

# Air Force Life Cycle Management Center



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## Assessing Limited Pedigree da/dN Crack Propagation Data

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- **A-10 Structures and Aero Team**



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# Overview



## ■ Motivation

## ■ Data Evaluation Example

### ■ 7050

- Fixed Ranges of  $da/dN$  (Scatter of 2)
- Fixed Ranges of  $\Delta K$  (+/- 5%)
  - $R = 0.1, 0.4, 0.8$

### ■ 2024

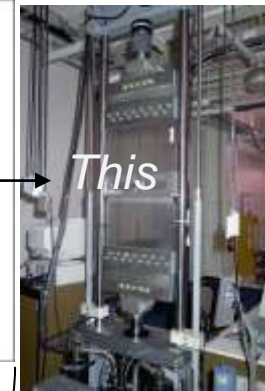
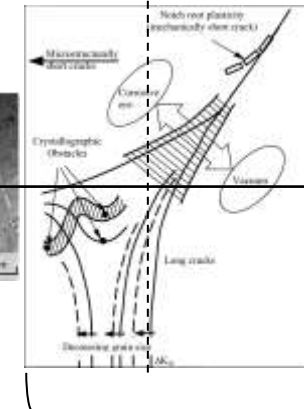
- Fixed Ranges of  $\Delta K$  (+/- 5%)
  - $R = 0.1$

## ■ Observations

## ■ Concluding Remarks



*Is the earth flat or spherical?  
Turns out that even though evidence  
has suggested otherwise since about  
Aristotle's time, it is a recurring debate!?*





# Motivation



- **Access to  $da/dN$  v  $\Delta K$  data in the public domain is limited to**
  - MMPDS, Damage Tolerance Design Handbook, Software/models, AFMAT, etc.
- **Data pedigree is often minimal**
  - $\Delta K_{Applied}$  or  $\Delta K_{Effective}$ , etc.
  - Constant Amplitude (CA), K-Decreasing, K-Increasing, etc.
  - Product Form, thickness range, test environment, etc.
- **Modification and sustainment engineers and DERs struggle to acquire data**
  - Limited expertise in
    - Testing and data evaluation
    - Scatter, test invalidities (angle, symmetry, etc.)
  - Limited funding



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# Specific Example



## ■ 7050-T7451 L-T

### ■ What we know about this data set

- 7050 is a controlled damage tolerant alloy: Composition and processing are controlled to meet specific requirements, e.g. strength, toughness, and corrosion resistance to include stress corrosion
- Stress Ratio
- Range of  $\Delta K$
- Grain orientation

### ■ What we do not know

- Specimen configuration
- Product form thickness
- Testing environment and frequency
- Test Type (CA, K-Decreasing, etc.)
- Test data supplier
- Producer/lot variation

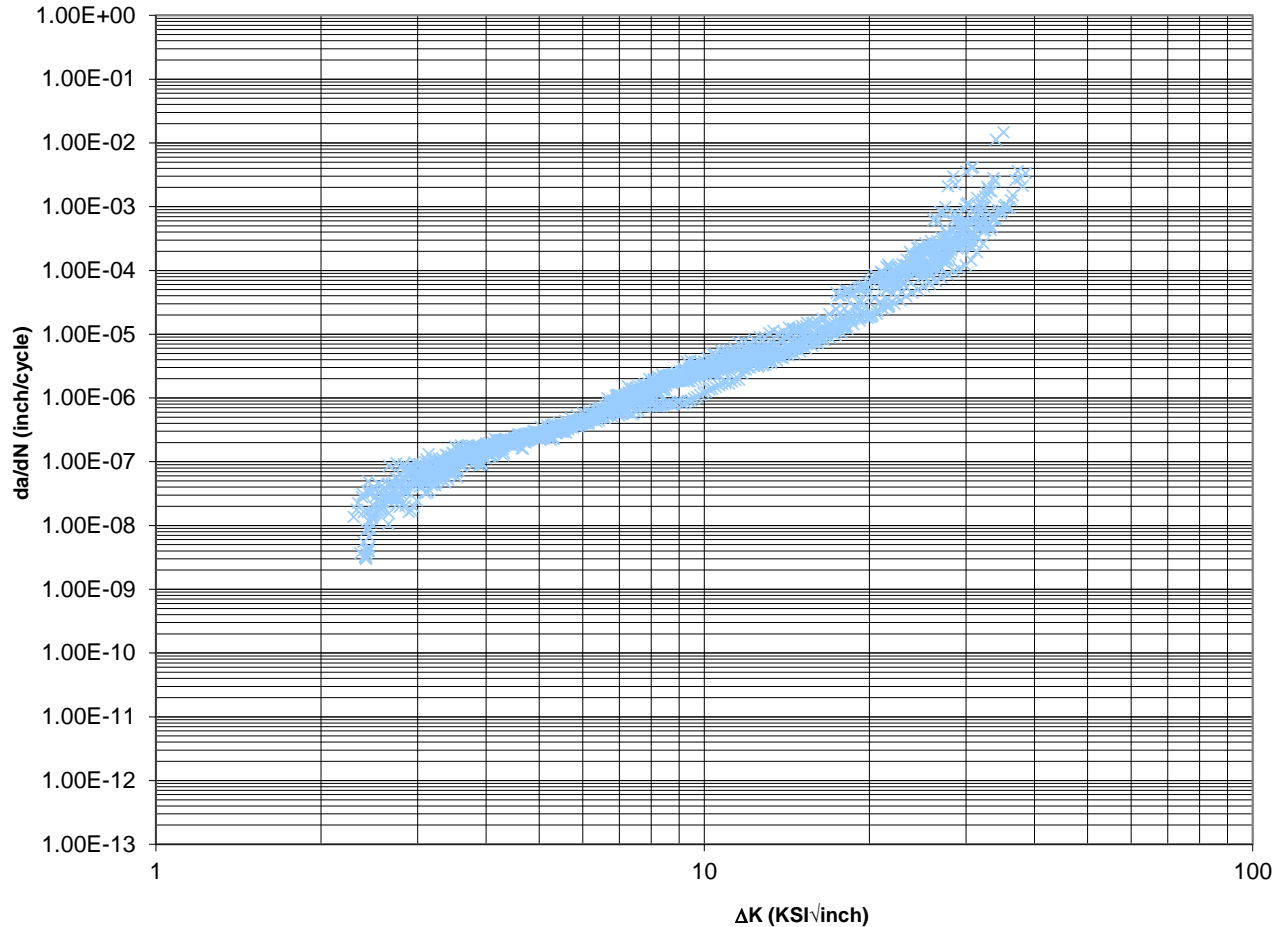


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# Generic Test Data Example



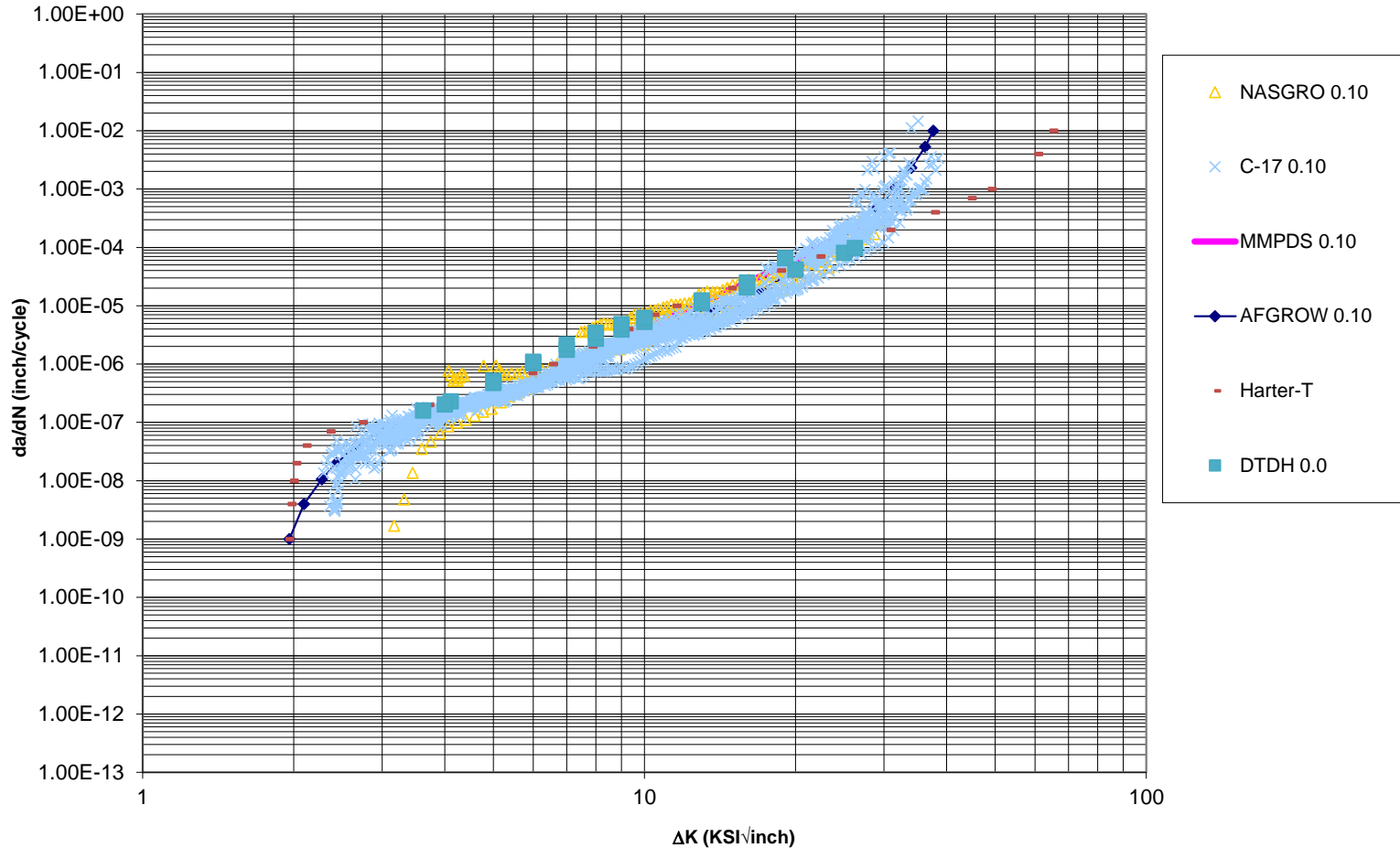
7050-T7451 L-T Crack Growth Rates  
(C-17 Only)



# Data Comparison (Establishing 'Confidence')



7050-T7451 L-T Crack Growth Rates  
(C-17 compared to Other Sources)



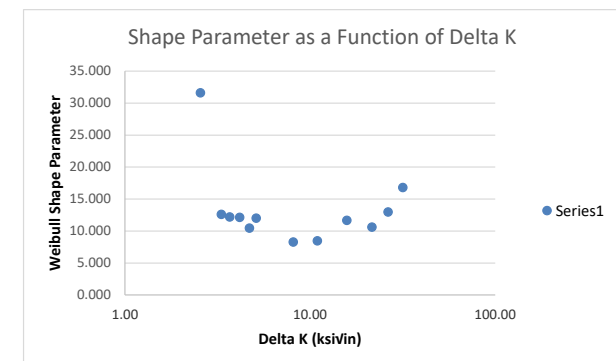


# Option 1: Analyze $\Delta K$ in Ranges of $da/dN$



- Choose ranges of  $da/dN$  (Max/Min = 2) to analyze from total population of data
  - No justification on scatter factor of 2 (folklore)
- Develop Maximum Likelihood Estimation (MLE)<sup>4</sup> distribution parameters for  $\Delta K$  within each given range of  $da/dN$ 
  - 1388 data pairs across 13  $da/dN$  ranges
  - Approximately 107  $\Delta K$  points/ $da/dN$
- Resulting shape parameters as a function of  $\Delta K$

Fixed $da/dN$ Midpoint $da/dN$	Weibull parameters for Delta K	
	Delta K	Shape
1.920E-08	2.56	31.615
6.940E-08	3.32	12.620
1.130E-07	3.68	12.200
1.650E-07	4.17	12.143
2.240E-07	4.70	10.457
2.770E-07	5.11	12.020
1.176E-06	8.12	8.285
3.050E-06	10.95	8.458
9.490E-06	15.81	11.675
4.460E-05	21.59	10.608
1.650E-04	26.35	12.993
6.110E-04	31.69	16.812
1.750E-03	34.18	13.136

