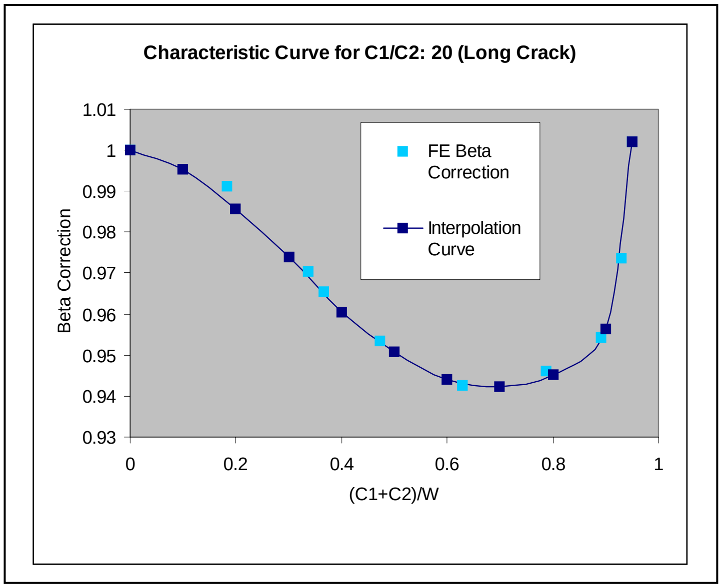



Characteristic Curve for C1/C2: 3 (Long Crack)
FE Beta Correction
Interpolation Curve
Beta Correction
(C1+C2)/W
1.35
1.3
1.25
1.2
1.15
1.1
1.05
1
0.95
0.9
0
0.2
0.4
0.6
0.8
1


Characteristic Curve for C1/C2: 5 (Long Crack)
FE Beta Correction
Interpolation Curve
Beta Correction
(C1+C2)/W
1.25
1.2
1.15
1.1
1.05
1
0.95
0.9
0
0.2
0.4
0.6
0.8
1
1.2


Characteristic Curve for C1/C2: 7 (Long Crack)
FE Beta Correction
Interpolation Curve
Beta Correction
(C1+C2)/W
1.12
1.1
1.08
1.06
1.04
1.02
1
0.98
0.96
0.94
0.92
0
0.2
0.4
0.6
0.8
1


Characteristic Curve for C1/C2: 10 (Long Crack)
FE Beta Correction
Interpolation Curve
Beta Correction
(C1+C2)/W
1.06
1.04
1.02
1
0.98
0.96
0.94
0.92
0
0.2
0.4
0.6
0.8
1


Characteristic Curve for C1/C2: 20 (Long Crack)
Beta Correction
1.01
1
0.99
0.98
0.97
0.96
0.95
0.94
0.93
0
0.2
0.4
0.6
0.8
1
(C1+C2)/W
FE Beta Correction
Interpolation Curve

Appendix E
Unequal Cracks at a Hole in a Plate

E1. FE Solutions for a Centered Hole

E1.1 Cases

Case 1: W: 40.0 in., B: 20.0 in. and D: 0.25 in.

C1	C2	(C1+C2)/W	FE-K1	FE-K2
0.15	3	0.07875	2.304	2.31
0.25	5	0.13125	2.973	2.976
0.39	7.8	0.20475	3.751	3.784
0.5	10	0.2625	4.33	4.421
0.65	13	0.34125	5.218	5.5
0.8	16	0.42	6.318	7.407
0.85	17	0.44625	6.772	8.605
0.9	18	0.4725	7.532	10.656
0.3	3	0.0825	2.377	2.362
0.4	4	0.11	2.725	2.718
0.5	5	0.1375	3.031	3.039
0.8	8	0.22	3.899	3.925
0.95	9.5	0.26125	4.31	4.374
1.2	12	0.33	5.027	5.243
1.35	13.5	0.37125	5.487	5.917
1.5	15	0.4125	6.031	6.832
1.65	16.5	0.45375	6.68	8.264
1.8	18	0.495	7.69	11.12
0.4	2.8	0.08	2.235	2.33
0.6	4.2	0.12	2.84	2.845
0.8	5.6	0.16	3.27	3.295
1.1	7.7	0.22	3.899	3.935
1.4	9.8	0.28	4.481	4.593
1.7	11.9	0.34	5.103	5.363
2	14	0.4	5.783	6.388
2.2	15.4	0.44	6.316	7.39
2.5	17.5	0.5	7.357	10.264
0.5	2.5	0.075	2.264	2.261
1	5	0.15	3.175	3.176
1.6	8	0.24	4.084	4.124
2	10	0.3	4.663	4.788
2.5	12.5	0.375	5.374	5.79
2.8	14	0.42	5.922	6.604
3.2	16	0.48	6.816	8.311
3.5	17.5	0.525	7.674	10.758
0.6	1.8	0.06	1.97	1.99

C1	C2	(C1+C2)/W	FE-K1	FE-K2
1	3	0.1	2.586	2.587
2	6	0.2	3.621	3.678
3	9	0.3	4.538	4.694
4	12	0.4	5.55	5.942
5	15	0.5	6.811	7.988
6	18	0.6	8.463	13.67
0.8	1.2	0.05	1.825	1.821
2	3	0.125	2.894	2.895
4	6	0.25	4.1	4.138
6	9	0.375	5.266	5.373
8	12	0.5	6.639	6.939
10	15	0.625	8.207	9.606
12	18	0.75	10.415	16.713
1	1	0.05	1.841	1.841
3	3	0.15	3.176	3.176
4	4	0.2	3.688	3.688
5	5	0.25	4.134	4.134
8	8	0.4	5.584	5.584
12	12	0.6	8.097	8.097
15	15	0.75	11.355	11.355
16	16	0.8	13.158	13.158
18	18	0.9	20.35	20.35
0.8	1.6	0.06	2.044	2.038
2	4	0.15	3.1708	3.177
4	8	0.3	4.644	4.692
6	12	0.45	6.092	6.493
7	14	0.525	6.942	7.847
8	16	0.6	7.993	10.089
9	18	0.675	9.507	15.225
0.4	1.6	0.05	1.879	1.871
1	4	0.125	2.891	2.897
2	8	0.25	4.181	4.221
3	12	0.375	5.407	5.7
3.5	14	0.4375	6.115	6.814
4	16	0.5	6.996	8.609
4.5	18	0.5625	8.344	12.684

Case 2: W: 40.0 in., B: 20.0 in. and D: 2.5 in.

C1	C2	(C1+C2)/W	FE-K1	FE-K2
0.5	10	0.2625	4.878	4.99
0.8	16	0.42	7.186	9.278
0.9	18	0.4725	9.16	17.125
1	10	0.275	5.127	5.149
1.25	12.5	0.34375	5.897	6.263
1.8	18	0.495	9.526	18.16
1	7	0.2	4.382	4.286
2	14	0.4	6.501	7.469
2.5	17.5	0.5	8.824	14.825
1	5	0.15	3.892	3.765
2.5	12.5	0.375	6.075	6.638
3.5	17.5	0.525	9.057	15.562
3	9	0.3	5.215	5.336
5	15	0.5	7.581	9.563
6	18	0.6	10.477	22.506
4	6	0.25	4.751	4.727
10	15	0.625	9.161	11.428
12	18	0.75	12.638	27.988
5	5	0.25	4.717	4.717
15	15	0.75	13.43	13.43
18	18	0.9	33.03	33.03

Case 3: W: 40.0 in., B: 20.0 in. and D: 5.0 in.

C1	C2	(C1+C2)/W	FE-K1	FE-K2
0.5	10	0.2625	5.143	5.872
0.7	14	0.3675	6.738	8.718
0.85	17	0.44625	9.665	22.485
1	10	0.275	5.77	5.992
1.25	12.5	0.34375	6.66	7.502
1.7	17	0.4675	10.469	23.527
1	7	0.2	5.013	4.989
2	14	0.4	7.566	9.238
2.4	16.8	0.48	10.219	20.809
1	5	0.15	4.548	4.476
2.5	12.5	0.375	7.035	7.889
3.4	17	0.51	11.003	25.732
3	9	0.3	6.04	6.134
5	15	0.5	8.801	12.385
5.5	16.5	0.55	10.427	20.196
4	6	0.25	5.464	5.452
10	15	0.625	10.613	14.751
11	16.5	0.6875	12.549	24.495
5	5	0.25	5.447	5.447
15	15	0.75	17.207	17.207
17	17	0.85	40.964	40.964

Case 4: W: 20.0 in., B: 10.0 in. and D: 0.25 in.

C1	C2	(C1+C2)/W	FE-K1	FE-K2
0.2	4	0.21	2.723	2.714
0.3	6	0.315	3.515	3.618
0.45	9	0.4725	5.405	7.779
0.2	2	0.11	1.954	1.93
0.5	5	0.275	3.19	3.241
0.9	9	0.495	5.52	8.224
0.4	2.8	0.16	2.358	2.362
0.8	5.6	0.32	3.487	3.636
1.2	8.4	0.48	5.026	6.53
0.6	3	0.18	2.511	2.512
1.2	6	0.36	3.765	3.989
1.8	9	0.54	5.832	8.951
0.8	2.4	0.16	2.369	2.366
2	6	0.4	4.015	4.282
3	9	0.6	6.267	10.121
1	1.5	0.125	2.103	2.099
4	6	0.5	4.745	5.013
6	9	0.75	7.664	12.48

C1	C2	(C1+C2)/W	FE-K1	FE-K2
1	1	0.1	1.884	1.884
5	5	0.5	4.816	4.816
9	9	0.9	14.868	14.868

Case 5: W: 8.0 in., B: 4.0 in. and D: 0.25 in.

