Stress Intensity Solutions for Continuing Damage

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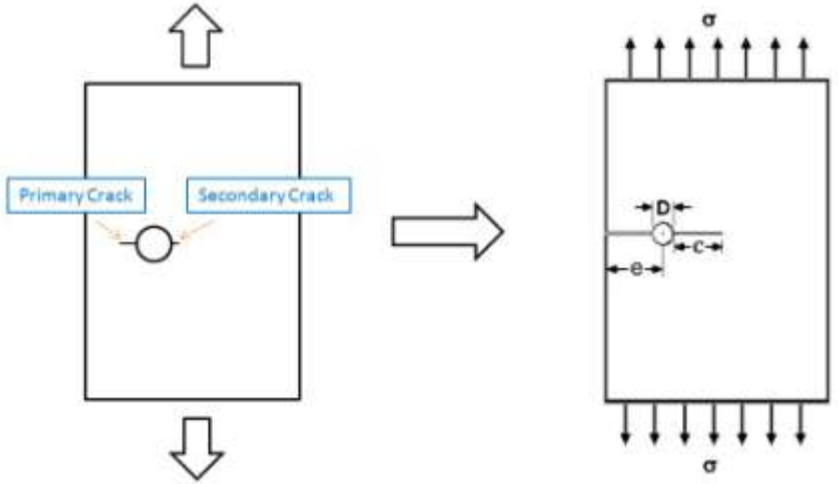
Continuing Damage

When an initial crack grows to a physical boundary (free edge, adjacent hole, etc.), standard LEFM methods can not be used to continue to predict crack growth in the structure unless a secondary crack is assumed to exist.

One of the most common continuing damage scenario is the case of a crack growing from a fastener hole to the near edge of a plate.

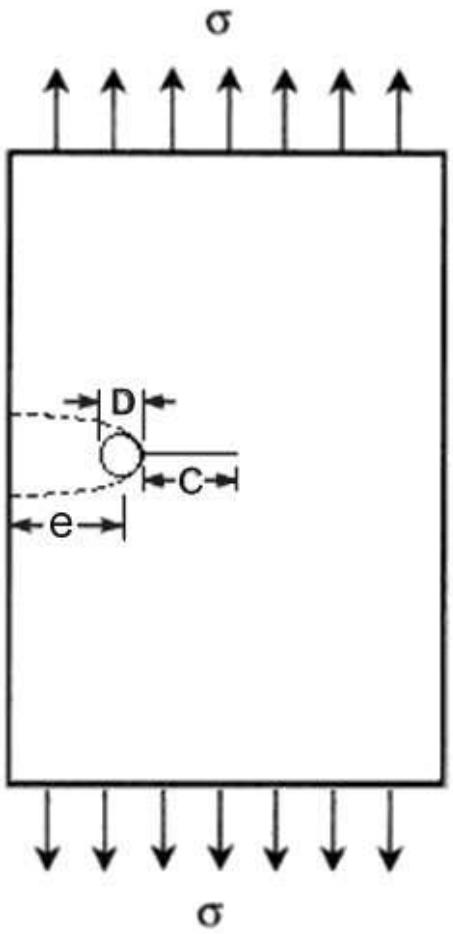
When the near ligament is severed, the resulting geometry is equivalent to a U-shaped notch.



Standard practice assumes an initial 0.05 corner crack (primary crack) on the near side of a hole and an 0.005 corner crack (secondary crack) on the opposite side of the hole.