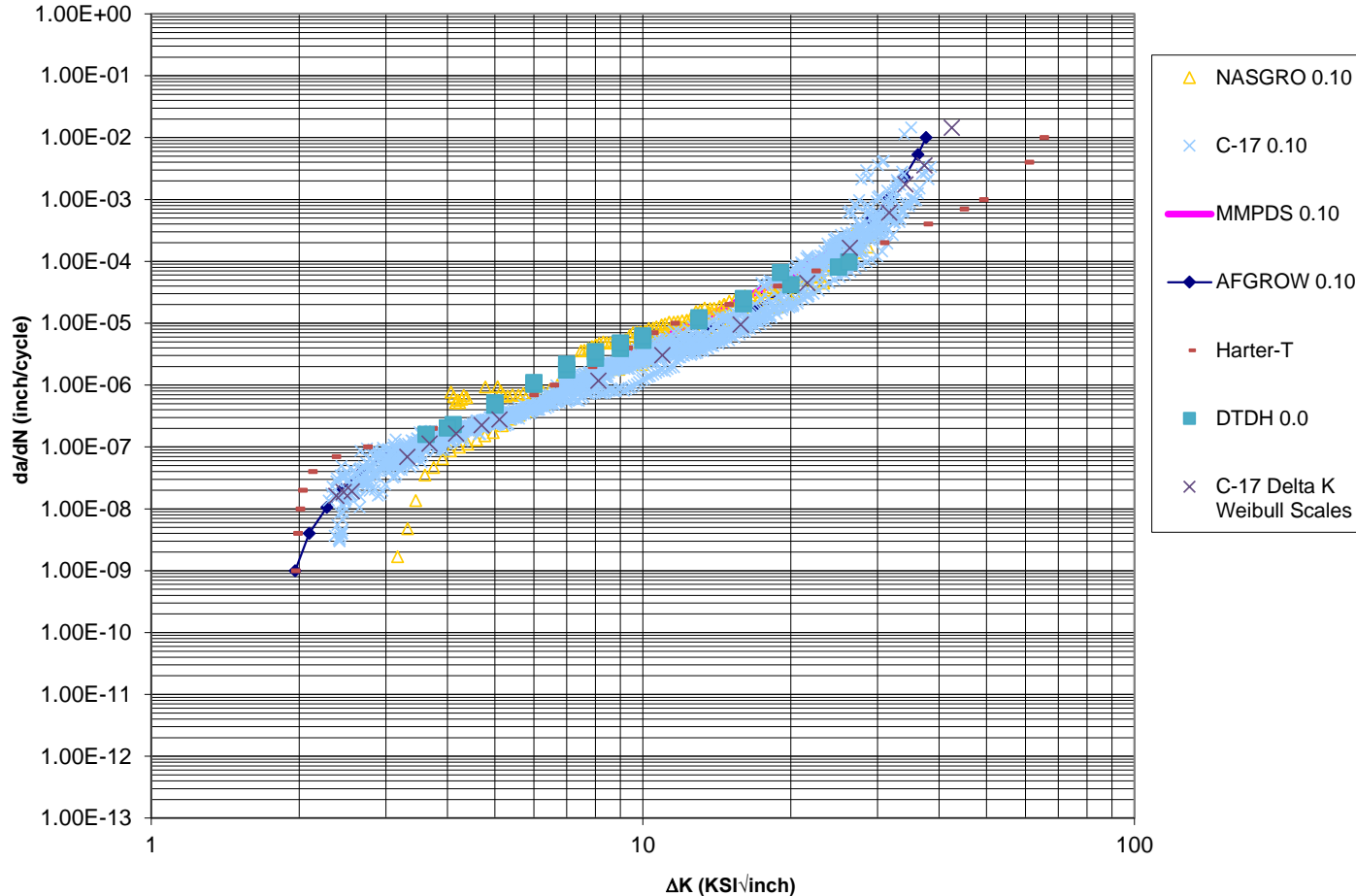




# Characterizing C-17 Data (2-P Weibull $R = 0.1 \Delta K$ )



7050-T7451 L-T Crack Growth Rates  
(Weibull  $da/dN$  MLE Analysis)



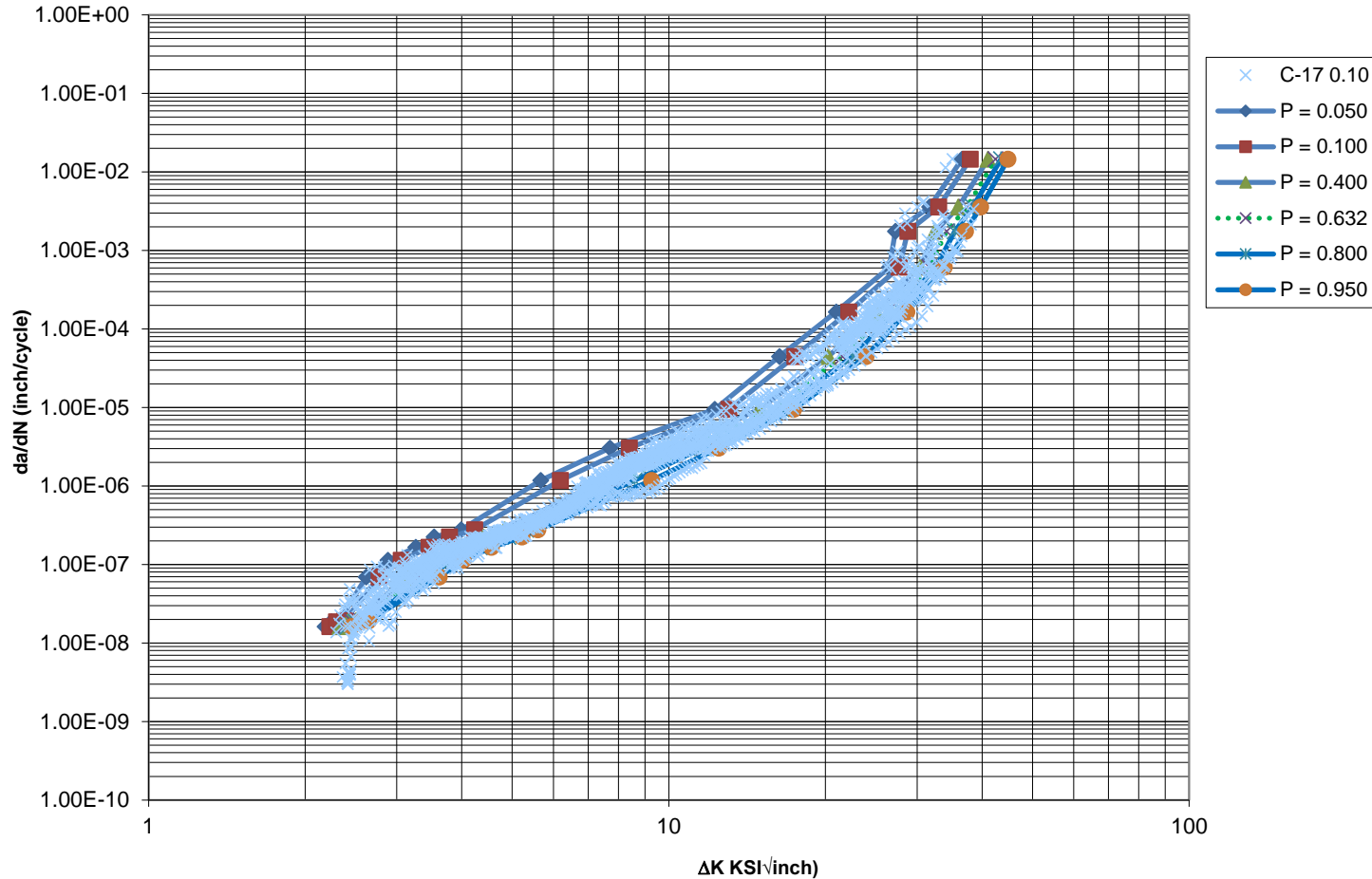


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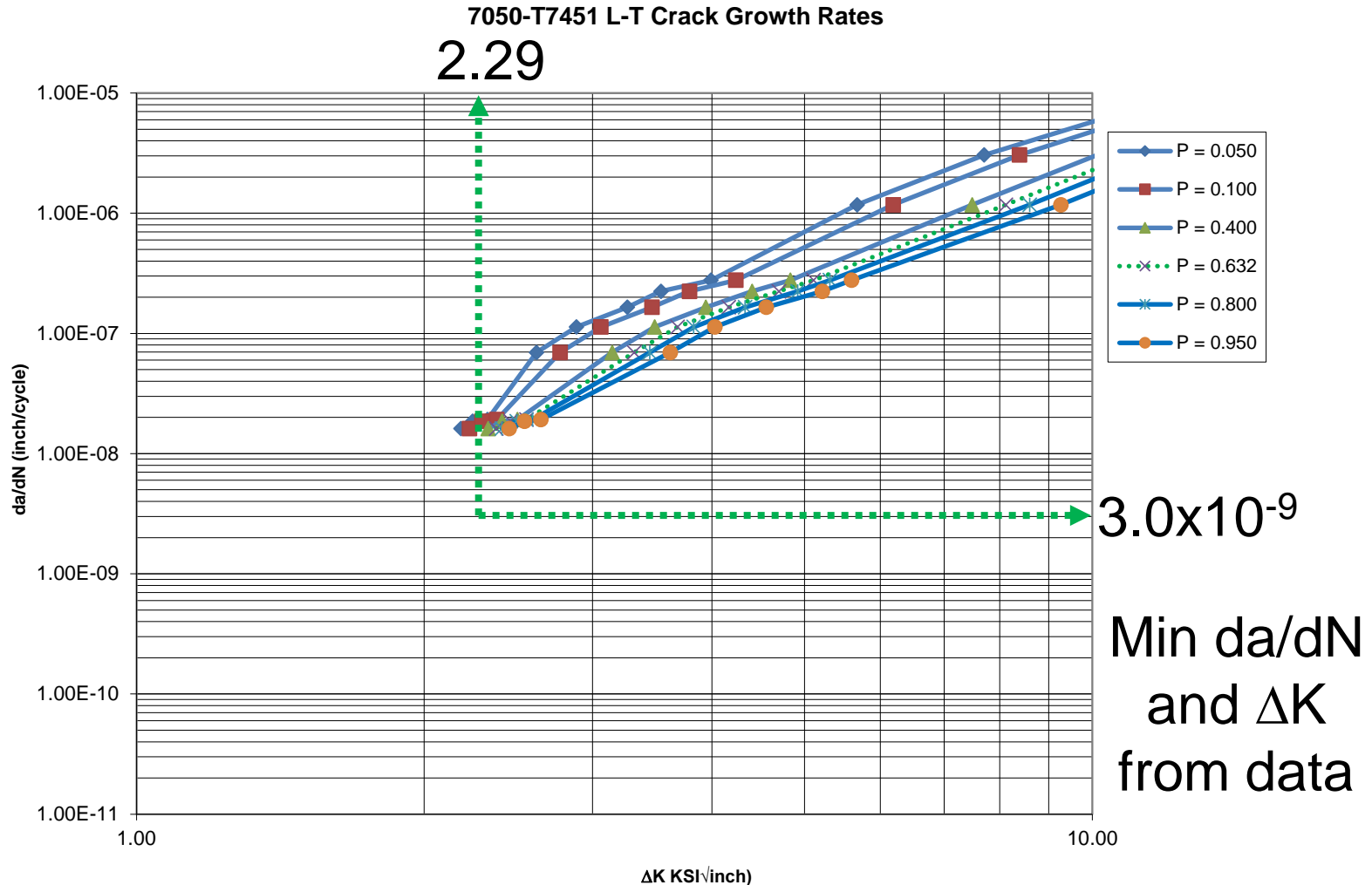
# Constant Probability Curves ( $\Delta K R = 0.1$ )