C1	C2	(C1+C2)/W	FE-K1	FE-K2
0.1	2	0.2625	2.019	2.108
0.15	3	0.39375	2.752	3.196
0.19	3.8	0.49875	4.463	10.2
0.2	2	0.275	2.103	2.164
0.3	3	0.4125	2.863	3.315
0.36	3.6	0.495	3.73	5.951
0.3	2.1	0.3	2.225	2.292
0.4	2.8	0.4	2.732	3.058
0.5	3.5	0.5	3.589	5.332
0.4	2	0.3	2.209	2.276
0.6	3	0.45	2.992	3.551
0.72	3.6	0.54	3.924	6.555
0.8	2.4	0.4	2.642	2.833
1	3	0.5	3.206	3.863
1.2	3.6	0.6	4.186	7.251
1	1.5	0.3125	2.239	2.255
2	3	0.625	3.88	4.632
2.5	3.75	0.78125	5.738	13.69
0.8	0.8	0.2	1.764	1.764
2.8	2.8	0.7	4.765	4.765
3.7	3.7	0.925	13.66	13.66

Case 6: W: 4.0 in., B: 2.0 in. and D: 0.25 in.

C1	C2	(C1+C2)/W	FE-K1	FE-K2
0.05	1	0.2625	1.5423	1.5962
0.08	1.6	0.42	2.2622	2.934
0.09	1.8	0.4725	2.9385	5.4352
0.1	1	0.275	1.6334	1.6375
0.125	1.25	0.34375	1.856	1.984
0.18	1.8	0.495	2.981	5.7702
0.1	0.7	0.2	1.378	1.352
0.2	1.4	0.4	2.0552	2.365
0.25	1.75	0.5	2.7909	4.67
0.1	0.5	0.15	1.226	1.196
0.25	1.25	0.375	1.922	2.099
0.35	1.75	0.525	2.868	4.916
0.3	0.9	0.3	1.654	1.689

C1	C2	(C1+C2)/W	FE-K1	FE-K2
0.5	1.5	0.5	2.403	3.033
0.6	1.8	0.6	3.321	7.1475
0.4	0.6	0.25	1.5055	1.499
1	1.5	0.625	2.893	3.616
1.2	1.8	0.75	4.03	8.94
0.5	0.5	0.25	1.495	1.495
1.5	1.5	0.75	4.257	4.257
1.8	1.8	0.9	10.486	10.486

Case 7: W: 4.0 in., B: 2.0 in. and D: 0.5 in.

C1	C2	(C1+C2)/W	FE-K1	FE-K2
0.05	1	0.2625	1.6406	1.857
0.07	1.4	0.3675	2.128	2.795
0.085	1.7	0.44625	3.054	7.185
0.1	1	0.275	1.817	1.894
0.125	1.25	0.34375	2.11	2.354
0.17	1.7	0.4675	3.304	7.518
0.1	0.7	0.2	1.579	1.586
0.2	1.4	0.4	2.388	2.91
0.24	1.68	0.48	3.243	6.635
0.1	0.5	0.15	1.434	1.418
0.25	1.25	0.375	2.222	2.523
0.34	1.7	0.51	3.488	8.22
0.3	0.9	0.3	1.914	1.961
0.5	1.5	0.5	2.779	3.903
0.55	1.65	0.55	3.321	6.441
0.4	0.6	0.25	1.726	1.722
1	1.5	0.625	3.385	4.655
1.1	1.65	0.6875	3.959	7.814
0.5	0.5	0.25	1.7203	1.7203
1.5	1.5	0.75	5.435	5.435
1.7	1.7	0.85	13.092	13.092

Case 8: W: 4.0 in., B: 2.0 in. and D: 1.0 in.

C1	C2	(C1+C2)/W	FE-K1	FE-K2
0.04	0.8	0.21	1.521	2.265
0.05	1	0.2625	1.787	2.626
0.07	1.4	0.3675	2.619	5.897
0.1	1	0.275	2.152	2.668
0.125	1.25	0.34375	2.602	3.847
0.14	1.4	0.385	3.092	6.041
0.1	0.7	0.2	1.895	2.159
0.15	1.05	0.3	2.414	2.908

C1	C2	(C1+C2)/W	FE-K1	FE-K2
0.2	1.4	0.4	3.266	6.201
0.1	0.5	0.15	1.734	1.94
0.2	1	0.3	2.449	2.812
0.28	1.4	0.42	3.418	6.409
0.2	0.6	0.2	2.074	2.102
0.3	0.9	0.3	2.479	2.675
0.47	1.41	0.47	3.681	7.286
0.4	0.6	0.25	2.278	2.3
0.7	1.05	0.4375	3.139	3.563
0.94	1.41	0.5875	4.46	8.546
0.5	0.5	0.25	2.27	2.27
1	1	0.5	3.769	3.769
1.4	1.4	0.7	9.26	9.26

Case 9: W: 4.0 in., B: 2.0 in. and D: 2.0 in.

C1	C2	(C1+C2)/W	FE-K1	FE-K2
0.04	0.8	0.21	2.211	5.534
0.045	0.9	0.23625	2.487	7.5
0.07	0.7	0.1925	2.533	4.67
0.09	0.9	0.2475	3.178	7.554
0.09	0.63	0.18	2.655	4.288
0.12	0.84	0.24	3.324	6.149
0.14	0.7	0.21	3.205	4.74
0.18	0.9	0.27	3.984	7.718
0.2	0.6	0.2	3.408	4.267
0.3	0.9	0.3	4.589	8.001
0.3	0.45	0.1875	3.595	3.845
0.6	0.9	0.375	5.719	8.812
0.4	0.4	0.2	3.832	3.832
0.9	0.9	0.45	9.702	9.702

E2. FE Solutions for an Offset Hole

E2.1 Cases

Case 1: W: 40.0 in., B: 12.5 in. and D: 0.25 in.

C1	C2	(C1+C2)/W	FE-K1	FE-K2
0.25	5	0.13125	2.999	2.97
0.5	10	0.2625	4.35	4.296
0.8	16	0.42	5.831	5.839
1.1	22	0.5775	7.599	8.705
1.34	26.8	0.7035	11.385	28.03
0.4	4	0.11	2.736	2.716
0.8	8	0.22	3.938	3.901
1.2	12	0.33	5.003	4.923
1.8	18	0.495	6.624	6.75
2.3	23	0.6325	8.354	10.1
2.68	26.8	0.737	11.758	29.614
0.5	3.5	0.1	2.605	2.589
1	7	0.2	3.742	3.706
2	14	0.4	5.695	5.594
3	21	0.6	7.894	8.55
3.8	26.6	0.76	11.696	26.451
0.5	2.5	0.075	2.255	2.278
1	5	0.15	3.205	3.191
2.5	12.5	0.375	5.499	5.353
4	20	0.6	8.006	8.217
5.3	26.5	0.795	12.205	26.093
0.6	1.8	0.06	2.047	2.038
2	6	0.2	3.767	3.721
4	12	0.4	5.915	5.622
6	18	0.6	8.537	7.887
8.8	26.4	0.88	15.583	27.258
0.8	1.2	0.05	1.873	1.872
4	6	0.25	4.359	4.2505
6	9	0.375	5.961	5.509
8	12	0.5	8.272	6.905
11.5	17.25	0.71875	22.924	10.916
1	1	0.05	1.882	1.88
5	5	0.25	4.425	4.286
8	8	0.4	6.873	5.981
10	10	0.5	10.213	7.326
11.8	11.8	0.59	22.2	10.004

Case 2: W: 40.0 in., B: 12.5 in. and D: 2.5 in.

C1	C2	(C1+C2)/W	FE-K1	FE-K2
0.8	16	0.42	6.551	6.45
1	20	0.525	7.76	8.361
1.2	24	0.63	9.638	14.221
1	10	0.275	5.204	4.93
2	20	0.55	8.12	8.653
2.4	24	0.66	10.031	14.842
2	14	0.4	6.457	6.155
2.5	17.5	0.5	7.543	7.464
3.4	23.8	0.68	10.338	14.66
2.5	12.5	0.375	6.25	5.901
4	20	0.6	9.023	9.28
4.8	24	0.72	11.296	15.99
4	12	0.4	6.729	6.201
6	18	0.6	9.831	8.782
8	24	0.8	15.278	17.631
4	6	0.25	5.1	4.81
8	12	0.5	10.136	7.712
10	15	0.625	18.529	10.119
7	7	0.35	7.097	6.042
10	10	0.5	14.852	8.784

Case 3: W: 40.0 in., B: 12.5 in. and D: 5.0 in.

C1	C2	(C1+C2)/W	FE-K1	FE-K2
0.8	16	0.42	7.228	7.248
1	20	0.525	8.65	9.851
1.2	24	0.63	11.301	22.704
1	10	0.275	5.989	5.614
2	20	0.55	9.392	10.195
2.4	24	0.66	12.152	23.599
2	14	0.4	7.562	6.894
2.5	17.5	0.5	8.824	8.456
3.4	23.8	0.68	12.452	22.182
2.5	12.5	0.375	7.393	6.652
4	20	0.6	10.634	10.843
4.8	24	0.72	13.798	25.389
4	12	0.4	8.049	6.975
6	18	0.6	12.131	10.061
8	24	0.8	20.833	27.736
4	6	0.25	6.205	5.628
8	12	0.5	13.768	9.098
9	13.5	0.5625	20.581	10.616
7	7	0.35	9.023	7.14
9	9	0.45	16.966	9.422

Case 4: W: 40.0 in., B: 5.0 in. and D: 2.5 in.

C1	C2	(C1+C2)/W	FE-K1	FE-K2
1.2	24	0.63	14.142	9.425
1.5	30	0.7875	18.059	14.23
2	20	0.55	14.71	8.983
3	30	0.825	31.309	15.709
2	14	0.4	10.89	7.442
3	21	0.6	23.051	10.719
2	10	0.3	8.416	6.303
3	15	0.45	16.699	9.122
2	6	0.2	6.15	5.039
3	9	0.3	10.833	7.014
2	3	0.125	4.597	4.065
3	4.5	0.1875	7.171	5.329
2	2	0.1	4.096	3.734
3	3	0.15	6.094	4.757

Case 5: W: 20.0 in., B: 6.25 in. and D: 0.25 in.