Handbook Solution

Through Crack at a Semi-Elliptical Notch

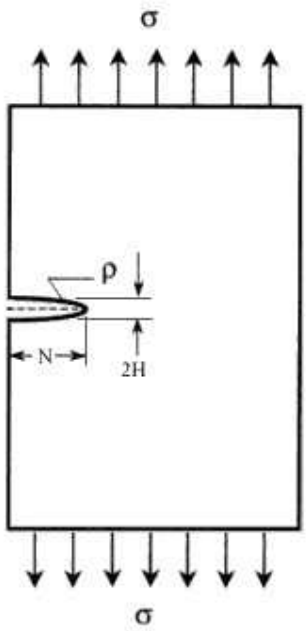


Ref: Tada, Paris, & Irwin, “The Stress Analysis of Cracks Handbook”

Elliptical Notch Solution

(Thru-Crack/Semi-Infinite Plate)

K_T * 1.1221

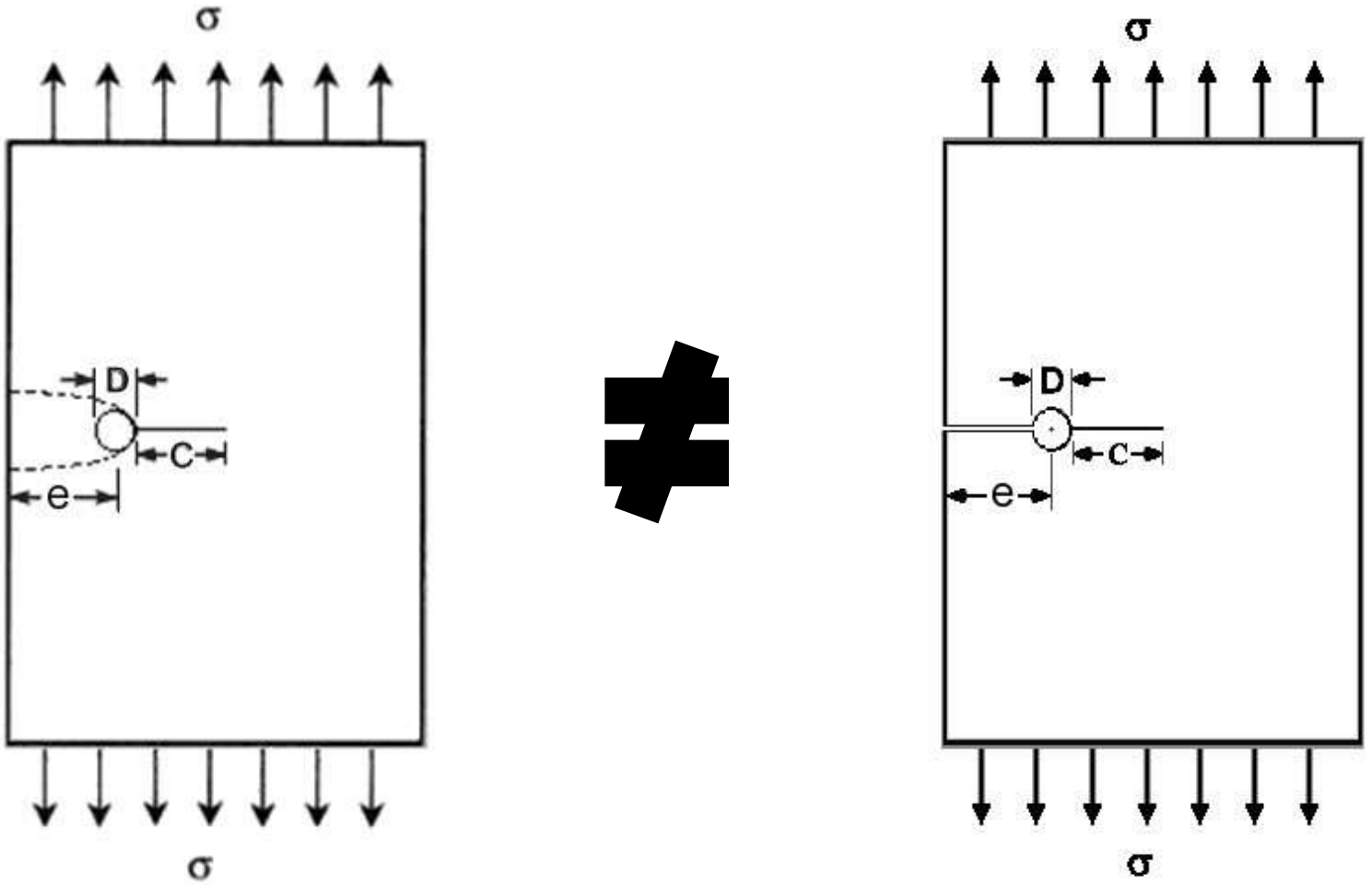


	Betas										
	H/N										
C/(C+N)	0.15	0.2	0.25	0.33	0.4	0.5	0.625	0.75	1	1.5	2
0	16.094	12.359	10.12	7.9516	6.7678	5.6539	4.7651	4.1743	3.4385	2.707	2.3433
0.025	9.4	7.95	7.05	6.023	5.462	4.815	4.205	3.8	3.22	2.592	2.28
0.05	5.075	5.045	5.025	4.95	4.619	4.26	3.803	3.5	3.065	2.5	2.22
0.1	3.592	3.592	3.592	3.592	3.524	3.46	3.214	3.05	2.775	2.328	2.105
0.15	2.925	2.925	2.925	2.925	2.902	2.88	2.772	2.7	2.525	2.174	2
0.2	2.59	2.59	2.59	2.59	2.59	2.59	2.512	2.46	2.31	2.046	1.915
0.3	2.004	2.004	2.004	2.004	2.004	2.004	2.004	2.004	1.999	1.832	1.75
0.4	1.75	1.75	1.75	1.75	1.75	1.75	1.75	1.75	1.76	1.656	1.605
0.5	1.553	1.553	1.553	1.553	1.553	1.553	1.553	1.553	1.553	1.505	1.469
0.6	1.405	1.405	1.405	1.405	1.405	1.405	1.405	1.405	1.405	1.392	1.385
0.7	1.315	1.315	1.315	1.315	1.315	1.315	1.315	1.315	1.315	1.312	1.31
0.8	1.24	1.24	1.24	1.24	1.24	1.24	1.24	1.24	1.24	1.24	1.24
0.9	1.175	1.175	1.175	1.175	1.175	1.175	1.175	1.175	1.175	1.175	1.175
1	1.1221	1.1221	1.1221	1.1221	1.1221	1.1221	1.1221	1.1221	1.1221	1.1221	1.1221