7050-T7451 L-T Crack Growth Rates



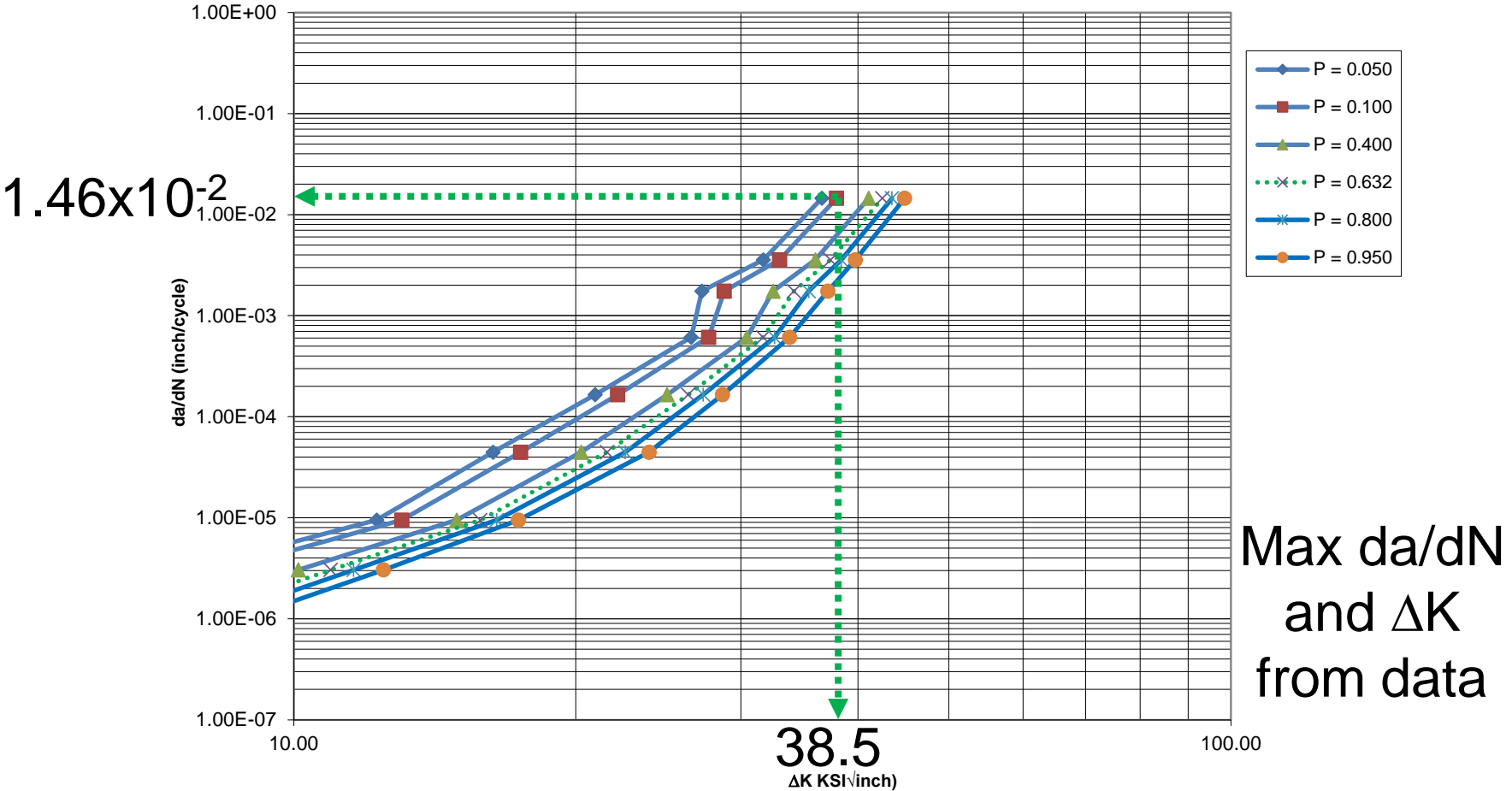
# Constant Probability (Low da/dN R = 0.1)



# Constant Probability (High da/dN R = 0.1)



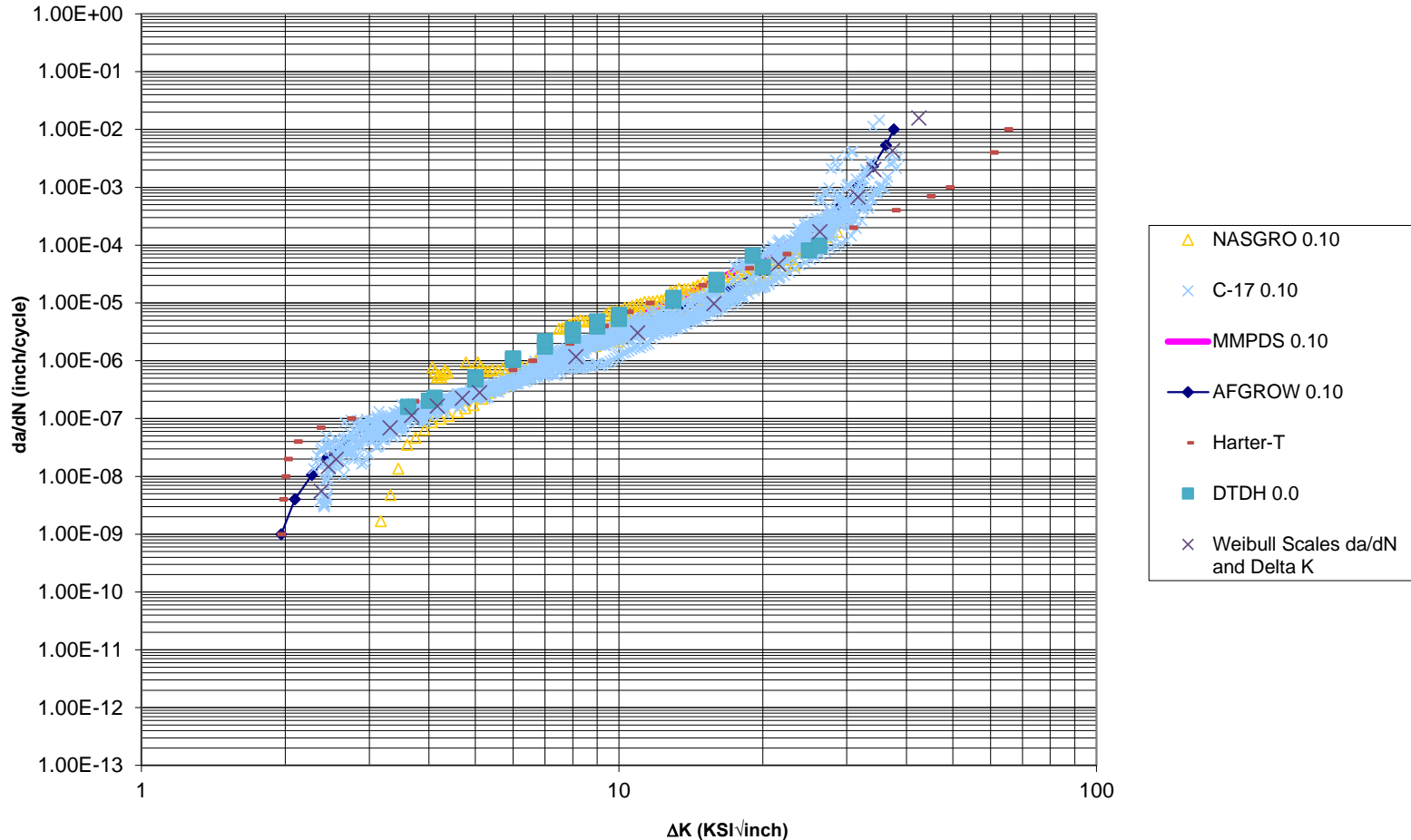
7050-T7451 L-T Crack Growth Rates



# 63% Look-up Curve ( $R = 0.1$ )



7050-T7451 L-T Crack Growth Rates  
(Weibull  $\Delta K$  MLE Analysis)

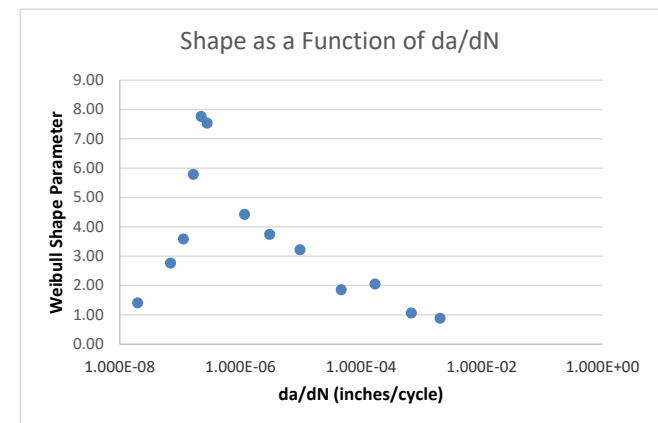


# Option 2: Analyze da/dN in Ranges of $\Delta K$ (R = 0.1)



- Choose ranges of  $\Delta K$  (+/- 5%) to analyze from total population of data
  - 1616 data pairs
  - 5% based on loads accuracy
- Develop Maximum Likelihood Estimation (MLE) distribution parameters for da/dN within each chosen range of  $\Delta K$ 
  - 799 data pairs across 13  $\Delta K$  ranges
  - Approximately 61 da/dN points/ $\Delta K$  range
- Resulting shape parameters as a function of da/dN

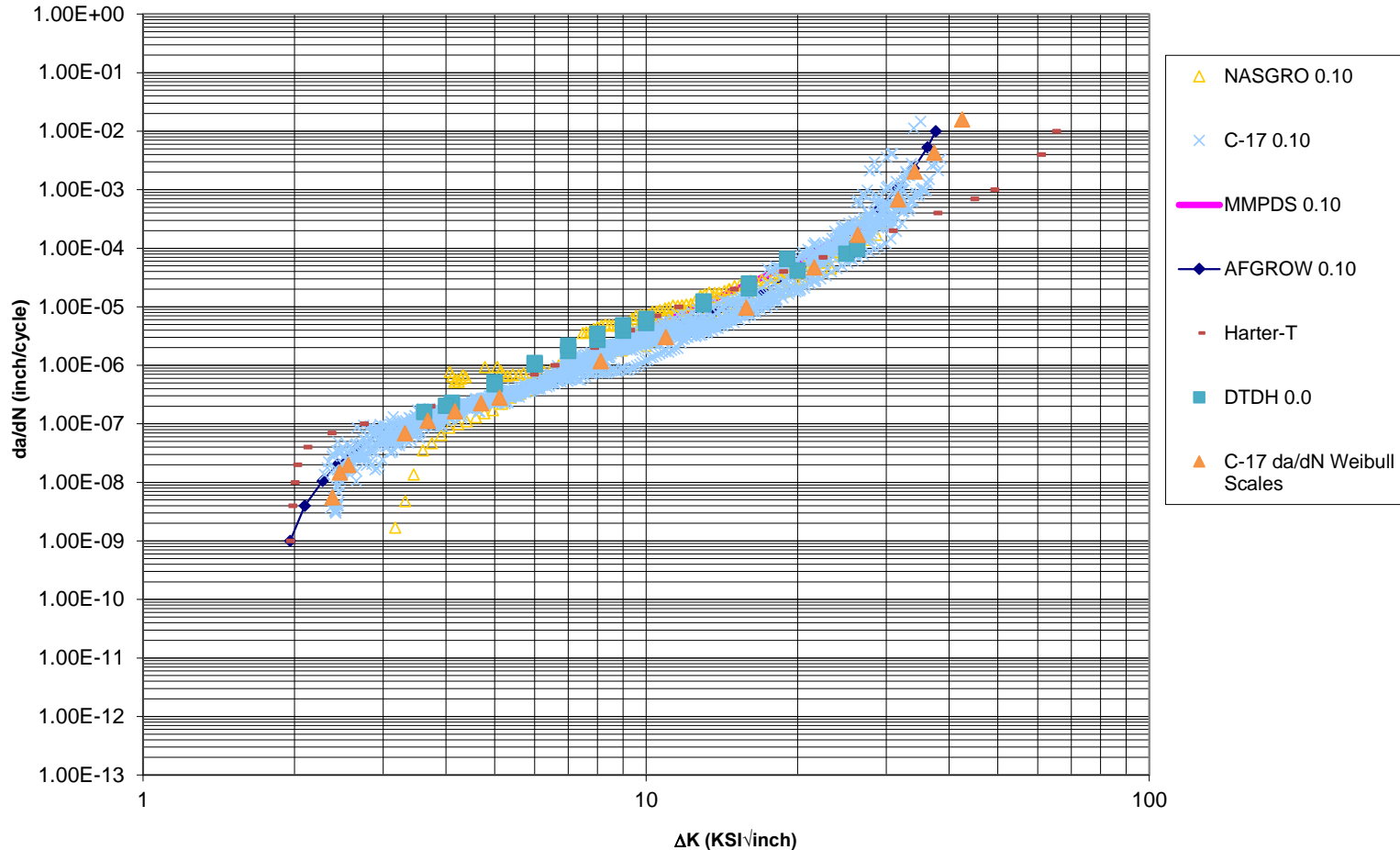
Weibull Parameters for da/dN at Fixed $\Delta K$		Target for da/dN Distributions	
Shape	Scale	$\Delta K$	Range
1.41	1.985E-08	2.50	2.375-2.625
2.77	6.936E-08	3.00	2.850-3.150
3.59	1.136E-07	3.50	3.325-3.675
5.79	1.650E-07	4.00	3.800-4.200
7.77	2.240E-07	4.50	4.275-4.725
7.54	2.810E-07	5.00	4.750-5.250
4.42	1.176E-06	7.50	7.125-7.875
3.74	3.049E-06	10.00	9.500-10.500
3.22	9.724E-06	15.00	14.250-15.750
1.85	4.698E-05	20.00	19.000-21.000
2.05	1.701E-04	25.00	23.750-26.250
1.07	6.814E-04	30.00	28.500-31.500
0.88	2.047E-03	35.00	33.250-36.750