C1	C2	(C1+C2)/W	FE-K1	FE-K2
0.2	4	0.21	2.766	2.698
0.4	8	0.42	4.227	4.143
0.65	13	0.6825	7.304	13.056
0.4	4	0.22	2.857	2.809
1	10	0.55	5.286	5.569
1.3	13	0.715	7.66	13.841
0.5	3.5	0.2	2.706	2.654
1.2	8.4	0.48	4.752	4.625
1.8	12.6	0.72	7.383	11.174
0.8	4	0.24	2.953	2.961
2	10	0.6	5.88	5.969
2.6	13	0.78	8.402	15.066
0.8	2.4	0.16	2.432	2.4
2	6	0.4	4.337	4.038
3	9	0.6	6.258	5.65
1	1.5	0.125	2.099	2.095
4	6	0.5	6.093	4.956
5.8	8.7	0.725	19.301	8.123
1	1	0.1	1.916	1.875
3	3	0.3	3.683	3.469
5.8	5.8	0.58	14.967	6.994

Case 6: W: 20.0 in., B: 2.5 in. and D: 0.25 in.

C1	C2	(C1+C2)/W	FE-K1	FE-K2
0.2	4	0.21	3.083	2.85
0.5	10	0.525	6.621	5.22
0.68	13.6	0.714	8.844	6.938
0.4	4	0.22	3.25	3.013
1	10	0.55	7.849	5.637
1.35	13.5	0.7425	11.453	7.45
0.5	3.5	0.2	3.08	2.832
1.2	8.4	0.48	7.328	5.213
1.8	12.6	0.72	15.01	7.706
0.8	4	0.24	3.656	3.216
1.4	7	0.42	6.866	4.93
1.8	9	0.54	11.014	6.219
0.8	2.4	0.16	2.755	2.516
1.4	4.2	0.28	4.67	3.729
1.8	5.4	0.36	6.951	4.758
1	1.5	0.125	2.31	2.229
1.4	2.1	0.175	3.196	2.748
1.8	2.7	0.225	4.429	3.377
1	1	0.1	2.112	1.979
1.4	1.4	0.14	2.705	2.409
1.8	1.8	0.18	3.668	2.982

Case 7: W: 8.0 in., B: 2.5 in. and D: 0.25 in.

C1	C2	(C1+C2)/W	FE-K1	FE-K2
0.1	2	0.2625	2.049	1.976
0.15	3	0.39375	2.637	2.55
0.25	5	0.65625	4.34	6.9
0.2	2	0.275	2.134	2.062
0.3	3	0.4125	2.699	2.654
0.5	5	0.6875	4.491	7.222
0.3	2.1	0.3	2.256	2.143
0.5	3.5	0.5	3.149	3.153
0.7	4.9	0.7	4.463	6.614
0.4	2	0.3	2.238	2.206
0.8	4	0.6	3.746	3.884
1	5	0.75	5.022	7.839
0.8	2.4	0.4	2.791	2.601
1.2	3.6	0.6	4.049	3.67
1.6	4.8	0.8	6.096	6.798
1	1.5	0.3125	2.437	2.24
1.5	2.25	0.46875	3.664	3.065
2.1	3.15	0.65625	7.948	4.385
0.8	0.8	0.2	1.855	1.779
1.6	1.6	0.4	3.345	2.809
2.1	2.1	0.525	6.301	3.761

Case 8: W: 8.0 in., B: 1.0 in. and D: 0.25 in.

C1	C2	(C1+C2)/W	FE-K1	FE-K2
0.1	2	0.2625	2.448	2.1
0.2	4	0.525	4.461	3.374
0.27	5.4	0.70875	6.028	4.555
0.2	2	0.275	2.623	2.236
0.45	4.5	0.61875	6.23	4.047
0.54	5.4	0.7425	8.182	4.966
0.2	1.4	0.2	2.097	1.839
0.4	2.8	0.4	3.95	3.026
0.7	4.9	0.7	10.55	4.909
0.3	1.5	0.225	2.34	2.023
0.5	2.5	0.375	4.026	2.936
0.7	3.5	0.525	7.642	4.14
0.3	0.9	0.15	1.794	1.597
0.6	1.8	0.3	3.592	2.616
0.7	2.1	0.35	4.86	3.059
0.3	0.45	0.09375	1.402	1.352
0.5	0.75	0.15625	1.983	1.784
0.7	1.05	0.21875	3.072	2.318
0.3	0.3	0.075	1.264	1.237
0.5	0.5	0.125	1.723	1.594
0.7	0.7	0.175	2.555	2.03

Case 9: W: 4.0 in., B: 1.25 in. and D: 0.25 in.

C1	C2	(C1+C2)/W	FE-K1	FE-K2
0.08	1.6	0.42	2.0731	2.028
0.1	2	0.525	2.453	2.638
0.12	2.4	0.63	3.035	4.529
0.1	1	0.275	1.645	1.556
0.2	2	0.55	2.557	2.74
0.24	2.4	0.66	3.172	4.722
0.2	1.4	0.4	2.041	1.945
0.25	1.75	0.5	2.385	2.357
0.34	2.38	0.68	3.269	4.646
0.25	1.25	0.375	1.976	1.872
0.4	2	0.6	2.863	2.95
0.48	2.4	0.72	3.569	5.079
0.4	1.2	0.4	2.127	1.989
0.6	1.8	0.6	3.121	2.774
0.8	2.4	0.8	4.832	5.597
0.4	0.6	0.25	1.621	1.518
0.8	1.2	0.5	3.206	2.436
1	1.5	0.625	5.908	3.2
0.7	0.7	0.35	2.235	1.906
1	1	0.5	4.702	2.771

Case 10: W: 4.0 in., B: 0.5 in. and D: 0.25 in.

C1	C2	(C1+C2)/W	FE-K1	FE-K2
0.08	1.6	0.42	2.997	2.255
0.12	2.4	0.63	4.472	2.988
0.13	2.6	0.6825	4.869	3.34
0.1	1	0.275	2.209	1.805
0.2	2	0.55	4.651	2.862
0.26	2.6	0.715	7.144	3.735
0.1	0.7	0.2	1.781	1.528
0.2	1.4	0.4	3.442	2.371
0.3	2.1	0.6	7.291	3.397
0.1	0.5	0.15	1.508	1.334
0.2	1	0.3	2.66	1.99
0.3	1.5	0.45	5.279	2.9
0.1	0.3	0.1	1.243	1.137
0.2	0.6	0.2	1.943	1.593
0.3	0.9	0.3	3.427	2.216
0.1	0.15	0.0625	1.042	0.9923
0.2	0.3	0.125	1.452	1.283
0.3	0.45	0.1875	2.267	1.663
0.1	0.1	0.05	0.9739	0.946
0.2	0.2	0.1	1.295	1.181
0.3	0.3	0.15	1.93	1.5

E3. AFGROW vs. Handbook SIF Values

Infinite Plate Case					
C1/R	C2/R	Rooke K1 [13]	Rooke K2 [13]	AFGROW K1	AFGROW K2
0.5	0.5	2.296	2.296	2.286	2.286
0.5	1	2.467	2.442	2.469	2.404
0.5	2	2.774	2.695	2.800	2.659
0.5	5	3.513	3.432	3.615	3.431
0.5	10	4.456	4.428	4.666	4.432
1	1	2.609	2.609	2.570	2.570
1	2	2.912	2.843	2.873	2.803
1	5	3.653	3.547	3.635	3.544
1	10	4.615	4.518	4.633	4.519
2	2	3.118	3.118	3.071	3.071
2	5	3.815	3.765	3.761	3.759
2	10	4.745	4.686	4.690	4.690
5	5	4.352	4.352	4.340	4.340
5	10	5.176	5.168	5.166	5.168
10	10	5.880	5.880	5.881	5.881

Appendix F
Internal Crack growing toward a Hole in a Plate

F1. Cases
(Holes are centered in the plate for the following 29 cases)
Case 1: W: 40.0 in., D: 0.25 in., d: 2.0 in. and R/d: 0.0625

C	d-(C+R)/R	C/d	FE-K(Left)	FE-K(Right)
0.075	14.4	0.0375	0.4858	0.4856
0.15	13.8	0.075	0.6882	0.6864
0.3	12.6	0.15	0.9721	0.97
0.45	11.4	0.225	1.1905	1.188
0.75	9	0.375	1.5404	1.537
0.9	7.8	0.45	1.6909	1.685
1.2	5.4	0.6	1.964	1.951
1.5	3	0.75	2.233	2.188
1.7	1.4	0.85	2.504	2.338

Case 2: W: 40.0 in., D: 0.25 in., d: 1.5 in. and R/d: 0.0833

C	d-(C+R)/R	C/d	FE-K(Left)	FE-K(Right)
0.075	10.4	0.05	0.4867	0.4863
0.15	9.8	0.1	0.6877	0.6872
0.3	8.6	0.2	0.9719	0.9703
0.45	7.4	0.3	1.193	1.189
0.6	6.2	0.4	1.381	1.375
0.75	5	0.5	1.547	1.539
1	3	0.666667	1.815	1.782
1.2	1.4	0.8	2.102	1.958

Case 3: W: 40.0 in., D: 0.25 in., d: 1.0 in. and R/d: 0.125

C	d-(C+R)/R	C/d	FE-K(Left)	FE-K(Right)
0.075	6.4	0.075	0.4882	0.4874
0.15	5.8	0.15	0.6893	0.6879
0.2	5.4	0.2	0.7965	0.7941
0.3	4.6	0.3	0.9789	0.9732
0.35	4.2	0.35	1.061	1.0516
0.4	3.8	0.4	1.137	1.124
0.5	3	0.5	1.2857	1.2586
0.7	1.4	0.7	1.604	1.501

Case 4: W: 40.0 in., D: 0.25 in., d: 0.75 in. and R/d: 0.167

C	d-(C+R)/R	C/d	FE-K(Left)	FE-K(Right)
0.05	4.6	0.066667	0.4036	0.402
0.1	4.2	0.133333	0.5693	0.567
0.15	3.8	0.2	0.6996	0.6936
0.2	3.4	0.266667	0.8114	0.8014
0.25	3	0.333333	0.9093	0.8957
0.375	2	0.5	1.136	1.099
0.5	1	0.666667	1.454	1.279
0.55	0.6	0.733333	1.666	1.3503