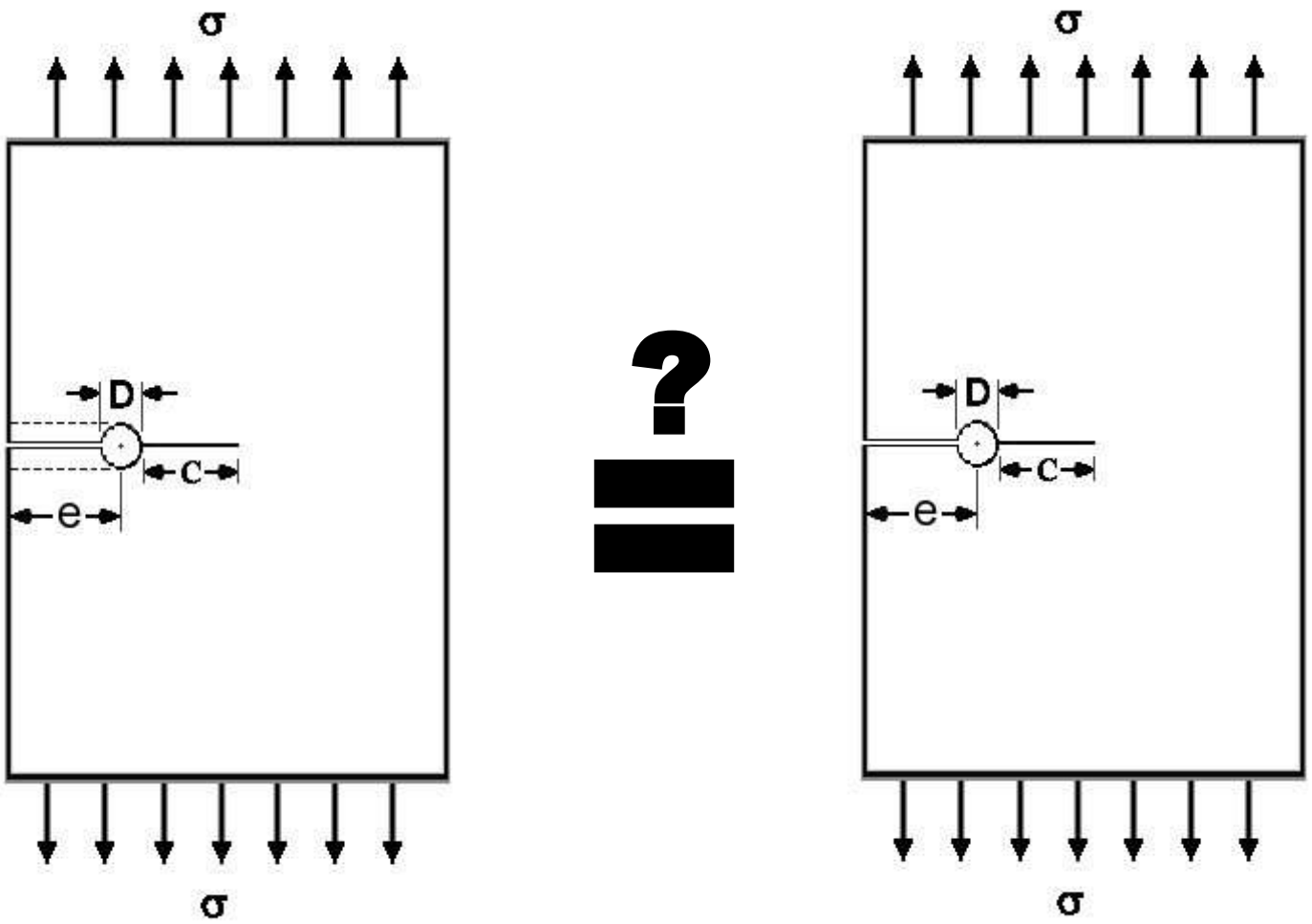
Edge Crack Beta

Stress Intensity Factor, $K = \sigma \sqrt{\pi c}\, \beta$

Problem #1

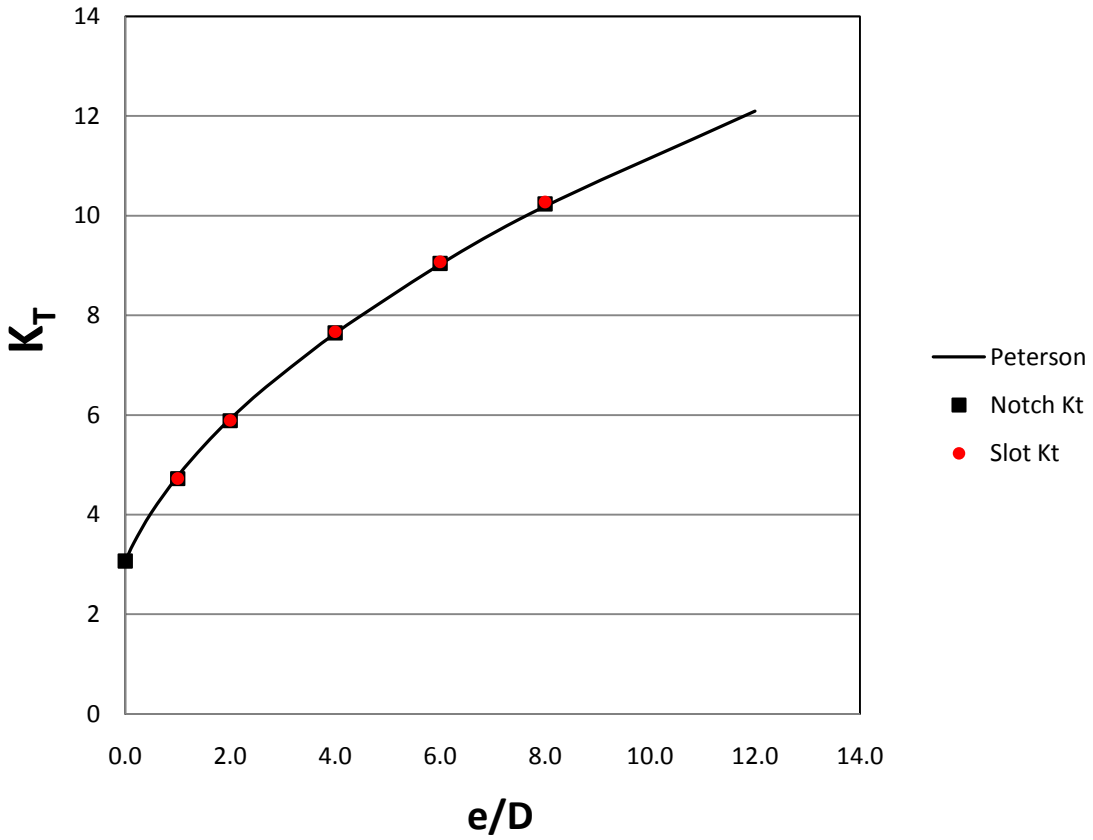


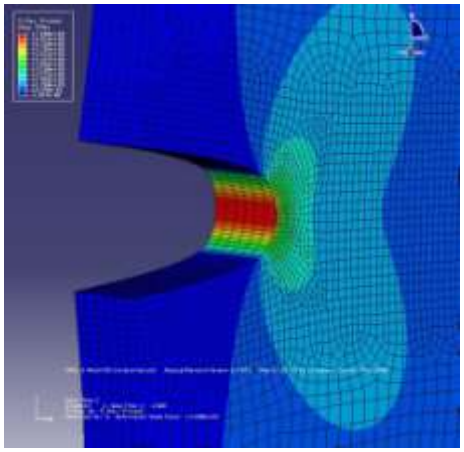
Problem #2

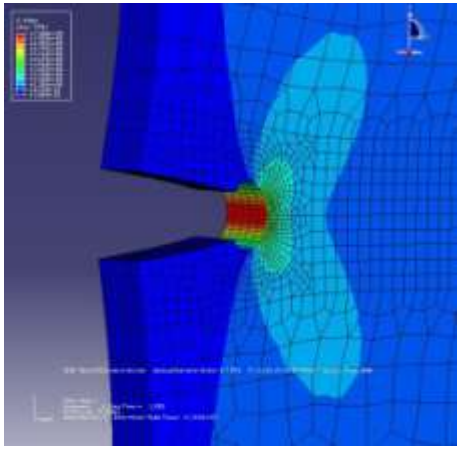


Although the stress concentration factor (Kt) for a U-Shaped notch is known, it would be helpful to verify that the Kt for a U-shaped notch is equivalent to the Kt for a hole when the primary crack has grown through the near ligament

Semi-Infinite Plate K_T (Notch vs. Slot)