# Characterizing C-17 Data (2-P Weibull R = 0.1 da/dN)



7050-T7451 L-T Crack Growth Rates  
(Weibull da/dN MLE Analysis)



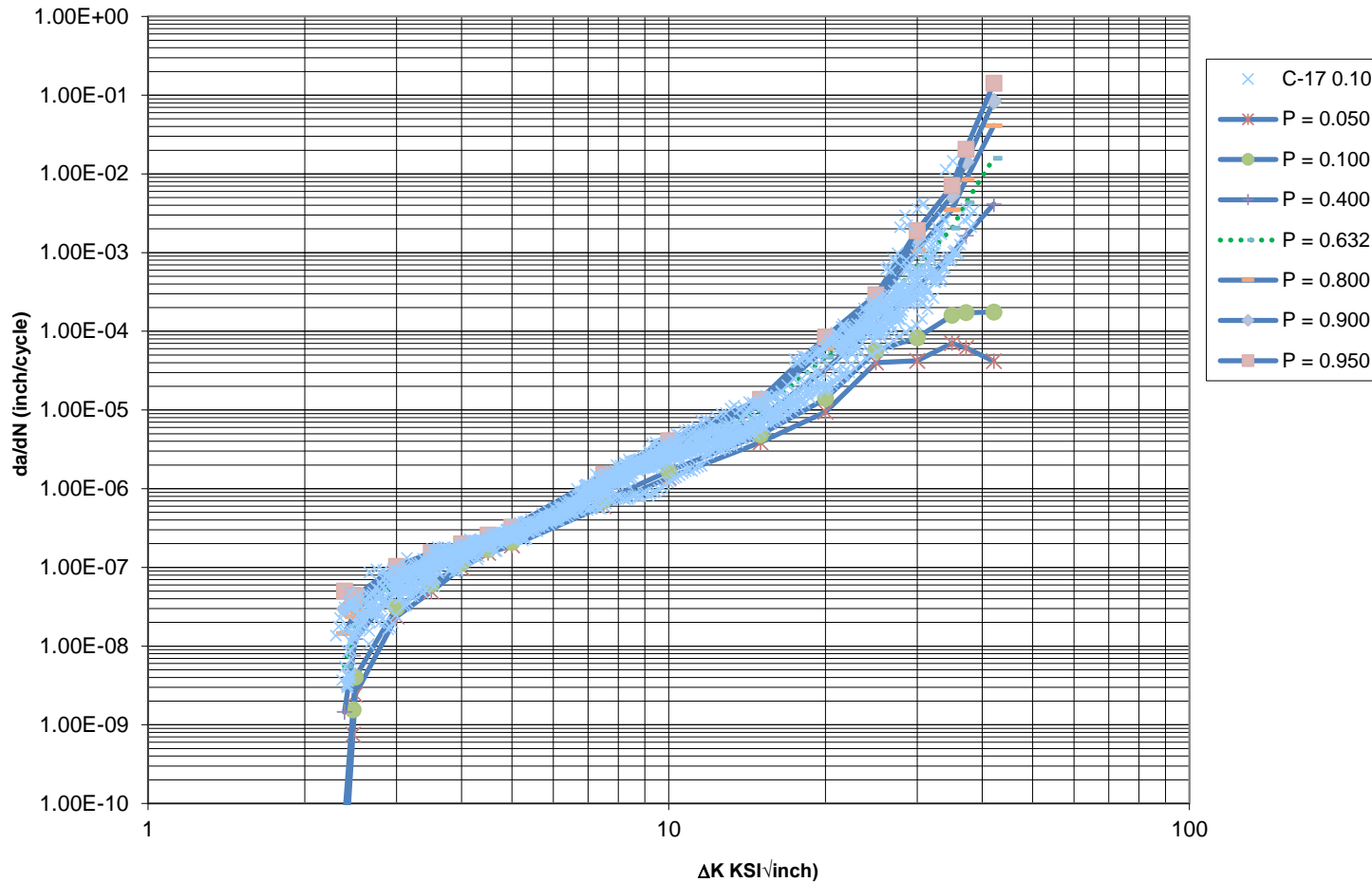


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# Constant Probability Curves ( $R = 0.1$ da/dN)

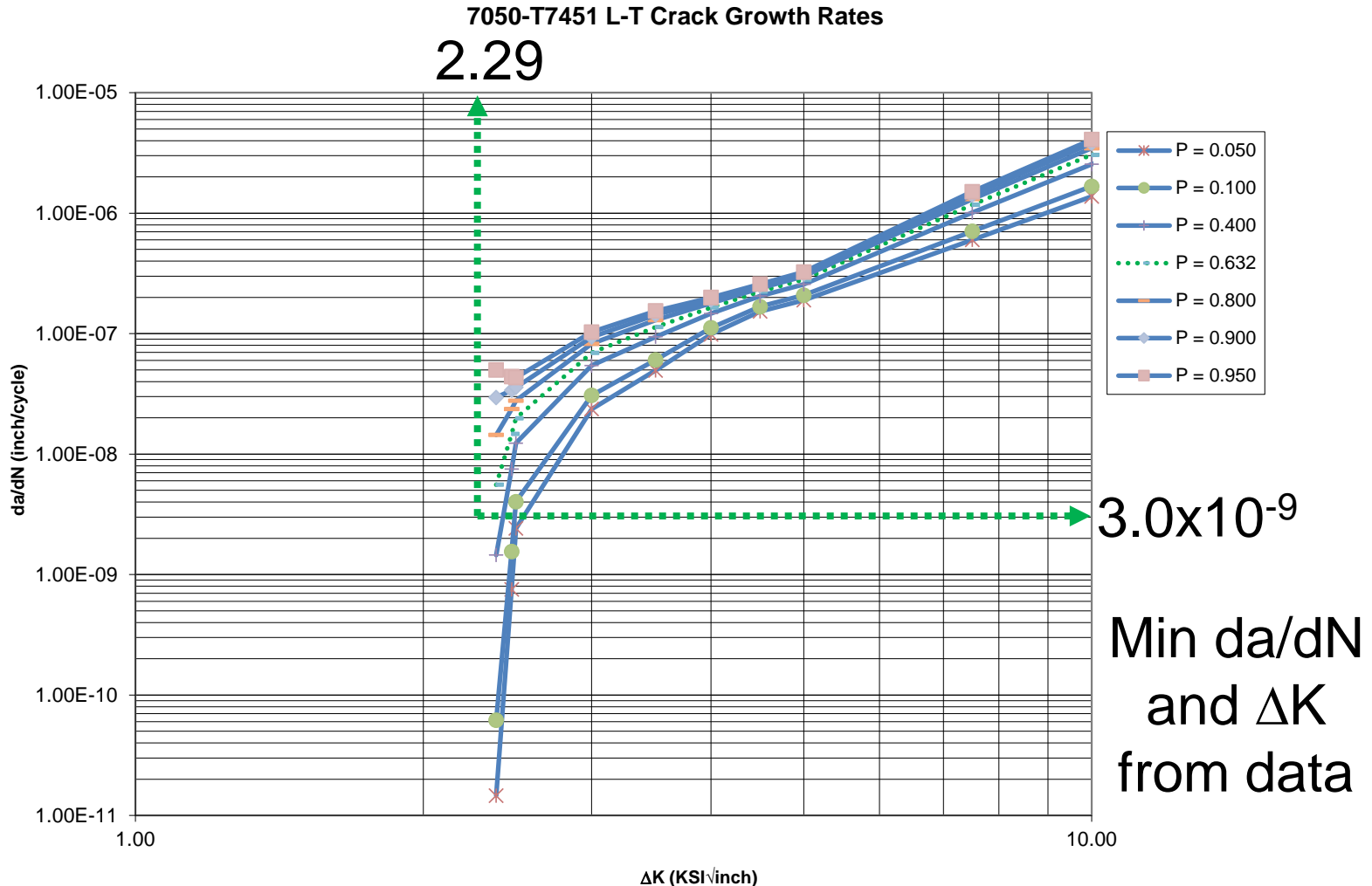


7050-T7451 L-T Crack Growth Rates





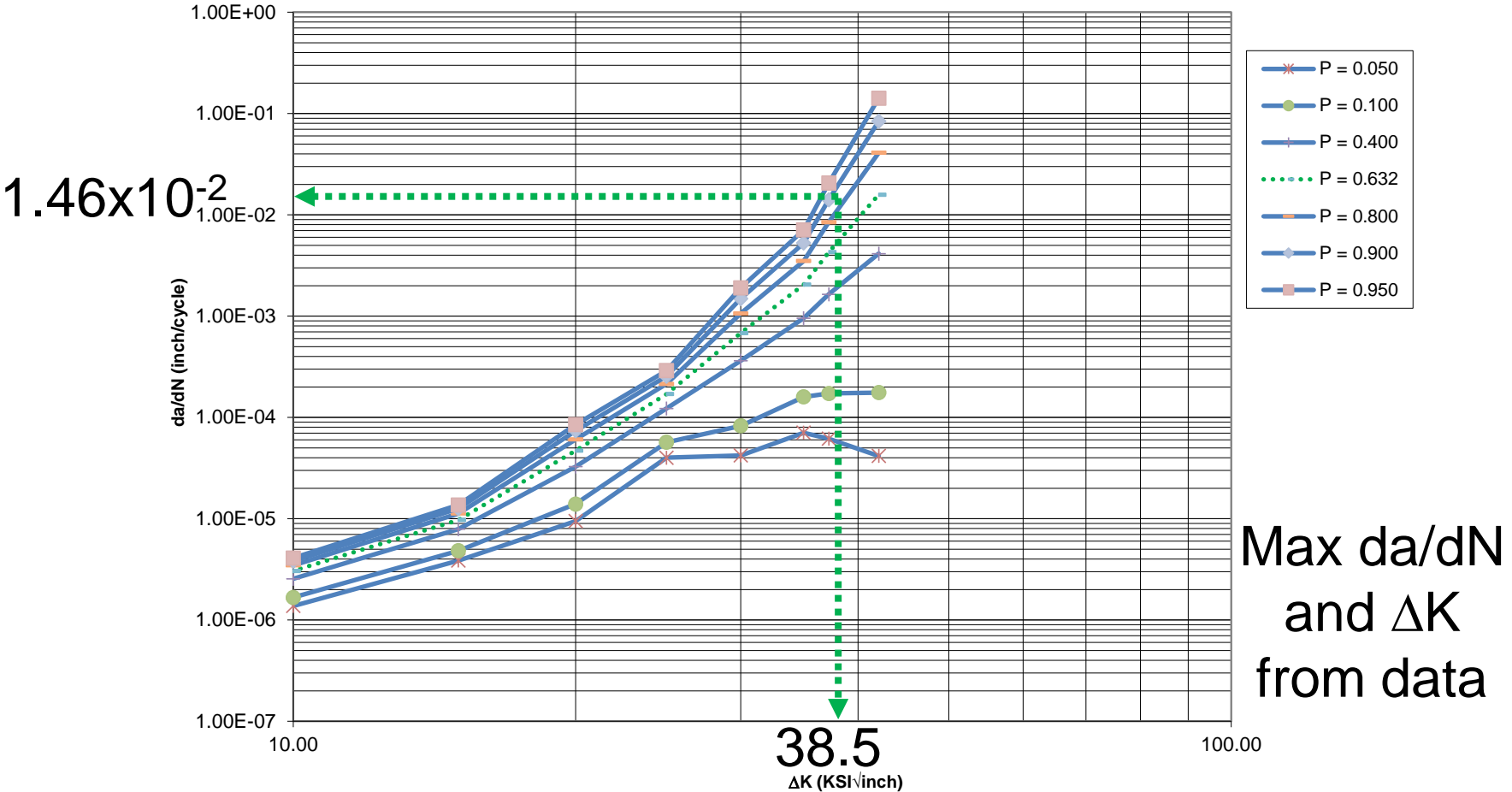
# Constant Probability (Low $\Delta K R = 0.1$ )