Case 5: W: 40.0 in., D: 0.25 in., d: 0.6 in. and R/d: 0.2083

C	d-(C+R)/R	C/d	FE-K(Left)	FE-K(Right)
0.05	3.4	0.083333	0.4062	0.4046
0.1	3	0.166667	0.5763	0.5712
0.15	2.6	0.25	0.7091	0.6987
0.2	2.2	0.333333	0.8256	0.8074
0.25	1.8	0.416667	0.936	0.9045
0.3	1.4	0.5	1.0493	0.9927
0.35	1	0.583333	1.183	1.076
0.4	0.6	0.666667	1.416	1.158
0.44	0.28	0.733333	1.775	1.231

Case 6: W: 40.0 in., D: 0.25 in., d: 0.5 in. and R/d: 0.25

C	d-(C+R)/R	C/d	FE-K(Left)	FE-K(Right)
0.025	2.8	0.05	0.294	0.2914
0.05	2.6	0.1	0.4115	0.4089
0.075	2.4	0.15	0.505	0.4995
0.1	2.2	0.2	0.5857	0.5764
0.125	2	0.25	0.6589	0.6446
0.2	1.4	0.4	0.8587	0.8169
0.25	1	0.5	1.0038	0.9172
0.3	0.6	0.6	1.205	1.013
0.33	0.36	0.66	1.464	1.073

Case 7: W: 40.0 in., D: 0.25 in., d: 0.4 in. and R/d: 0.3125

C	d-(C+R)/R	C/d	FE-K(Left)	FE-K(Right)
0.025	2	0.0625	0.2971	0.2947
0.05	1.8	0.125	0.4231	0.4147
0.075	1.6	0.1875	0.5226	0.5074
0.1	1.4	0.25	0.6119	0.5862
0.15	1	0.375	0.7808	0.7221
0.175	0.8	0.4375	0.8751	0.783
0.2	0.6	0.5	1.004	0.8416
0.24	0.28	0.6	1.319	0.9396

Case 8: W: 40.0 in., D: 0.25 in., d: 0.35 in. and R/d: 0.357143

C	d-(C+R)/R	C/d	FE-K(Left)	FE-K(Right)
0.025	1.6	0.071429	0.3044	0.3004
0.05	1.4	0.142857	0.435	0.4227
0.075	1.2	0.214286	0.5426	0.5173
0.1	1	0.285714	0.6426	0.5979
0.125	0.8	0.357143	0.7446	0.6697
0.15	0.6	0.428571	0.8617	0.739
0.175	0.4	0.5	1.028	0.8067
0.2	0.2	0.571429	1.332	0.8845

Case 9: W: 40.0 in., D: 0.25 in., d: 0.3 in. and R/d: 0.41667

C	d-(C+R)/R	C/d	FE-K(Left)	FE-K(Right)
0.025	1.2	0.08333	0.3216	0.3133
0.035	1.12	0.11667	0.3841	0.3697
0.055	0.96	0.18333	0.4921	0.4622
0.075	0.8	0.25	0.5942	0.5398
0.1	0.6	0.33333	0.7287	0.6256
0.11	0.52	0.36667	0.7893	0.6581
0.12	0.44	0.4	0.8602	0.6921
0.125	0.4	0.41667	0.8993	0.7091
0.15	0.2	0.5	1.1847	0.7965

Case 10: W: 40.0 in., D: 0.25 in., d: 0.25 in. and R/d: 0.5

C	d-(C+R)/R	C/d	FE-K(Left)	FE-K(Right)
0.025	0.8	0.1	0.3524	0.3341
0.035	0.72	0.14	0.4254	0.3938
0.055	0.56	0.22	0.5626	0.4924
0.065	0.48	0.26	0.6347	0.5362
0.075	0.4	0.3	0.7146	0.5776
0.085	0.32	0.34	0.8024	0.6192
0.09	0.28	0.36	0.8552	0.6404
0.1	0.2	0.4	0.9873	0.6856
0.11	0.12	0.44	1.198	0.7394

Case 11: W: 40.0 in., D: 0.5 in., d: 2.5 in. and R/d: 0.1

C	d-(C+R)/R	C/d	FE-K(Left)	FE-K(Right)
0.15	8.4	0.06	0.6901	0.6901
0.6	6.6	0.24	1.383	1.3806
1.5	3	0.6	2.234	2.195
2	1	0.8	2.915	2.616

Case 12: W: 20.0 in., D: 0.25 in., d: 2.0 in. and R/d: 0.0625

C	d-(C+R)/R	C/d	FE-K(Left)	FE-K(Right)
0.075	14.4	0.0375	0.484	0.4838
0.15	13.8	0.075	0.6858	0.6848
0.3	12.6	0.15	0.9728	0.9711
0.45	11.4	0.225	1.193	1.191
0.75	9	0.375	1.546	1.543
0.9	7.8	0.45	1.698	1.693
1.2	5.4	0.6	1.978	1.967
1.5	3	0.75	2.269	2.216
1.7	1.4	0.85	2.549	2.407

Case 13: W: 20.0 in., D: 0.25 in., d: 1.5 in. and R/d: 0.0833

C	d-(C+R)/R	C/d	FE-K(Left)	FE-K(Right)
0.075	10.4	0.05	0.4847	0.4846
0.15	9.8	0.1	0.6871	0.6862
0.3	8.6	0.2	0.9737	0.9727
0.45	7.4	0.3	1.195	1.192
0.6	6.2	0.4	1.382	1.379
0.75	5	0.5	1.555	1.545
1	3	0.6666667	1.831	1.793
1.2	1.4	0.8	2.126	1.979

Case 14: W: 20.0 in., D: 0.25 in., d: 1.0 in. and R/d: 0.125

C	d-(C+R)/R	C/d	FE-K(Left)	FE-K(Right)
0.075	6.4	0.075	0.4886	0.4874
0.15	5.8	0.15	0.6918	0.6893
0.2	5.4	0.2	0.7997	0.7962
0.3	4.6	0.3	0.9834	0.9766
0.35	4.2	0.35	1.066	1.056
0.4	3.8	0.4	1.142	1.129
0.5	3	0.5	1.288	1.264
0.7	1.4	0.7	1.614	1.502

Case 15: W: 20.0 in., D: 0.25 in., d: 0.75 in. and R/d: 0.1667

C	d-(C+R)/R	C/d	FE-K(Left)	FE-K(Right)
0.05	4.6	0.0666667	0.4035	0.4019
0.1	4.2	0.1333333	0.5696	0.5666
0.15	3.8	0.2	0.6976	0.6933
0.2	3.4	0.2666667	0.8069	0.8002
0.25	3	0.3333333	0.906	0.8946
0.375	2	0.5	1.139	1.097
0.5	1	0.6666667	1.417	1.284
0.55	0.6	0.7333333	1.631	1.351

Case 16: W: 20.0 in., D: 0.25 in., d: 0.5 in. and R/d: 0.25

C	d-(C+R)/R	C/d	FE-K(Left)	FE-K(Right)
0.025	2.8	0.05	0.2955	0.2936
0.05	2.6	0.1	0.4153	0.4114
0.075	2.4	0.15	0.5064	0.5004
0.1	2.2	0.2	0.5853	0.5759
0.125	2	0.25	0.6555	0.6428
0.2	1.4	0.4	0.8557	0.8136
0.25	1	0.5	1.007	0.9175
0.3	0.6	0.6	1.212	1.013

Case 17: W: 16.0 in., D: 0.25 in., d: 2.0 in. and R/d: 0.0625

C	d-(C+R)/R	C/d	FE-K(Left)	FE-K(Right)
0.075	14.4	0.0375	0.4846	0.4845
0.15	13.8	0.075	0.6864	0.6858
0.3	12.6	0.15	0.9731	0.9725
0.45	11.4	0.225	1.193	1.192
0.75	9	0.375	1.553	1.547
0.9	7.8	0.45	1.707	1.702
1.2	5.4	0.6	1.991	1.984
1.5	3	0.75	2.28	2.247
1.7	1.4	0.85	2.583	2.423

Case 18: W: 16.0 in., D: 0.25 in., d: 1.5 in. and R/d: 0.0833

C	d-(C+R)/R	C/d	FE-K(Left)	FE-K(Right)
0.075	10.4	0.05	0.4853	0.485
0.15	9.8	0.1	0.6887	0.6869
0.3	8.6	0.2	0.9759	0.9746
0.45	7.4	0.3	1.198	1.194
0.6	6.2	0.4	1.388	1.382
0.75	5	0.5	1.5604	1.552
1	3	0.666667	1.836	1.8006
1.2	1.4	0.8	2.14	1.997

Case 19: W: 16.0 in., D: 0.25 in., d: 1.0 in. and R/d: 0.125

C	d-(C+R)/R	C/d	FE-K(Left)	FE-K(Right)
0.075	6.4	0.075	0.4889	0.4884
0.15	5.8	0.15	0.6922	0.6908
0.2	5.4	0.2	0.8004	0.7977
0.3	4.6	0.3	0.9848	0.9782
0.35	4.2	0.35	1.0657	1.0566
0.4	3.8	0.4	1.142	1.13
0.5	3	0.5	1.288	1.265
0.7	1.4	0.7	1.611	1.522

Case 20: W: 16.0 in., D: 0.25 in., d: 0.75 in. and R/d: 0.167

C	d-(C+R)/R	C/d	FE-K(Left)	FE-K(Right)
0.05	4.6	0.066667	0.4031	0.4017
0.1	4.2	0.133333	0.5703	0.5674
0.15	3.8	0.2	0.6985	0.6938
0.2	3.4	0.266667	0.8105	0.8015
0.25	3	0.333333	0.9107	0.8964
0.375	2	0.5	1.1405	1.102
0.5	1	0.666667	1.422	1.281
0.55	0.6	0.733333	1.647	1.381