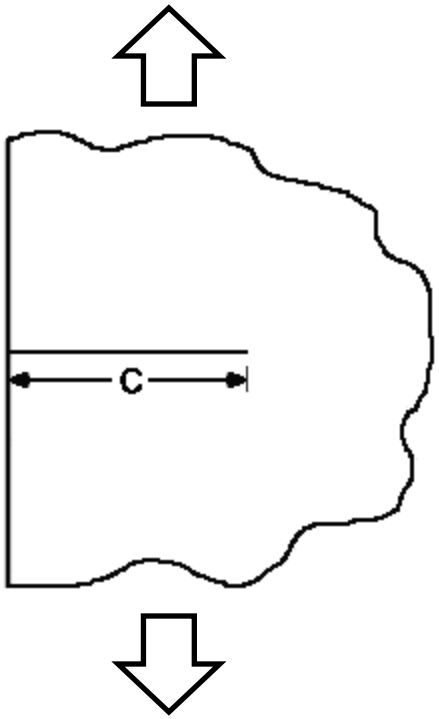


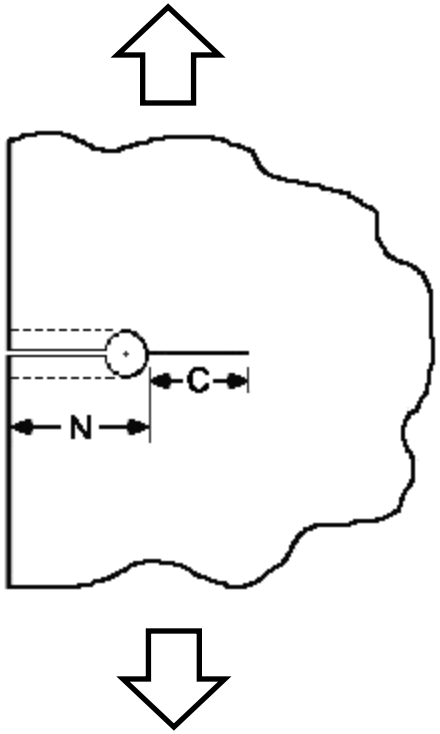


e/D	Kt (Notch)	Kt (Slot)	Kt (Peterson)*
8	10.236	10.275	10.280
6	9.044	9.075	9.185
4	7.649	7.669	7.709
2	5.885	5.89	5.920
1	4.724	4.726	4.775
0	3.071	N/A	3.065

* values interpolated from Peterson

Equivalent Edge Crack Solution





$$K = \sigma \sqrt{\pi C}\, \beta$$

$$\beta = 1.1221$$

$$\sigma \sqrt{\pi\,(C + N)}\,(1.1221) = \sigma \sqrt{\pi\, C}\, \beta$$

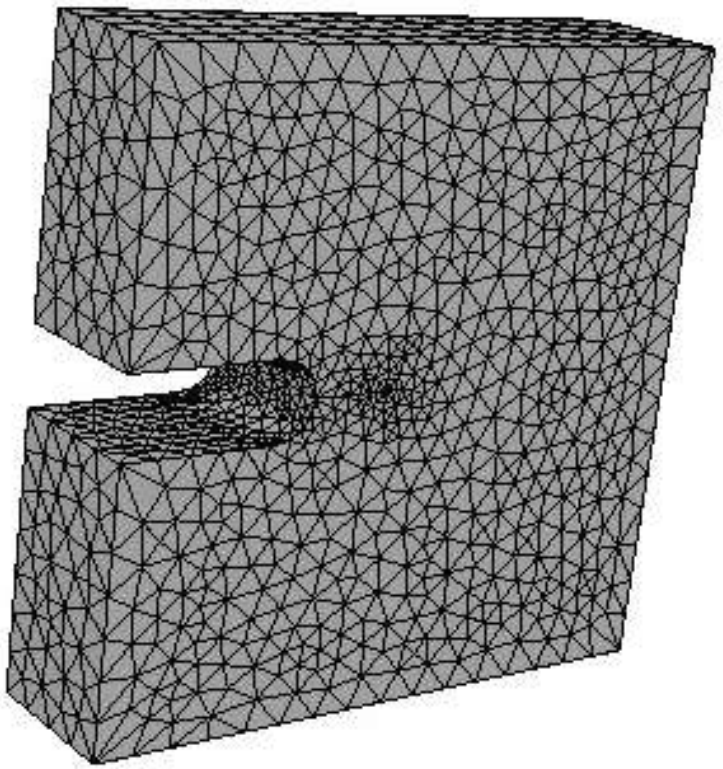
$$\beta = \frac{1.1221}{\sqrt{C/(C + N)}}$$