# Constant Probability (High $\Delta K$ $R = 0.1$ )



7050-T7451 L-T Crack Growth Rates



# Scatter in da/dN (R = 0.1)

## ■ Is a factor of 2 sufficient?

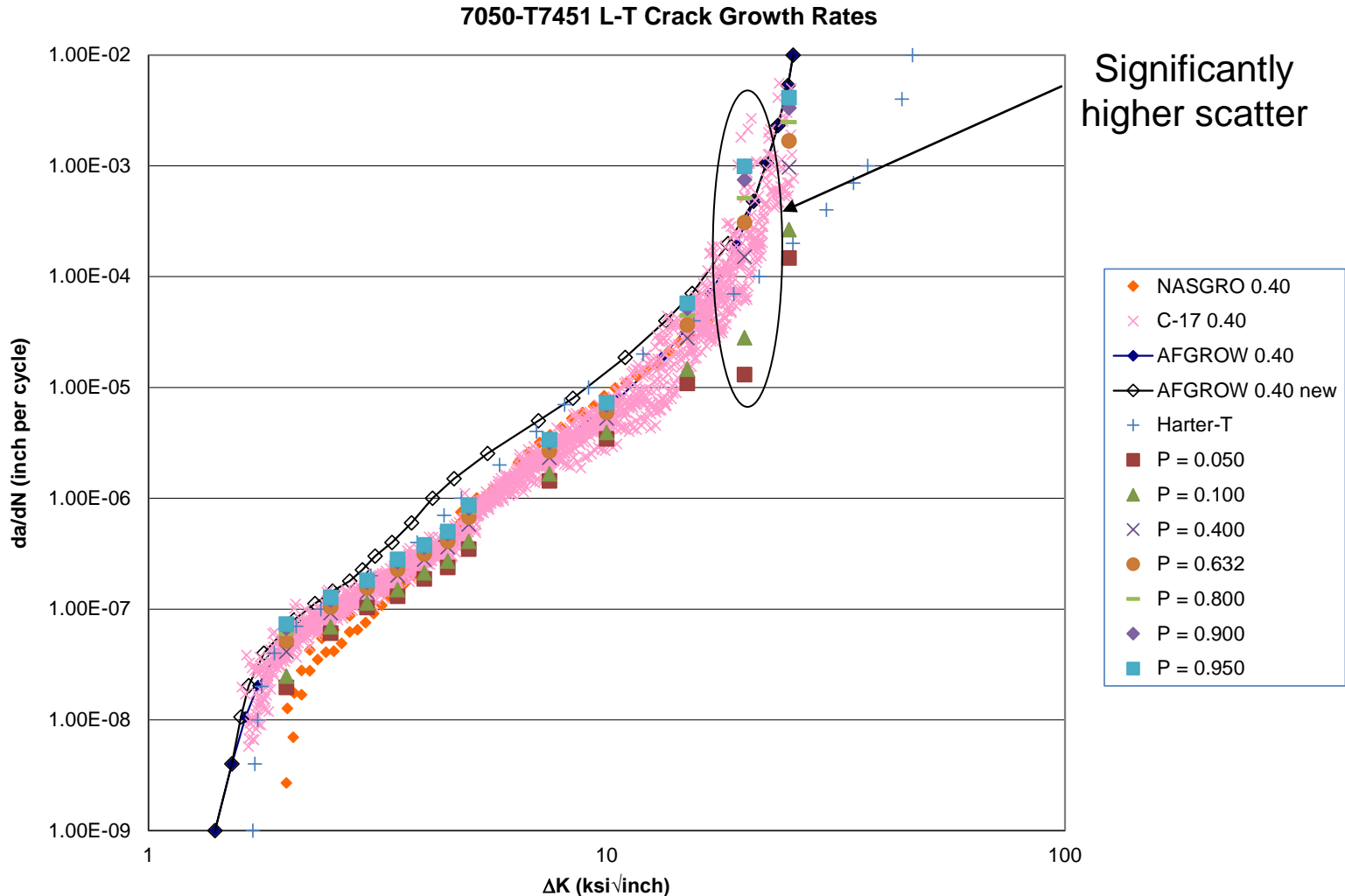
- At  $1.65 \times 10^{-7}$  in/cycle it captures 90% of the data
- At  $1.18 \times 10^{-6}$  in/cycle it captures 82% of the data
- In order to capture more than 80% of the data at the lowest and highest growth rates, scatter factors of 17 and 95, respectively, result

Middle Growth Rates Scatter						
		Probability		Bottom		
	$1.65\text{E-}07$	0.63		$1.99\text{E-}08$		
Scatter = 2	$1.10\text{E-}07$	0.09		$3.76\text{E-}09$	Scatter = 17	
	$2.20\text{E-}07$	0.99	90%	$6.46\text{E-}08$		90%
		Probability		Top		
	$1.18\text{E-}06$	0.63		$2.05\text{E-}03$		
Scatter = 2	$7.84\text{E-}07$	0.15		$1.43\text{E-}04$	Scatter = 95	
	$1.57\text{E-}06$	0.97	82%	$1.35\text{E-}02$		82%

- Only about 1 decade of data fits within a scatter factor of 2



# Characterizing C-17 Data (2-P Weibull R = 0.4 da/dN)

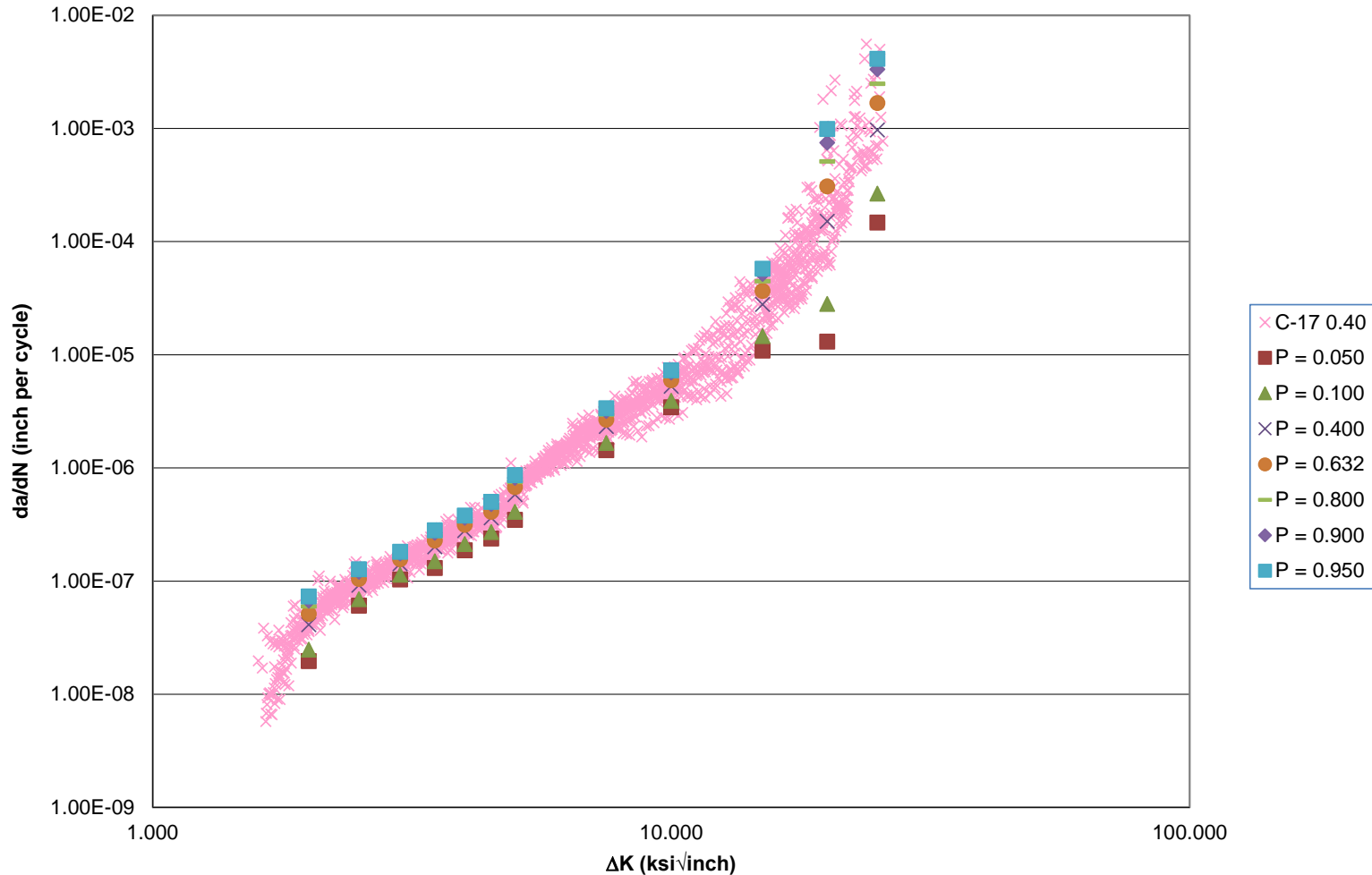


1601 total data pairs 17 tests  
653 da/dN points analyzed across 12  $\Delta K$  ranges 20

