MODIFICATION OF STRESS INTENSITY FACTOR EQUATION FOR CORNER CRACKS FROM A HOLE UNDER REMOTE BENDING

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The stress intensity factor equations for several typical crack configurations and loading conditions, developed by Newman and Raju [1-3], have been widely used in various damage tolerance analyses, and combined into a number of popular computer programs for life prediction of fatigue crack propagation. These equations greatly facilitate the damage tolerance analysis by not only providing the stress intensity factor solutions in a closed form, but also extending the solutions beyond the range in which the finite element solutions were available.

All the equations have been used successfully with one exception: small corner cracks ($a/t < 0.2$) emanating from a hole under remote bending [3]. Figure 1 shows the configuration in question and the definition of related parameters to be used. A recent study using a three-dimensional weight function method [4], which provides independent solutions for corner cracks emanating from a hole, has shown that significant error in the equation exists for small corner cracks under remote bending. The reason for the loss of accuracy was an invalid assumption made on the limiting behavior as $a/t \rightarrow 0$. Figure 2 shows an example of comparisons between the equation [3] and the weight function solutions [4]. It is noted that, at the time of the equation development [1-3], many engineering judgments and assumptions were necessary because available solutions were far from sufficient for developing equations, especially for small corner cracks from a hole subjected to remote bending.

The objective of the present work is to improve the accuracy of the bending equation for corner cracks emanating from holes [3] using a modification based on the weight function solutions [4]. The modifications are carried out in such a way that the change in the functional form of the original equation [3] will be kept to a minimum. The applicable range of the modified equation will stay the same as before.

The nomenclature in [3] is followed and given below for convenience.

a, c = semi-axes of a quarter-elliptical crack

b = half plate width

F_{ch} = dimensionless stress intensity factor for corner crack at a hole under bending

h = half plate height

H_{ch} = bending multiplier for corner crack at a hole

K = stress intensity factor

Q = shape factor of an ellipse

r = hole radius

S_b = remote outer fiber bending stress, $3M/b/t^2$

S_t = remote tensile stress

t = plate thickness

φ = parametric angle of an elliptical crack

To start with, the original equation [3] is listed in the following.

$$K = (S_t + H_{ch} S_b)\sqrt{\pi a / Q}\, F_{ch}(a/c, a/t, r/t, r/b, c/b, \phi) \tag{1}$$

$$F_{ch} = [M_1 + M_2 * (\frac{a}{t})^2 + M_3 * (\frac{a}{t})^4] g_1 g_2 g_3 g_4 f_\phi f_w \tag{2}$$

S

For $a/c \leq 1$:

$$M_1 = 1.13 - 0.09\frac{a}{c} \tag{3}$$

S

$$M_2 = -0.54 + \frac{0.89}{0.2 + \frac{a}{c}} \tag{4}$$

S

$$M_3 = 0.5 - \frac{1}{0.65 + \frac{a}{c}} + 14(1 - \frac{a}{c})^{24} \tag{5}$$

S

$$g_1 = 1 + [0.1 + 0.35(\frac{a}{t})^2](1 - \sin\phi)^2 \tag{6}$$

S

$$g_2 = \frac{1 + 0.358\lambda + 1.425\,\lambda^2 - 1.578\,\lambda^3 + 2.156\,\lambda^4}{1 + 0.13\,\lambda^2} \qquad (7)$$ S

$$\lambda = \frac{1}{1 + (c/r)\cos\{[0.85 - 0.25(a/t)^{0.25}]\phi\}} \qquad (8)$$ S

$$g_3 = (1 + 0.04\frac{a}{c})[1 + 0.1(1 - \cos\phi)^2][0.85 + 0.15(\frac{a}{t})^{0.25}] \qquad (9)$$ S

$$g_4 = 1 - 0.7(1 - \frac{a}{t})(\frac{a}{c} - 0.2)(1 - \frac{a}{c}) \qquad (10)$$ S

$$f_\phi = [(a/c)^2\cos^2\phi + \sin^2\phi]^{0.25} \qquad (11)$$ S

$$f_w = \{\sec(\frac{\pi r}{2b})\sec[\frac{\pi(2r + nc)}{4(b - c) + 2nc}\sqrt{\frac{a}{t}}]\}^{0.5} \qquad (12)$$ S

$$H_{ch} = H_1 + (H_2 - H_1)\sin^p\phi \qquad (13)$$ Same S

v = a/t
xnr = a/c

$$p = 0.1 + 1.3\frac{a}{t} + 1.1\frac{a}{c} - 0.7(\frac{a}{c})(\frac{a}{t}) \qquad (14)\ \mathrm{S}$$

$$H_i(\frac{a}{c}, \frac{a}{t}) = 1 + G_{i1}(\frac{a}{c}) * (\frac{a}{t}) + G_{i2}(\frac{a}{c}) * (\frac{a}{t})^2 + G_{i3}(\frac{a}{c}) * (\frac{a}{t})^3 \quad i = 1,2 \qquad (15)$$

$H_1 = 1 + G_{11}(\frac{a}{t}) + G_{12}(a/t)^2 + G_{13}(a/t)^3$

$H_2 = 1 + G_{21}(a/t) + G_{22}(a/t)^2 + G_{23}(a/t)^3$

$$G_{ij} = t_{ij0} + t_{ij1} * (\frac{a}{c}) + t_{ij2} * (\frac{a}{c})^2 \qquad i = 1,2; \qquad j = 1,2,3 \qquad (16)$$