Case 21: W: 16.0 in., D: 0.25 in., d: 0.5 in. and R/d: 0.25

C	d-(C+R)/R	C/d	FE-K(Left)	FE-K(Right)
0.025	2.8	0.05	0.2948	0.2929
0.05	2.6	0.1	0.415	0.4116
0.075	2.4	0.15	0.5069	0.5011
0.1	2.2	0.2	0.5864	0.5768
0.125	2	0.25	0.6612	0.6439
0.2	1.4	0.4	0.8626	0.8174
0.25	1	0.5	1.008	0.9181
0.3	0.6	0.6	1.2201	1.024

Case 22: W: 8.0 in., D: 0.25 in., d: 2.0 in. and R/d: 0.0625

C	d-(C+R)/R	C/d	FE-K(Left)	FE-K(Right)
0.075	14.4	0.0375	0.4868	0.4866
0.15	13.8	0.075	0.6892	0.6891
0.3	12.6	0.15	0.9838	0.9823
0.45	11.4	0.225	1.216	1.222
0.75	9	0.375	1.6302	1.662
0.9	7.8	0.45	1.826	1.894
1.2	5.4	0.6	2.256	2.462
1.5	3	0.75	2.799	3.376
1.7	1.4	0.85	3.449	4.551

Case 23: W: 8.0 in., D: 0.25 in., d: 1.5 in. and R/d: 0.0833

C	d-(C+R)/R	C/d	FE-K(Left)	FE-K(Right)
0.075	10.4	0.05	0.4871	0.4868
0.15	9.8	0.1	0.6889	0.6897
0.3	8.6	0.2	0.9813	0.9823
0.45	7.4	0.3	1.212	1.213
0.6	6.2	0.4	1.417	1.422
0.75	5	0.5	1.609	1.619
1	3	0.666667	1.937	1.955
1.2	1.4	0.8	2.314	2.261

Case 24: W: 8.0 in., D: 0.25 in., d: 1.0 in. and R/d: 0.125

C	d-(C+R)/R	C/d	FE-K(Left)	FE-K(Right)
0.075	6.4	0.075	0.4896	0.4894
0.15	5.8	0.15	0.6946	0.6937
0.2	5.4	0.2	0.8037	0.8004
0.3	4.6	0.3	0.9886	0.984
0.35	4.2	0.35	1.072	1.064
0.4	3.8	0.4	1.153	1.139
0.5	3	0.5	1.304	1.282
0.7	1.4	0.7	1.648	1.548

Case 25: W: 8.0 in., D: 0.25 in., d: 0.75 in. and R/d: 0.167

C	d-(C+R)/R	C/d	FE-K(Left)	FE-K(Right)
0.05	4.6	0.066667	0.4023	0.4017
0.1	4.2	0.133333	0.5705	0.5676
0.15	3.8	0.2	0.7017	0.6955
0.2	3.4	0.266667	0.8128	0.8051
0.25	3	0.333333	0.9139	0.9002
0.375	2	0.5	1.147	1.109
0.5	1	0.666667	1.435	1.297
0.55	0.6	0.733333	1.654	1.372

Case 26: W: 8.0 in., D: 0.25 in., d: 0.5 in. and R/d: 0.25

C	d-(C+R)/R	C/d	FE-K(Left)	FE-K(Right)
0.025	2.8	0.05	0.2922	0.2906
0.05	2.6	0.1	0.4143	0.4107
0.075	2.4	0.15	0.5083	0.5017
0.1	2.2	0.2	0.5899	0.579
0.125	2	0.25	0.6628	0.6467
0.2	1.4	0.4	0.8671	0.8209
0.25	1	0.5	1.0153	0.9233
0.3	0.6	0.6	1.225	1.021

Case 27: W: 4.0 in., D: 0.25 in., d: 1.0 in. and R/d: 0.125

C	d-(C+R)/R	C/d	FE-K(Left)	FE-K(Right)
0.075	6.4	0.075	0.4925	0.4921
0.15	5.8	0.15	0.7018	0.7003
0.2	5.4	0.2	0.8168	0.8159
0.3	4.6	0.3	1.024	1.027
0.35	4.2	0.35	1.121	1.131
0.4	3.8	0.4	1.219	1.238
0.5	3	0.5	1.422	1.473
0.7	1.4	0.7	1.967	2.163

Case 28: W: 4.0 in., D: 0.25 in., d: 0.75 in. and R/d: 0.167

C	d-(C+R)/R	C/d	FE-K(Left)	FE-K(Right)
0.05	4.6	0.066667	0.40403	0.4032
0.1	4.2	0.133333	0.5737	0.5715
0.15	3.8	0.2	0.7071	0.7021
0.2	3.4	0.266667	0.823	0.8159
0.25	3	0.333333	0.9301	0.9196
0.375	2	0.5	1.187	1.162
0.5	1	0.666667	1.523	1.416
0.55	0.6	0.733333	1.774	1.536

Case 29: W: 4.0 in., D: 0.25 in., d: 0.5 in. and R/d: 0.25

C	d-(C+R)/R	C/d	FE-K(Left)	FE-K(Right)
0.025	2.8	0.05	0.2919	0.2907
0.05	2.6	0.1	0.4149	0.4112
0.075	2.4	0.15	0.5104	0.5034
0.1	2.2	0.2	0.5922	0.5815
0.125	2	0.25	0.6677	0.6512
0.2	1.4	0.4	0.876	0.8299
0.25	1	0.5	1.0291	0.9374
0.3	0.6	0.6	1.246	1.044

F2. Beta Correction for a Through Crack Growing toward a Hole

Table 1: Beta Correction Table for the Crack Tip Growing toward the Hole

	R/d					
C/Cmax	0.0625	0.1250	0.1667	0.2458	0.3125	0.4167
0	1.002	1.0082	1.015	1.0338	1.063	1.132
0.1	1.0021	1.0083	1.016	1.036	1.0643	1.14
0.2	1.0023	1.0087	1.0176	1.0388	1.068	1.156
0.3	1.0026	1.0095	1.02	1.0443	1.08	1.18
0.4	1.0035	1.0115	1.026	1.0537	1.098	1.21
0.5	1.00555	1.018	1.033	1.0662	1.12	1.25
0.6	1.0088	1.028	1.046	1.093	1.159	1.32
0.7	1.014	1.045	1.0711	1.149	1.23	1.42
0.8	1.026	1.074	1.126	1.24	1.35	1.576
0.9	1.075	1.19	1.3	1.47	1.634	1.96
1	1.38	1.68	1.82	2.13	2.38	2.81

Table 1: Continued

	R/d				
C/Cmax	0.5000	0.7000	0.8000	0.9000	0.9700
0	1.22	1.6	1.95	2.38	2.8
0.1	1.2355	1.631	1.974	2.4065	2.84
0.2	1.2577	1.681	2.015	2.44909	2.92
0.3	1.29	1.743	2.083	2.5035	3.02
0.4	1.329	1.824	2.176	2.5824	3.11
0.5	1.39	1.935	2.299	2.704	3.24
0.6	1.47	2.05	2.42	2.84	3.41
0.7	1.579	2.22	2.61	3.06	3.63
0.8	1.7615	2.47	2.904	3.4065	4.1
0.9	2.27	3.109	3.69	4.37	5.3
1	3.21	4.34	5.2	6.2	7.8

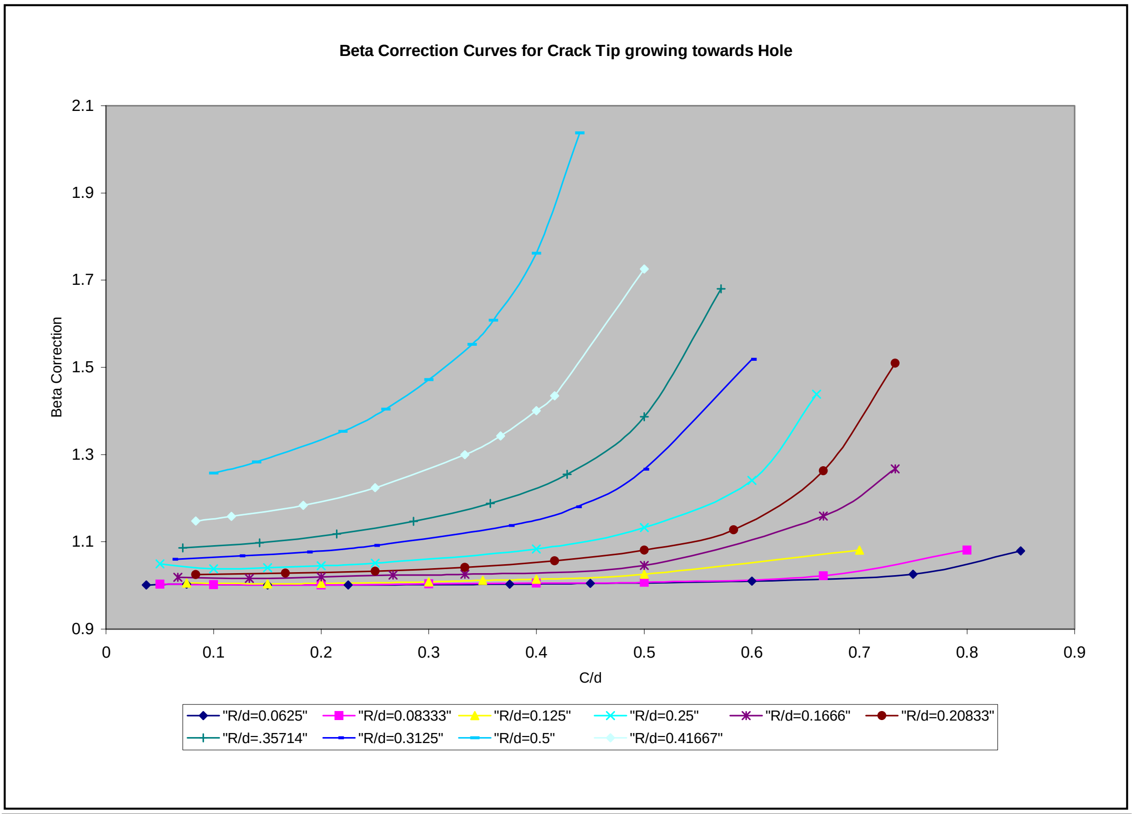
Table 2: Beta Correction Table for the Crack Tip Growing away from the Hole