# Constant Probability ( $R = 0.4 \text{ da/dN}$ )



7050-T7451 L-T Crack Growth Rates





# Scatter in da/dN (R = 0.4)



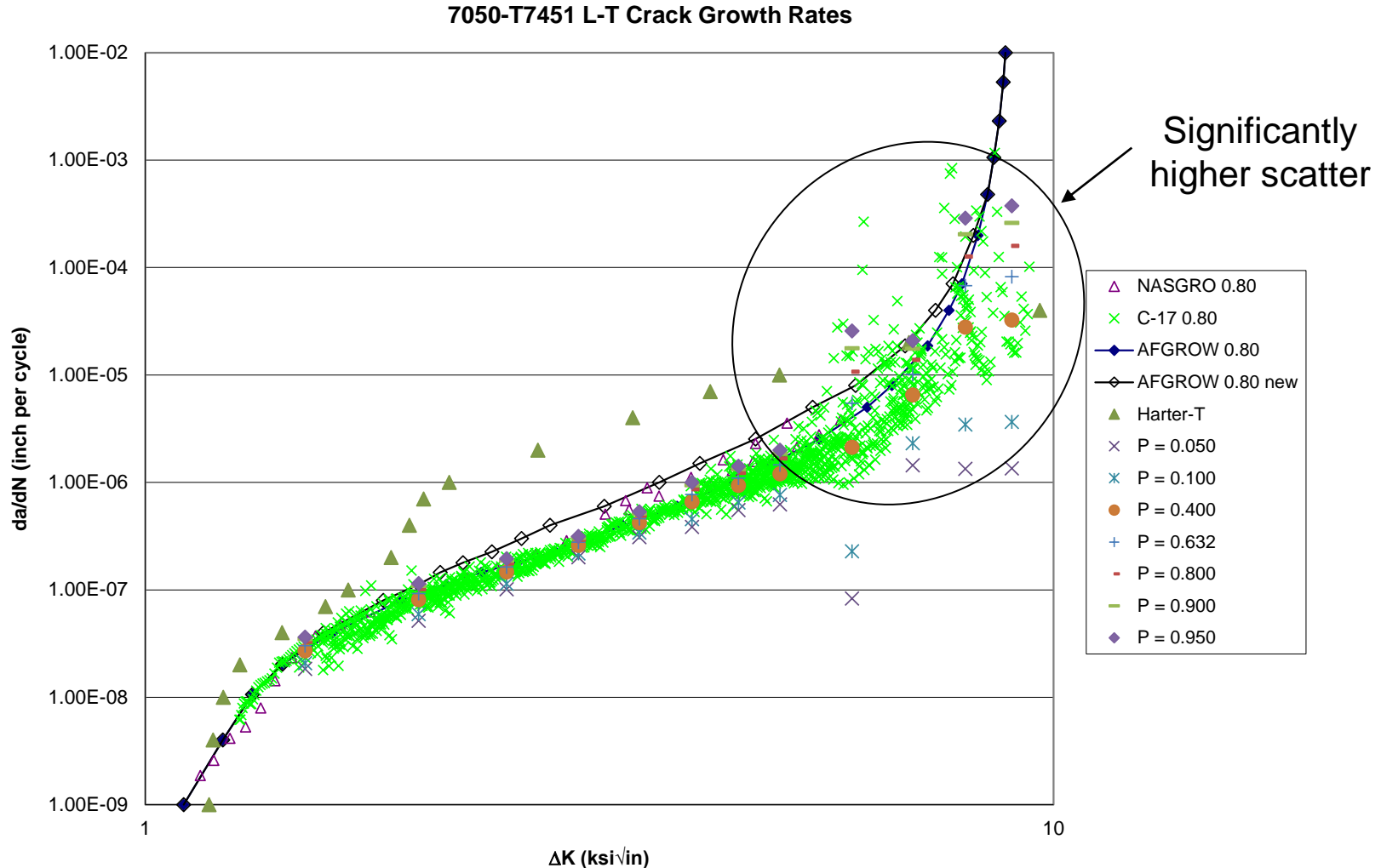
## ■ Is a factor of two sufficient?

- At  $1.04 \times 10^{-7}$  in/cycle it captures 89% of the data
- At  $5.97 \times 10^{-6}$  in/cycle it captures 89% of the data
- In order to capture more than 80% of the data at the lowest and highest growth rates, scatter factors of 3 and 21, respectively, result

Middle Growth Rates Scatter						
		Probability		Bottom		
	1.04E-07	0.63		5.13E-08		
Scatter = 2	6.95E-08	0.10		2.50E-08	Scatter = 3	
	1.39E-07	0.99	89%	8.56E-08		89%
		Probability		Top		
	5.97E-06	0.63		1.68E-03		
Scatter = 2	3.98E-06	0.11		2.80E-04	Scatter = 21	
	7.95E-06	0.99	89%	6.00E-03		89%

- About 1.5 decades near the lower da/dN range falls within the scatter factor of 2

# Characterizing C-17 Data (2-P Weibull R = 0.8 da/dN)

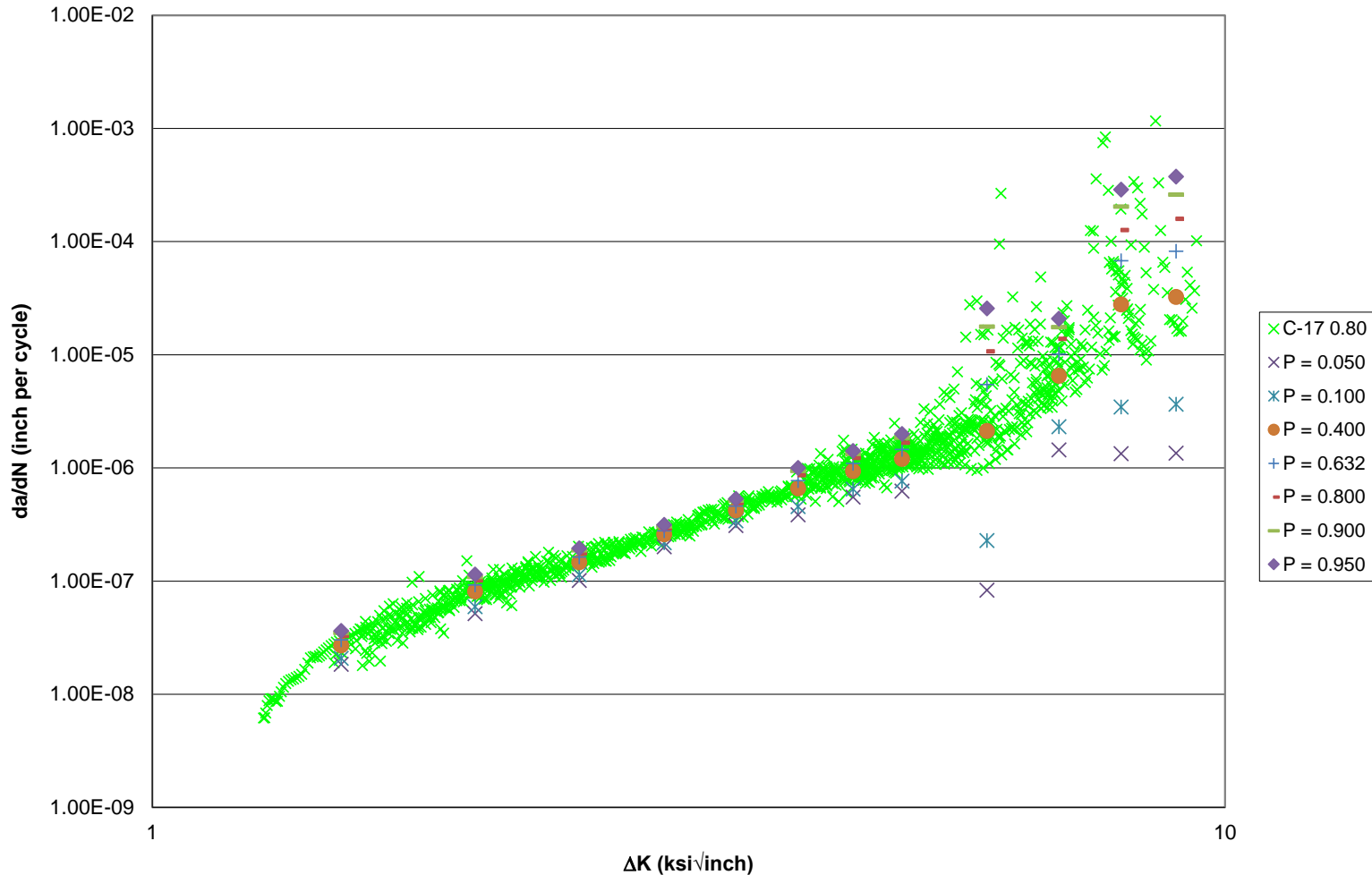


1280 total data pairs across 17 tests  
767 da/dN points analyzed across 12  $\Delta K$  ranges

# Constant Probability ( $R = 0.8$ da/dN)



7050-T7451 L-T Crack Growth Rates







# Scatter in da/dN (R = 0.8)



## ■ Is a factor of two sufficient?

- At  $3.03 \times 10^{-8}$  in/cycle it captures 91% of the data
- At  $4.61 \times 10^{-7}$  in/cycle it captures 95% of the data
- In order to capture more than 80% of the data at the lowest and highest growth rates, scatter factors of 2 and 1315, respectively, result

Middle Growth Rates Scatter						
		Probability		Bottom		
	3.03E-08	0.63		3.03E-08		
Scatter = 2	2.02E-08	0.08		2.02E-08	Scatter = 2	
	4.04E-08	1.00	91%	4.04E-08		91%
		Probability		Top		
	4.61E-07	0.63		8.23E-05		
Scatter = 2	3.07E-07	0.05		1.23E-06	Scatter = 1315	
	6.15E-07	1.00	95%	1.62E-03		95%

- Only the lowest decade da/dN falls within the scatter factor of 2



# Observations

- **Scatter across the range of  $da/dN$  is not consistent**
  - A scatter factor of two for this data set has limited applicability
  - Only about 1-1.5 decades of data on the lower end of the  $da/dN$  range fit across all three stress ratios
- **Characterizing behavior based on fixed ranges of  $da/dN$ , therefore, is complicated by the inconsistent scatter observed**
- **Weibull shape parameter behavior between stress ratios for distributions of  $da/dN$  for fixed  $\Delta K$  differs significantly**
  - $R = 0.4$  and  $0.8$  data tends to converge on very narrow distributions
  - $\Delta K_{\text{Applied}}$  and  $\Delta K_{\text{Effective}}$  affect

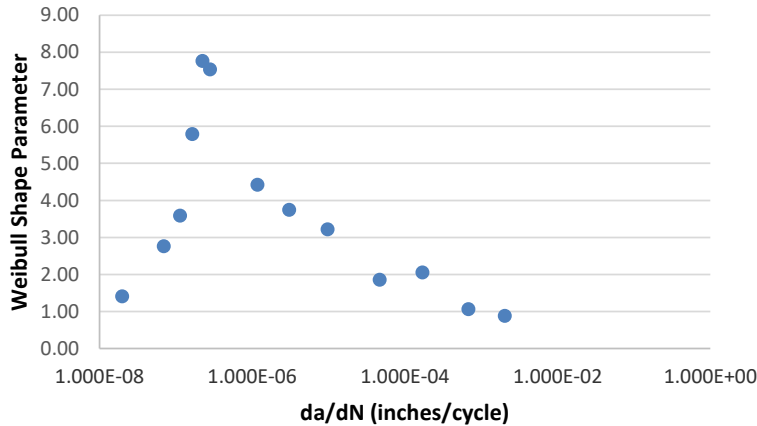


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# Shape Parameter Summary (7050)