For a/c > 1:

$$M_1 = (c/a)^{0.5}(1 + 0.04c/a) \qquad (17)\ \mathrm{S}$$

$$M_2 = 0.2(c/a)^4 \qquad (18)\ \mathrm{S}$$

$$M_3 = -0.11(c/a)^4 \qquad (19)\ \mathrm{S}$$

$$g_1 = 1 + [0.1 + 0.35(c/a)(a/t)^2](1 - \sin\phi)^2 \qquad (20)\ \mathrm{S}$$

$$g_3 = (1.13 - 0.09c/a)[1 + 0.1(1 - \cos\phi)^2][0.85 + 0.15(a/t)^{0.25}] \qquad (21)\ \mathrm{S}$$

$$g_4 = 1 \qquad (22)$$

S

$$f_\phi = [(c/a)^2 \sin^2\phi + \cos^2\phi]^{0.25} \qquad (23)$$

S

$$p = 0.2 + \frac{c}{a} + 0.6 * (\frac{a}{t}) \qquad (24)$$

S

$$G_{ij} = q_{ij\,0} + q_{ij\,1} * (c/a) \qquad i = 1,2; \qquad j = 1,2,3 \qquad (25)$$

Equation (1) shows that the bending solution is obtained by combining a bending multiplier, H_{ch}, with the tension solution, F_{ch}. (For tension, all the equations related to F_{ch} will be the same as shown here, with only one exception of eq.(8).) Thus, of interest to the present work are the equations (13)-(16), (24) and (25), among which eqs.(15), (16) and (25) are necessary to modify.

It is also noted that the above bending related equations do not contain r/t as a variable in them, which implies that r/t has the same effect on bending solution as it does on tension. However, the weight function solutions [4] show that it is helpful to introduce r/t into the bending related equations. This is achieved by introducing two fine-tuning functions, R_i (i=1,2), into the H_i functions.

Without going into the detail, the modified equations are given below, using the same equation numbers with an 'm' attached.

$$H_i(\frac{a}{c},\frac{a}{t}) = [G_{i0}(\frac{a}{c}) + G_{i1}(\frac{a}{c})*(\frac{a}{t}) + G_{i2}(\frac{a}{c})*(\frac{a}{t})^2 + G_{i3}(\frac{a}{c})*(\frac{a}{t})^3]*R_i(\frac{r}{t}) \quad i=1,2 \tag{15m}$$

$$G_{ij} = t_{ij0} + t_{ij1}*(\frac{a}{c}) + t_{ij2}*(\frac{a}{c})^2 \qquad i=1,2; \qquad j=0,1,2,3 \tag{16m}$$

$$G_{ij} = q_{ij0} + q_{ij1}*(\frac{a}{c}) \qquad i=1,2; \qquad j=0,1,2,3 \tag{25m}$$

The fine tuning functions in eq.(15m) are:

$$R_1(\frac{r}{t}) = (\frac{r}{t})^{-0.09} \tag{R1}$$

$$R_2(\frac{r}{t}) = (\frac{r}{t})^{-0.13} \tag{R1}$$

Though the same functional form is used in the modified equation, attention should be paid to eq.(25m), in which the variable is a/c, instead of c/a as in eq.(25). The coefficients in eqs.(16m) and

(25m) are listed in Table 1. The values for the coefficients are completely different from those in eqs.(16) and (25).

Table 1. Coefficients of eqs.(15m) and (25m)

	0	1	2
t_{10k}	0.548	0.830	-0.633
t_{11k}	1.230	-2.902	1.488
t_{12k}	-0.605	-0.308	0.963
t_{13k}	-0.981	2.805	-2.015
t_{20k}	0.584	0.658	-0.513
t_{21k}	1.134	-4.344	2.464
t_{22k}	-3.550	5.701	-2.657
t_{23k}	1.944	-2.652	0.901
q_{10k}	0.770	-0.0266	
q_{11k}	-0.0366	-0.162	
q_{12k}	-0.244	0.336	
q_{13k}	-0.116	-0.0955	
q_{20k}	0.727	0.00165	
q_{21k}	-0.551	-0.207	
q_{22k}	-0.732	0.258	
q_{23k}	0.334	-0.168	

The modification to bending equation is now complete, which is obtained by replacing eqs.(15), (16) and (25) respectively with eqs.(15m), (16m) and (25m). Figure 3 shows examples of the comparison between the modified equation and the weight function solutions [4].

In summary, the modified bending equation, maintaining the same functional form as before, provides accurate solutions throughout the entire applicable range.

References

[1] J.C. Newman, Jr. and I.S. Raju, Analyses of surface cracks in finite plates under tension and bending loads. NASA TP-1578 (1979).

[2] J.C. Newman Jr.and I.S. Raju, Stress intensity factor equations for cracks in three-dimensional finite bodies. ASTM STP 791,(Edited by J.C. Lewis and G. Sines), I-238 - I-256 (1983).

[3] J.C. Newman Jr.and I.S. Raju, Stress intensity factor equations for cracks in three-dimensional finite bodies subjected to tension and bending loads. in Computational Methods in the Mechanics of Fracture, (Edited by S.N. Atluri), 312-334 (1986).

[4] W. Zhao, J.C. Newman, Jr., M.A. Sutton, X.R. Wu and K.N. Shivakumar, Analysis of corner cracks at hole by a 3-D weight function method with stresses from finite element analysis. NASA TM-110144 (1995).

[5] I.S. Raju and J.C. Newman, Jr., Stress intensity factors for two symmetric corner cracks. ASTM STP 677, C.W. Smith, Ed., 411-430 (1979).