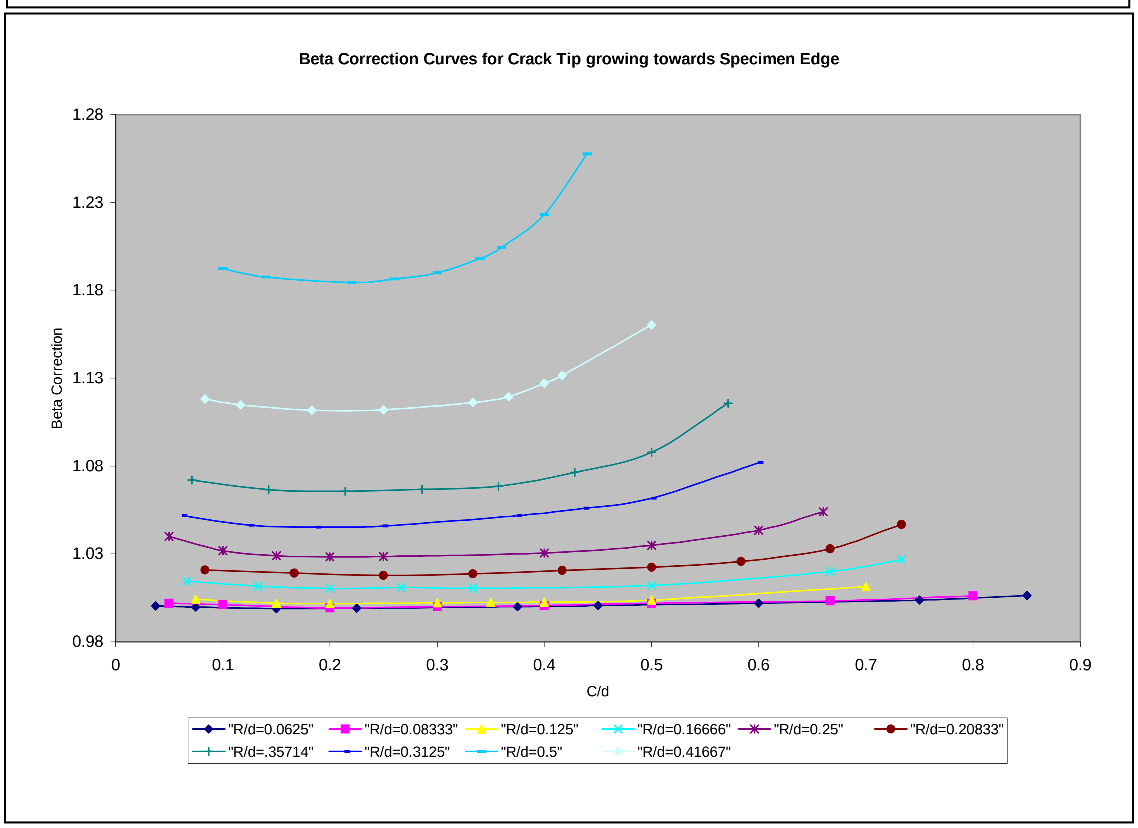
	R/d					
C/Cmax	0.0625	0.1250	0.1667	0.2458	0.3125	0.4167
0	1.0019	1.00612	1.015	1.03455	1.058	1.131
0.1	1.0004	1.00261	1.01132	1.0279	1.0493	1.1192
0.2	1.00003	1.00214	1.0101	1.0253	1.046	1.1149
0.3	1	1.00185	1.0099	1.0247	1.045	1.1127
0.4	1.00022	1.00231	1.01055	1.0262	1.0466	1.1157
0.5	1.0007	1.00295	1.011	1.0283	1.0496	1.1196
0.6	1.00151	1.0043	1.0132	1.0314	1.054	1.1268
0.7	1.0026	1.00791	1.0164	1.0375	1.0627	1.1388
0.8	1.0039	1.0115	1.022	1.0444	1.0732	1.1571
0.9	1.0062	1.0165	1.029	1.0609	1.0922	1.182
1	1.01	1.0244	1.04	1.081	1.1208	1.215

Table 2: Continued

	R/d				
C/Cmax	0.5000	0.7000	0.8000	0.9000	1.0000
0	1.2141	1.605	1.93	2.389	2.985
0.1	1.1948	1.575	1.9075	2.359	2.949
0.2	1.1885	1.566	1.8879	2.346	2.925
0.3	1.186	1.56	1.8862	2.335	2.909
0.4	1.19	1.574	1.903	2.348	2.929
0.5	1.1955	1.588	1.926	2.366	2.95
0.6	1.204	1.597	1.929	2.387	2.984
0.7	1.2182	1.62	1.97	2.415	3.05
0.8	1.2322	1.638	1.9935	2.49	3.16
0.9	1.2544	1.655	2.021	2.576	3.3
1	1.2908	1.691	2.056	2.73	3.5

F3. Characteristic Plots for an Internal Crack growing toward a Hole





F4. Handbook and FE Comparison to AFGROW

F4.1 Handbook SIF Comparisons for an Infinite Plate Case (Crack Tip Growing to the Hole)

Case 1: R/d: 0.1

C	C/(d-R)	AFGROW	Rooke & Cartwright
0.15	0.066667	1.005	1
0.6	0.266667	1.006	1.008
1.5	0.666667	1.026	1.024
2	0.888889	1.111	1.13

Case 2: R/d: 0.20833

C	C/(d-R)	AFGROW	Rooke & Cartwright
0.1	0.1052632	1.025	1.025
0.2	0.2105263	1.028	1.03
0.3	0.3157895	1.033	1.035
0.4	0.4210526	1.041	1.04
0.5	0.5263158	1.052	1.055
0.6	0.6315789	1.078	1.081
0.7	0.7368421	1.132	1.128
0.8	0.8421053	1.238	1.24

Case 3: R/d: 0.5

C	C/(d-R)	AFGROW	Rooke & Cartwright
0.05	0.2	1.258	1.261
0.07	0.28	1.283	1.285
0.11	0.44	1.351	1.355
0.13	0.52	1.405	1.407
0.15	0.6	1.470	1.478
0.17	0.68	1.556	1.575
0.18	0.72	1.604	1.604
0.2	0.8	1.762	1.79
0.22	0.88	2.131	2.04

F4.2 StressCheck Comparison to AFGROW
(Plate Width = 40 in., Hole Dia. = 0.25 in.)

(The hole and crack are offset in the following 4 cases)

Case 1: B = 30 in., d = 5.0625

		AFGROW		**StressCheck**	
C	C/(d-R)	K(edge)	K(hole)	K(edge)	K(hole)
0.246875	0.05	0.882	0.8821	0.8493	0.8556
0.49375	0.1	1.251	1.251	1.2620	1.2510
0.740625	0.15	1.54	1.538	1.5460	1.5370
0.9875	0.2	1.791	1.786	1.7880	1.7840
1.234375	0.25	2.022	2.012	2.0190	2.0100
1.48125	0.3	2.243	2.224	2.2390	2.2180
1.728125	0.35	2.46	2.427	2.4520	2.4210
1.975	0.4	2.68	2.627	2.6710	2.6170
2.221875	0.45	2.907	2.826	2.9020	2.8120
2.46875	0.5	3.149	3.026	3.1220	3.0080
2.715625	0.55	3.412	3.232	3.3760	3.2070
2.9625	0.6	3.704	3.443	3.6560	3.4100
3.209375	0.65	4.037	3.662	3.9710	3.6230
3.45625	0.7	4.425	3.896	4.3380	3.8470
3.703125	0.75	4.891	4.154	4.7790	4.0890
3.95	0.8	5.47	4.423	5.3350	4.3570
4.196875	0.85	6.225	4.711	6.0890	4.6690
4.44375	0.9	7.325	5.134	7.2590	5.0660
4.690625	0.95	9.501	5.902	9.6000	5.7630

Case 2: B = 35 in., d = 2.5625

		AFGROW		StressCheck	
C	C/(d-R)	K(edge)	K(hole)	K(edge)	K(hole)
0.121875	0.05	0.6199	0.6202	0.5997	0.6019
0.24375	0.1	0.8784	0.8791	0.8856	0.8800
0.365625	0.15	1.08	1.08	1.0780	1.0800
0.4875	0.2	1.255	1.254	1.2540	1.2520
0.609375	0.25	1.415	1.41	1.4150	1.4090
0.73125	0.3	1.566	1.556	1.5640	1.5530
0.853125	0.35	1.714	1.696	1.7090	1.6920
0.975	0.4	1.862	1.832	1.8580	1.8240
1.096875	0.45	2.014	1.966	2.0050	1.9560
1.21875	0.5	2.174	2.1	2.1550	2.0870
1.340625	0.55	2.346	2.237	2.3200	2.2180
1.4625	0.6	2.536	2.377	2.5000	2.3510
1.584375	0.65	2.749	2.519	2.6990	2.4890
1.70625	0.7	2.994	2.673	2.9280	2.6330
1.828125	0.75	3.283	2.843	3.2000	2.7880
1.95	0.8	3.63	3.016	3.5390	2.9610
2.071875	0.85	4.065	3.195	3.9940	3.1680
2.19375	0.9	4.663	3.509	4.6890	3.4540
2.315625	0.95	5.78	4.125	6.0970	4.0220