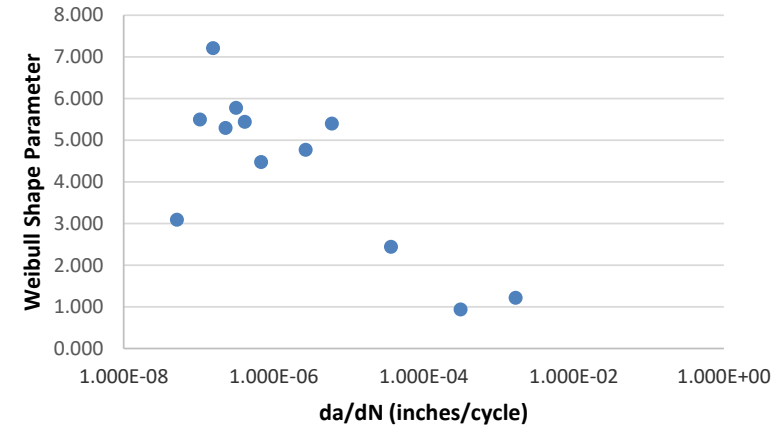


Shape as a Function of da/dN



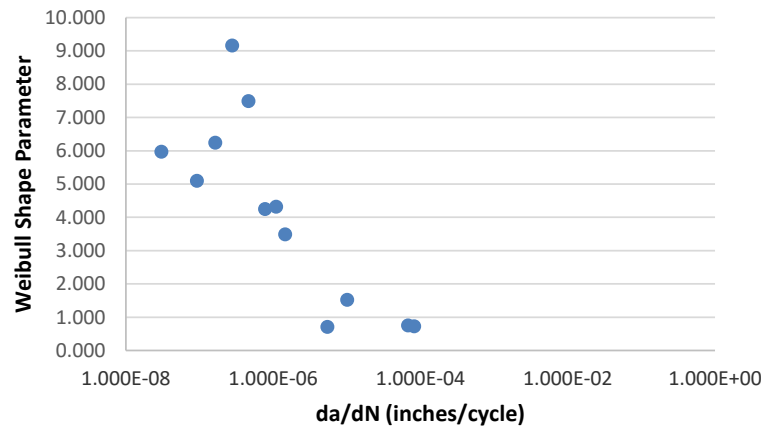
R = 0.1

Shape as a Function of da/dN



R = 0.4

Shape as a Function of da/dN



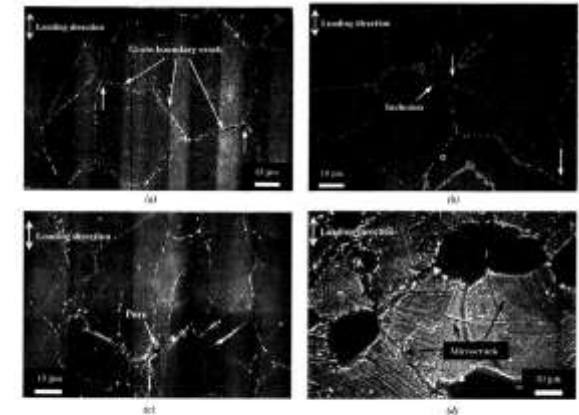
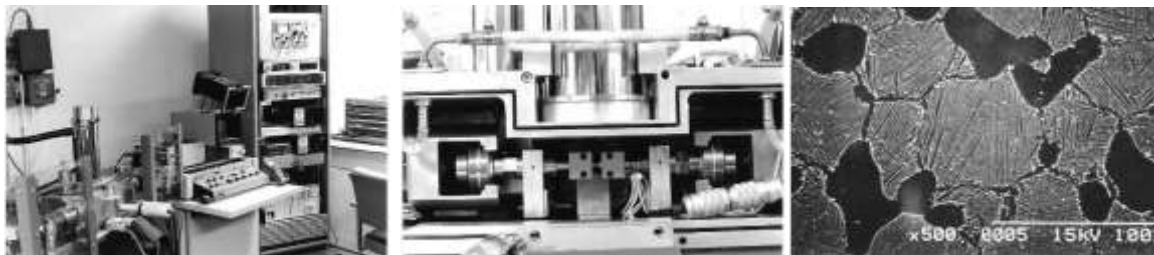
R = 0.8

■ Does this suggest that at about 0.15 <math>R < 0.25</math>  $\Delta K_{\text{applied}}$  approaches  $\Delta K_{\text{effective}}</math>?$

■ Coincidence?

# Additional Observations ( $da/dN$ $R = 0.1$ )

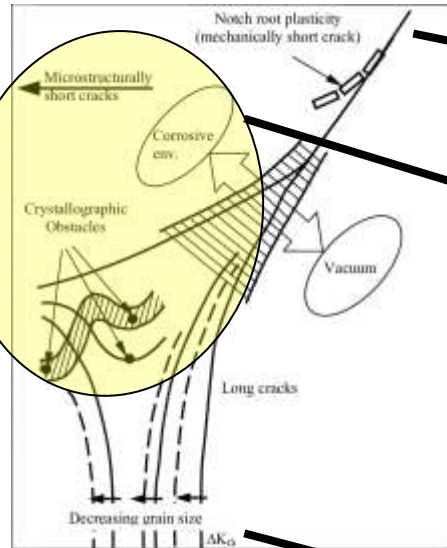
- Upper bound  $da/dN$  distribution is potentially indicative of testing specimen and methods
  - Net section yield
  - Instability
  
- Lower bound ( $R = 0.1$ )  $da/dN$  distribution might indicate transition from “short crack” behavior
  - Physically
  - Microstructurally



**Figure 3.12** Fatigue microcracks that nucleated due to various mechanisms. (a) Elevated temperature (700°C) grain boundary crack in Waspaloy. (b) Surface inclusion/slip band crack in Waspaloy. (c) Elevated temperature (500°C) surface pore/slip band in Waspaloy. (d) Cracking within  $\alpha$  and  $\beta$  phases in lamellar structure in titanium alloy IMB34 (courtesy of R. R. Stephens).

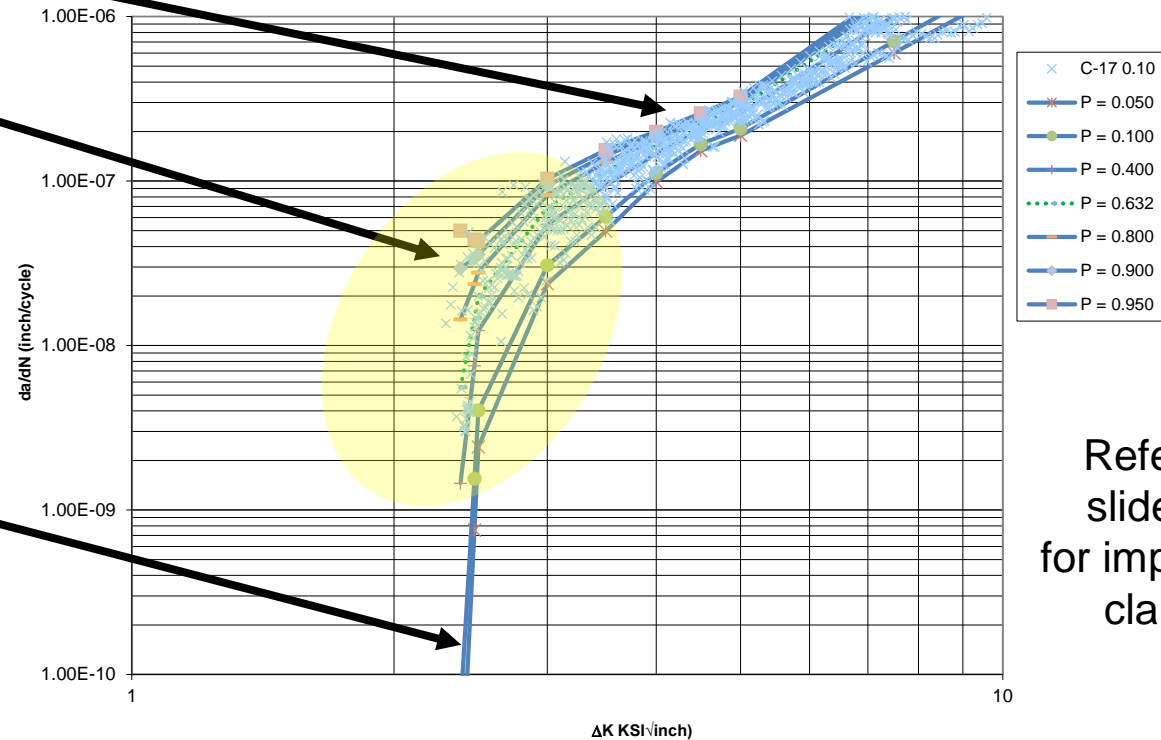
In-situ short crack growth test system circa 1990

# Transitioning from Short Crack Behavior?



6

7050-T7451 L-T Crack Growth Rates



Refer to slide 17 for improved clarity

- Iso-probabilities highlight potential transition from short crack, i.e. microstructural barriers, corrosion, etc. to long (through thickness) crack behavior
- Limited High R, short crack, data and observations



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# Further Considerations



- **“Long Crack Threshold”** for  $R = 0.1$  is defined by the NASGRO equation as approximately  $\Delta K = 3.1 \text{ ksi}\sqrt{\text{in}}$  at  $10^{-9} \text{ in/cycle}$  (Slide 15) would be expected to occur less than about 1/100,000 times (for distributions of  $da/dN$ )
- **Consistent loads accuracy, +/- 5%\***, is recommended (?) as the basis for defining fixed  $\Delta K$  range to determine distributions on  $da/dN$ 
  - \*Established practice within aerospace fatigue analysis
  - For a fixed crack size, geometry, applied force, and stress ratio, we calculate  $\Delta K$  and  $da/dN$  results
  - Should the tolerance be tighter? +/-2%, +/-1% (test instrumentation calibration ASTM E4 Force Calibration)?



# Alternative Example



## ■ 2024-T351 L-T

### ■ What we know about this data set

- Stress Ratio
- Range of  $\Delta K$
- Grain orientation
- Data developed between 1975 and 2015

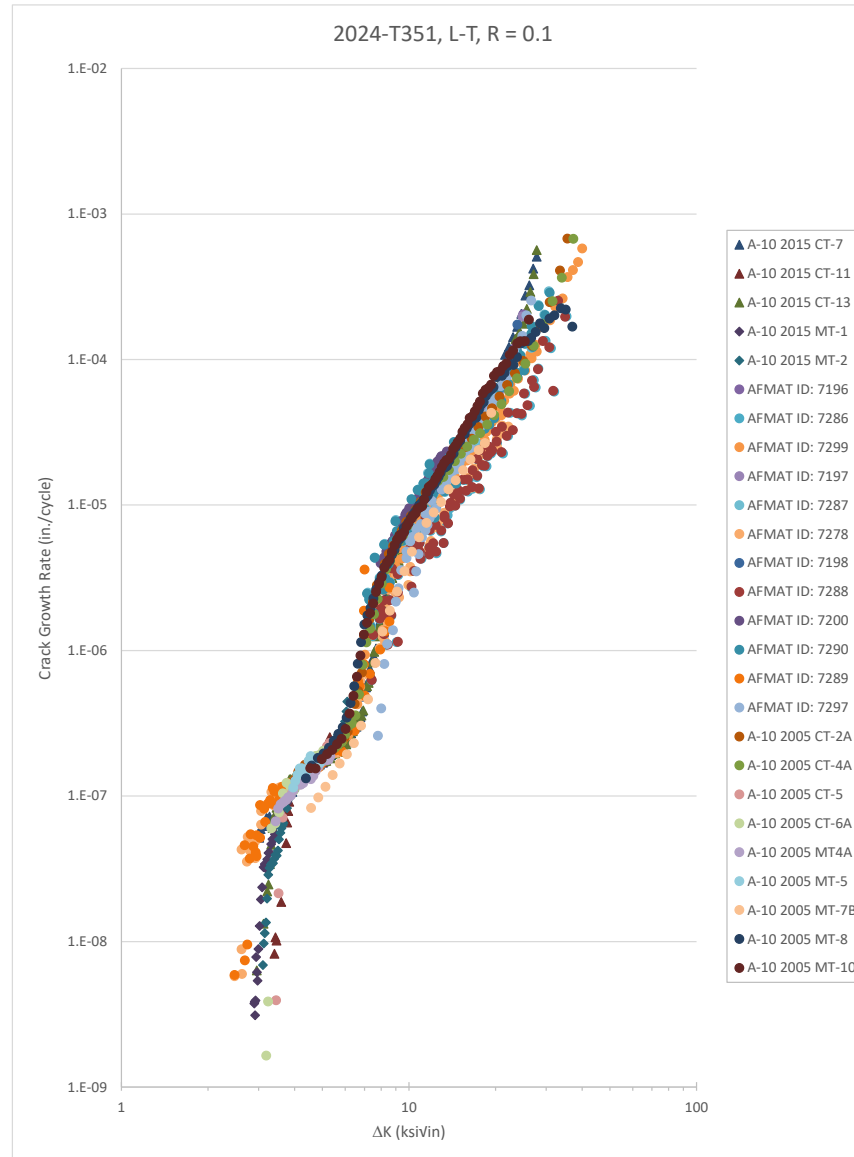
### ■ What we do not know

- Specimen configuration
- Product form thickness
- Testing environment and frequency
- Test Type (CA, K-Decreasing, etc.)
- Test data supplier
- Producer/lot variation