Case 3: B = 38 in., d = 1.0625

		AFGROW		StressCheck	
C	C/(d-R)	K(edge)	K(hole)	K(edge)	K(hole)
0.046875	0.05	0.3852	0.3868	0.3717	0.3747
0.09375	0.1	0.5452	0.5482	0.5567	0.5508
0.140625	0.15	0.67	0.6735	0.6716	0.6746
0.1875	0.2	0.7778	0.7811	0.7813	0.7833
0.234375	0.25	0.8757	0.8782	0.8856	0.8818
0.28125	0.3	0.968	0.9685	0.9737	0.9707
0.328125	0.35	1.058	1.054	1.0640	1.0580
0.375	0.4	1.147	1.139	1.1620	1.1420
0.421875	0.45	1.239	1.223	1.2450	1.2240
0.46875	0.5	1.334	1.308	1.3360	1.3070
0.515625	0.55	1.435	1.395	1.4360	1.3910
0.5625	0.6	1.547	1.485	1.5450	1.4770
0.609375	0.65	1.673	1.58	1.6660	1.5680
0.65625	0.7	1.817	1.683	1.8050	1.6670
0.703125	0.75	1.983	1.797	1.9700	1.7780
0.75	0.8	2.178	1.917	2.1760	1.9120
0.796875	0.85	2.416	2.058	2.4530	2.0890
0.84375	0.9	2.728	2.347	2.8790	2.3650
0.890625	0.95	3.285	2.921	3.7460	2.9500

Case 4: B = 39 in., d = 0.5625

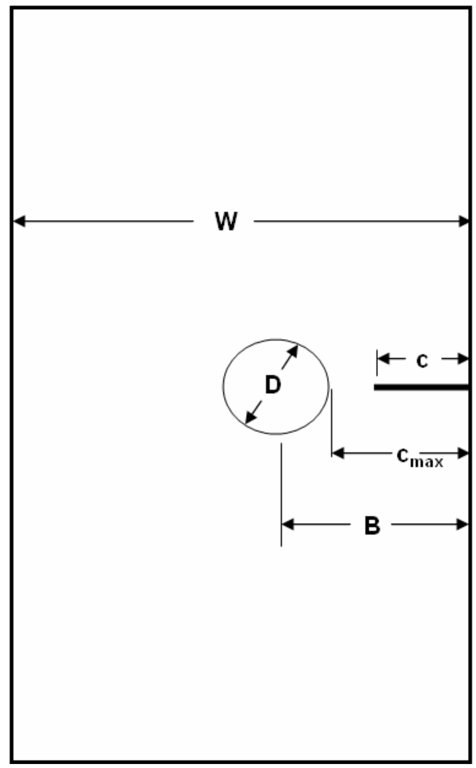
		AFGROW		StressCheck	
C	C/(d-R)	K(edge)	K(hole)	K(edge)	K(hole)
0.021875	0.05	0.269	0.2697	0.2601	0.2615
0.04375	0.1	0.3803	0.3825	0.3889	0.3880
0.065625	0.15	0.4667	0.4704	0.4686	0.4736
0.0875	0.2	0.5412	0.5462	0.5447	0.5509
0.109375	0.25	0.609	0.6148	0.6181	0.6210
0.13125	0.3	0.6729	0.6793	0.6772	0.6867
0.153125	0.35	0.7352	0.7418	0.7403	0.7502
0.175	0.4	0.7973	0.8031	0.8044	0.8143
0.196875	0.45	0.8604	0.8637	0.8676	0.8831
0.21875	0.5	0.9261	0.9253	0.9336	0.9390
0.240625	0.55	0.9965	0.9905	1.0040	1.0090
0.2625	0.6	1.074	1.062	1.0800	1.0770
0.284375	0.65	1.16	1.143	1.1660	1.1540
0.30625	0.7	1.259	1.235	1.2660	1.2440
0.328125	0.75	1.372	1.343	1.3860	1.3490
0.35	0.8	1.506	1.471	1.5380	1.4820
0.371875	0.85	1.671	1.638	1.7440	1.6660
0.39375	0.9	1.884	1.926	2.0620	1.9510
0.415625	0.95	2.254	2.423	2.7060	2.5180

Appendix G
Edge Crack Growing Toward a Hole

G1. Cases

W = 40 inches, Hole Dia. = 0.25

B=0.25					
C/Cmax	K	C	Beta	Afgrow	Correction
0.1	0.1724	0.0125	0.869976	0.2747	0.6276
0.2	0.2995	0.025	1.06869	0.3955	0.7573
0.3	0.4256	0.0375	1.23997	0.4969	0.8565
0.5	0.6998	0.0625	1.579279	0.6913	1.0123
0.75	1.1770	0.09375	2.168782	1.004	1.1723
0.9	1.7460	0.1125	2.936926	1.515	1.1525
0.95	2.1820	0.11875	3.572424	1.855	1.1763
0.98	2.9720	0.1225	4.790776	2.301	1.2916



B=0.5					
C/Cmax	K	C	Beta	Afgrow	Correction
0.1	0.3579	0.0375	1.042728	0.3996	0.8956
0.2	0.5348	0.075	1.101757	0.5668	0.9435
0.3	0.6824	0.1125	1.147857	0.6982	0.9774
0.5	0.9493	0.1875	1.236881	0.9215	1.0302
0.75	1.3460	0.28125	1.431937	1.263	1.0657
0.9	1.8740	0.3375	1.819943	1.713	1.0940
0.95	2.3360	0.35625	2.208109	2.109	1.1076

B=1.0					
C/Cmax	K	C	Beta	Afgrow	Correction
0.1	0.5758	0.0875	1.098228	0.5935	0.9702
0.2	0.8279	0.175	1.116565	0.8401	0.9855
0.3	1.0260	0.2625	1.129816	1.03	0.9961
0.5	1.3530	0.4375	1.154074	1.343	1.0074
0.75	1.7520	0.65625	1.220182	1.715	1.0216
0.9	2.2120	0.7875	1.406322	2.114	1.0464
0.95	2.6680	0.83125	1.650992	2.555	1.0442

B=2.0					
C/Cmax	K	C	Beta	Afgrow	Correction
0.1	0.9735	0.1875	1.268412	0.864	1.1267
0.2	1.2160	0.375	1.120322	1.224	0.9935
0.3	1.4950	0.5625	1.124618	1.502	0.9953
0.5	1.9470	0.9375	1.134502	1.952	0.9974
0.75	2.4380	1.40625	1.159919	2.442	0.9984
0.9	2.8600	1.6875	1.242136	2.826	1.0120
0.95	3.2620	1.78125	1.378944	3.253	1.0028

B=5.0					
C/Cmax	K	C	Beta	Afgrow	Correction
0.1	1.3880	0.4875	1.121572	1.394	0.9957
0.2	1.9710	0.975	1.126184	1.982	0.9945
0.3	2.4270	1.4625	1.132262	2.442	0.9939
0.5	3.1820	2.4375	1.149881	3.205	0.9928
0.75	4.0150	3.65625	1.184657	4.049	0.9916
0.9	4.5610	4.3875	1.228504	4.61	0.9894
0.95	4.9070	4.63125	1.286447	5.016	0.9783

B=10					
C/Cmax	K	C	Beta	Afgrow	Correction
0.1	1.9940	0.9875	1.132092	1.994	1.0000
0.2	2.8440	1.975	1.14175	2.858	0.9951
0.3	3.5390	2.9625	1.160049	3.564	0.9930
0.5	4.8130	4.9375	1.222045	4.835	0.9954
0.75	6.4460	7.40625	1.336338	6.461	0.9977
0.9	7.5620	8.8875	1.431106	7.604	0.9945
0.95	8.0660	9.38125	1.485775	8.201	0.9835

B=20					
C/Cmax	K	C	Beta	AFGROW	Correction
0.1	2.8180	1.9875	1.127749	2.867	0.9829
0.2	4.1960	3.975	1.187386	4.223	0.9936
0.3	5.4680	5.9625	1.263396	5.483	0.9973
0.5	8.3430	9.9375	1.493168	8.327	1.0019
0.75	13.4700	14.90625	1.968378	13.46	1.0007
0.9	18.0800	17.8875	2.411841	18.15	0.9961
0.95	20.1000	18.88125	2.609791	20.35	0.9877

B=30					
C/Cmax	K	C	Beta	AFGROW	Correction
0.01	1.09	0.29875	1.125117	1.089	1.0009
0.05	2.453	1.49375	1.132358	2.467	0.9943
0.1	3.5560	2.9875	1.160734	3.579	0.9936
0.2	5.4770	5.975	1.264151	5.491	0.9975
0.3	7.5690	8.9625	1.426425	7.558	1.0015

B=35					
C/Cmax	K	C	Beta	AFGROW	Correction
0.01	1.174	0.34875	1.121595	1.177	0.9975
0.05	2.658	1.74375	1.135633	2.675	0.9936
0.1	3.8860	3.4875	1.174007	3.91	0.9939
0.2	6.1400	6.975	1.31166	6.146	0.9990
0.3	8.7830	10.4625	1.53197	8.762	1.0024

B=38					
C/Cmax	K	C	Beta	AFGROW	Correction
0.01	1.224	0.37875	1.122096	1.227	0.9976
0.05	2.776	1.89375	1.138107	2.794	0.9936
0.1	4.0800	3.7875	1.182793	4.103	0.9944
0.2	6.5530	7.575	1.343303	6.554	0.9998
0.3	9.5850	11.3625	1.60428	9.559	1.0027

B=39					
C/Cmax	K	C	Beta	AFGROW	Correction
0.01	1.24	0.38875	1.122048	1.243	0.9976
0.05	2.814	1.94375	1.138751	2.833	0.9933
0.1	4.1440	3.8875	1.185795	4.167	0.9945
0.2	6.6940	7.775	1.354443	6.693	1.0001
0.3	9.8660	11.6625	1.629935	9.839	1.0027

B=39.5					
C/Cmax	K	C	Beta	AFGROW	Correction
0.01	1.248	0.39375	1.122094	1.251	0.9976
0.05	2.833	1.96875	1.139138	2.852	0.9933
0.1	4.1760	3.9375	1.18734	4.199	0.9945
0.2	6.7650	7.875	1.36009	6.764	1.0001
0.3	10.0100	11.8125	1.643192	9.982	1.0028