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# Generic Test Data Example



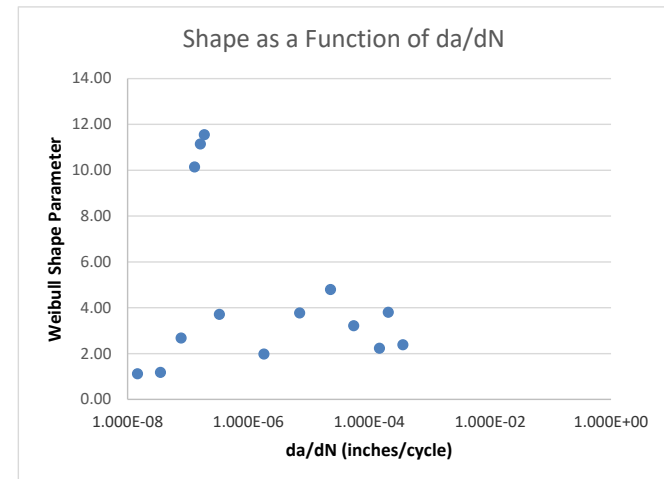


# da/dN within Discrete Ranges of $\Delta K$ (R = 0.1)



- Choose ranges of  $\Delta K$  (+/- 5%) to analyze from total population of data
  - 1625 data pairs
  - 5% based on loads accuracy
- Develop Maximum Likelihood Estimation (MLE) distribution parameters for da/dN within each chosen range of  $\Delta K$ 
  - 749 data pairs across 14  $\Delta K$  ranges
  - Approximately 54 da/dN points/ $\Delta K$  range
- Resulting shape parameters as a function of da/dN

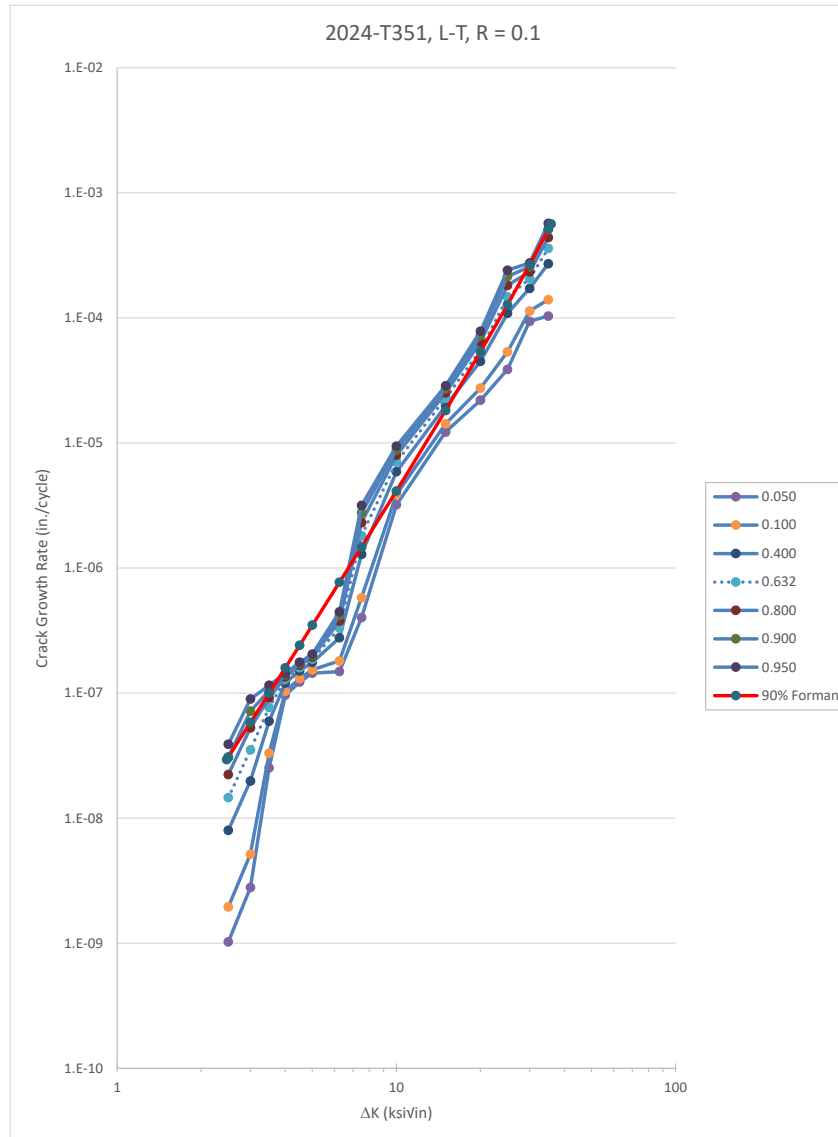
Weibull Parameters for da/dN at Fixed $\Delta K$		Target for da/dN Distributions	
Shape	Scale	$\Delta K$	Range
1.12	1.461E-08	2.50	2.375-2.625
1.17	3.521E-08	3.00	2.850-3.150
2.68	7.675E-08	3.50	3.325-3.675
10.14	1.290E-07	4.00	3.800-4.200
11.14	1.601E-07	4.50	4.275-4.725
11.54	1.868E-07	5.00	4.750-5.250
3.71	3.32E-07	6.25	5.938-6.563
1.97	1.815E-06	7.50	7.125-7.875
3.77	7.051E-06	10.00	9.500-10.500
4.79	2.277E-05	15.00	14.250-15.750
3.21	5.550E-05	20.00	19.000-21.000
2.23	1.472E-04	25.00	23.750-26.250
3.79	2.058E-04	30.00	28.500-31.500
2.38	3.600E-04	35.00	33.250-36.750



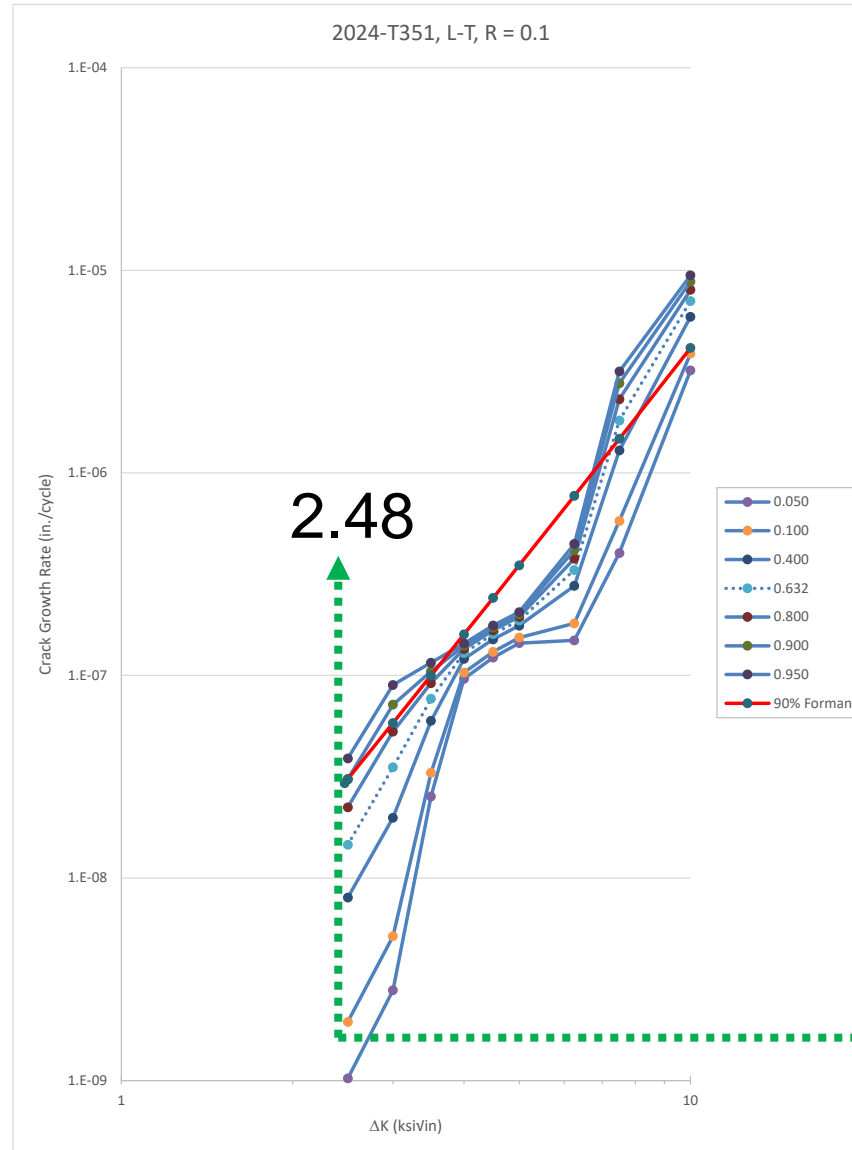


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# Constant Probability Curves ( $R = 0.1$ da/dN)



# Constant Probability (Low $\Delta K$ R = 0.1)

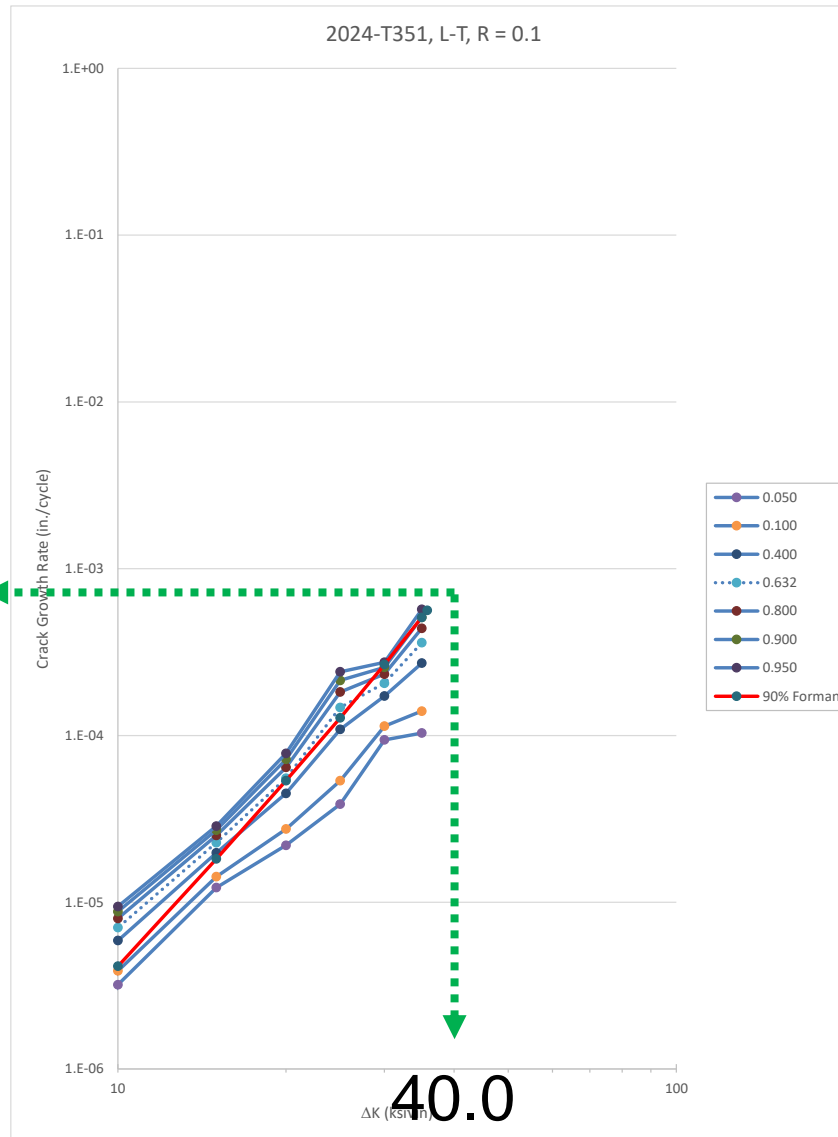


Min  $da/dN$   
and  $\Delta K$   
from data

# Constant Probability (High $\Delta K$ $R = 0.1$ )



$6.78 \times 10^{-4}$



Max  $da/dN$   
and  $\Delta K$   
from data



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# Concluding Remarks



- **Obtaining crack growth data and establishing a working 'confidence' in that data can be a tedious process**
- **Implementing consistency in how this data is further characterized for use in damage tolerance assessments is open to debate**
- **These examples highlighted the following**
  - da/dN scatter can vary widely across the range of  $\Delta K$
  - Statistical representations can provide insight into the applicability of established concepts and why challenges in predictions can occur
  - The behavior of the  $R = 0.1$  data, specifically, at low  $\Delta K$  values shares similarity to Short Crack behavior
    - Coincidence?



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# References and Related Bibliography



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Heinimann, M. B., Baumeister, W. J., Ramachandran, N. C., "Crack Growth of Corner Cracks at Offset Holes: Experimental Validation of Predictions from Commercial Codes," ICAF Proceedings 2003.