B=39.75					
C/Cmax	K	C	Beta	AFGROW	Correction
0.01	1.252	0.39625	1.122134	1.255	0.9976
0.05	2.843	1.98125	1.139547	2.862	0.9934
0.1	4.1910	3.9625	1.18784	4.215	0.9943
0.2	6.8010	7.925	1.363007	6.799	1.0003
0.3	10.0800	11.8875	1.649454	10.05	1.0030

G2. Finite Plate Beta Correction for an Edge Crack Growing to a Hole

Note: These corrections are applied to the single edge crack case without a hole

W/D=32	**B/W**					
C/Cmax	0.0319	0.0625	0.1	0.125	0.25	0.4
0	0.66700	0.91710	0.97500	0.98400	0.99810	1.00038
0.01	0.67800	0.91900	0.97600	0.98600	0.99935	1.00049
0.05	0.74500	0.93030	0.97650	0.98781	0.99980	1.00058
0.1	0.82767	0.94600	0.97940	0.98800	1.00020	1.00080
0.15	0.90000	0.96300	0.98700	0.99300	1.00070	1.00104
0.2	0.98070	0.98500	0.99080	0.99500	1.00100	1.00109
0.3	1.12555	1.01850	1.00540	1.00500	1.00160	1.00130
0.5	1.39000	1.09980	1.03000	1.02200	1.00574	1.00300
0.7	1.70400	1.20587	1.06480	1.04400	1.01527	1.00800
0.8	2.04900	1.32601	1.12000	1.08850	1.02760	1.01380
0.9	2.73180	1.69000	1.30000	1.21150	1.07200	1.04500
0.95	3.31479	1.99000	1.55000	1.42840	1.20548	1.14500
0.98	3.85000	2.52700	1.94500	1.80000	1.43500	1.33400

W/D=32	**B/W**				
C/Cmax	**0.5**	**0.75**	**0.875**	**0.9375**	**0.96875**
0	1.00058	1.00011	1.00009	1.00007	1.00006
0.01	1.00066	1.00015	1.00012	1.00009	1.00007
0.05	1.00070	1.00020	1.00018	1.00012	1.00000
0.1	1.00112	1.00030	1.00025	1.00020	1.00016
0.15	1.00111	1.00040	1.00035	1.00029	1.00025
0.2	1.00103	1.00063	1.00045	1.00037	1.00031
0.3	1.00110	1.00080	1.00059	1.00056	1.00050
0.5	1.00250	1.00150	1.00138	1.00127	1.00120
0.7	1.00542	1.00305	1.00318	1.00288	1.00277
0.8	1.00910	1.00734	1.00630	1.00604	1.00587
0.9	1.03526	1.02309	1.01740	1.01450	1.01790
0.95	1.12300	1.08000	1.05834	1.05610	1.05249
0.98	1.28900	1.19000	1.16500	1.15110	1.13820

W/D=16	B/W					
C/Cmax	0.0625	0.125	0.175	0.25	0.5	0.75
0	0.64300	0.89500	0.94500	0.95700	0.99000	0.98500
0.01	0.66880	0.90300	0.95100	0.96400	0.99300	0.98960
0.05	0.73700	0.92340	0.96400	0.97650	0.99169	0.98627
0.1	0.81528	0.94820	0.97600	0.98730	0.99110	0.98646
0.2	0.96600	0.99180	0.98900	0.99860	0.99535	0.99290
0.3	1.11420	1.02980	1.01600	1.00760	1.00060	0.99780
0.5	1.42470	1.10924	1.05400	1.03107	1.01080	1.00223
0.7	1.79000	1.23700	1.11700	1.07750	1.02630	1.00390
0.8	2.08300	1.36700	1.19100	1.13340	1.04900	1.01166
0.9	2.64300	1.65870	1.39700	1.29032	1.09000	1.03500
0.95	3.29800	2.03674	1.70000	1.54050	1.27590	1.19423
0.98	4.30000	2.70700	2.23500	1.99100	1.70000	1.55000

W/D=8	B/W					
C/Cmax	0.125	0.25	0.35	0.5	0.75	0.875
0	0.67800	0.89100	0.94600	0.97300	0.96000	0.93100
0.01	0.69838	0.90887	0.95900	0.97530	0.96190	0.93270
0.05	0.76923	0.94396	0.97400	0.98091	0.96310	0.93420
0.1	0.84868	0.96799	0.98500	0.98548	0.96546	0.94401
0.2	1.00380	1.01049	1.00800	0.99715	0.97672	0.96166
0.3	1.15510	1.04906	1.02700	1.00979	0.98791	0.97865
0.5	1.47630	1.13300	1.04100	1.01200	1.00074	1.00146
0.7	1.90250	1.27491	1.14000	1.08462	1.01576	1.02213
0.8	2.23170	1.42323	1.21700	1.14929	1.04129	1.03465
0.9	2.82820	1.76436	1.46100	1.33977	1.14930	1.10000
0.95	3.53230	2.21475	1.82300	1.65166	1.38360	1.28200
0.98	4.90000	2.95500	2.40600	2.10000	1.75400	1.61600

W/D=4	B/W		
C/Cmax	0.25	0.5	0.75
0	0.76200	0.90000	0.81200
0.01	0.78174	0.91050	0.82700
0.05	0.85524	0.93680	0.83899
0.1	0.93862	0.95740	0.85889
0.2	1.10088	1.00180	0.89094
0.3	1.25990	1.04290	0.92738
0.5	1.60706	1.13110	0.98802
0.7	2.10042	1.28600	1.05819
0.8	2.51120	1.47020	1.13432
0.9	3.30740	1.90330	1.35551
0.95	4.27940	2.54450	1.77078
0.98	6.00000	3.52000	2.43000

W/D=2	B/W		
C/Cmax	0.4	0.5	0.6
0	0.53950	0.71200	0.57490
0.01	0.56040	0.72925	0.58958
0.05	0.65210	0.79804	0.63970
0.1	0.77860	0.88743	0.71250
0.2	1.12510	1.05882	0.84276
0.3	1.48160	1.22640	0.97605
0.5	2.26360	1.59321	1.24551
0.7	3.51140	2.14995	1.58329
0.8	4.54360	2.66005	1.83686
0.9	6.54670	3.76750	2.40137
0.95	8.89160	5.28700	3.29298
0.98	12.20000	7.34996	4.55000

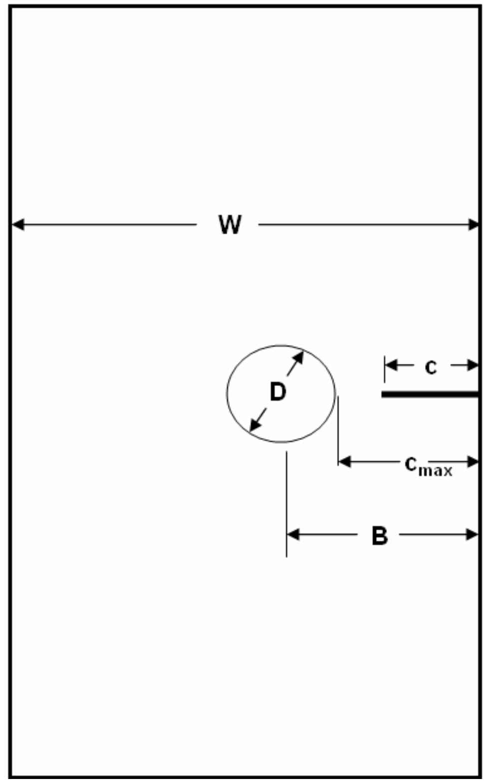
W/D=1.5	B/W
C/Cmax	0.5
0	0.56900
0.01	0.59542
0.05	0.72967
0.1	0.88956
0.2	1.23046
0.3	1.58384
0.5	2.39059
0.7	3.54787
0.8	4.50833
0.9	6.45507
0.95	9.05433
0.98	14.27140

G3. Edge Crack Growing Toward a Hole (Test Cases)

Comparison Between AFGROW and StressCheck

Case 1: W = 0.5, D = 0.25, B = 0.25

C	C/Cmax	AFGROW	StressCheck
0.00125	0.01	0.05131	0.0513
0.00625	0.05	0.1259	0.1259
0.0125	0.1	0.199	0.1991
0.025	0.2	0.3404	0.3404
0.0375	0.3	0.4919	0.4919
0.0625	0.5	0.8675	0.8675
0.0875	0.7	1.481	1.4811
0.1	0.8	2.038	2.0376
0.1125	0.9	3.195	3.1952
0.11875	0.95	4.712	4.7123



Case 2: W = 1.0, D = 0.25, B = 0.75

C	C/Cmax	AFGROW	StressCheck
0.1	0.16	0.5898	0.5916
0.2	0.32	1.012	1.0166
0.375	0.6	2.182	2.1772
0.5	0.8	4.018	3.9899
0.575	0.92	7.132	7.2366

Case 3: W = 1.0, D = 0.5, B = 0.6

C	C/Cmax	AFGROW	StressCheck
0.05	0.1429	0.3505	0.3408
0.1	0.2857	0.6412	0.6310
0.225	0.6429	1.768	1.7715

Case 4: W = 2.0, D = 0.5, B = 0.5

C	C/Cmax	AFGROW	StressCheck
0.0025	0.01	0.0776	0.0784
0.0125	0.05	0.1905	0.1906
0.025	0.1	0.2962	0.2960
0.05	0.2	0.4938	0.4934
0.075	0.3	0.6965	0.6968
0.125	0.5	1.165	1.1659
0.175	0.7	1.84	1.8404
0.2	0.8	2.38	2.3817
0.225	0.9	3.369	3.3319
0.2375	0.95	4.51	4.3863

Case 5: W = 4.0, D = 0.5, B = 3.0

C	C/Cmax	AFGROW	StressCheck
0.0275	0.01	0.3178	0.3176
0.1375	0.05	0.7197	0.7185
0.275	0.1	1.043	1.0428
0.55	0.2	1.602	1.6020
0.825	0.3	2.196	2.1962
1.375	0.5	3.804	3.8072
1.925	0.7	6.654	6.6584
2.2	0.8	9.167	9.1726
2.475	0.9	13.99	14.0080
2.6125	0.95	20.1	20.1190