Problem #3

Semi-Infinite Plate Betas												
	R/N											
C/(C+N)	0.01	0.02	0.04	0.0625	0.1	0.15	0.2	0.25	0.33333	0.5	0.75	1
0.000	25.9205	18.6830	13.5774	11.1368	8.9768	7.5068	6.6428	6.0425	5.3580	4.5389	3.8600	3.4392
0.003	18.1250	14.8375	11.0950	10.2450	8.5950	7.2365	6.4478	5.8677	5.2200	4.4753	3.8185	3.4108
0.00625	13.9750	12.4500	10.7300	9.4750	8.0625	6.9678	6.2525	5.6950	5.1050	4.4127	3.7750	3.3820
0.009	11.8000	10.9100	9.8650	8.8948	7.7175	6.7545	6.0965	5.5635	5.0095	4.3609	3.7393	3.3583
0.0125	10.0364	9.5000	9.0000	8.2500	7.3220	6.5000	5.9100	5.4100	4.9000	4.2965	3.6950	3.3310
0.01875	8.1947	8.0350	7.7150	7.3100	6.6985	6.0750	5.5900	5.1500	4.7225	4.1855	3.6180	3.2822
0.025	7.0968	7.0968	6.8500	6.6000	6.1920	5.7000	5.3050	4.9250	4.5750	4.0800	3.5450	3.2375
0.0375	5.7945	5.7945	5.7600	5.6700	5.4500	5.1450	4.8650	4.6000	4.3000	3.9000	3.4200	3.1535
0.050	5.0182	5.0182	5.0182	4.9500	4.8400	4.6500	4.4800	4.3125	4.0700	3.7255	3.3150	3.0738
0.0625	4.4884	4.4884	4.4884	4.4884	4.3750	4.2800	4.1700	4.0600	3.8595	3.5750	3.2200	2.9980
0.075	4.0973	4.0973	4.0973	4.0973	4.0350	3.9820	3.9220	3.8350	3.6750	3.4320	3.1300	2.9315
0.0875	3.7934	3.7934	3.7934	3.7934	3.7755	3.7415	3.7000	3.6375	3.5100	3.3050	3.0450	2.8676
0.100	3.5484	3.5484	3.5484	3.5484	3.5484	3.5325	3.5050	3.4600	3.3575	3.1880	2.9650	2.8040
0.125	3.1738	3.1738	3.1738	3.1738	3.1738	3.1680	3.1625	3.1500	3.0750	2.9650	2.7575	2.6800
0.150	2.8972	2.8972	2.8972	2.8972	2.8972	2.8960	2.8950	2.8925	2.8675	2.7575	2.6225	2.5600
0.175	2.6823	2.6823	2.6823	2.6823	2.6823	2.6823	2.6823	2.6800	2.6728	2.6215	2.5400	2.4495
0.200	2.5091	2.5091	2.5091	2.5091	2.5091	2.5091	2.5091	2.5091	2.5070	2.4750	2.4150	2.3490
0.250	2.2442	2.2442	2.2442	2.2442	2.2442	2.2442	2.2442	2.2442	2.2442	2.2375	2.2100	2.1739
0.300	2.0487	2.0487	2.0487	2.0487	2.0487	2.0487	2.0487	2.0487	2.0487	2.0420	2.0300	2.0185
0.350	1.8967	1.8967	1.8967	1.8967	1.8967	1.8967	1.8967	1.8967	1.8967	1.8967	1.8920	1.8870
0.400	1.7742	1.7742	1.7742	1.7742	1.7742	1.7742	1.7742	1.7742	1.7742	1.7742	1.7740	1.7730
0.450	1.6727	1.6727	1.6727	1.6727	1.6727	1.6727	1.6727	1.6727	1.6727	1.6727	1.6727	1.6695
0.500	1.5869	1.5869	1.5869	1.5869	1.5869	1.5869	1.5869	1.5869	1.5869	1.5869	1.5869	1.5869
0.550	1.5130	1.5130	1.5130	1.5130	1.5130	1.5130	1.5130	1.5130	1.5130	1.5130	1.5130	1.5130
0.600	1.4486	1.4486	1.4486	1.4486	1.4486	1.4486	1.4486	1.4486	1.4486	1.4486	1.4486	1.4486
0.700	1.3412	1.3412	1.3412	1.3412	1.3412	1.3412	1.3412	1.3412	1.3412	1.3412	1.3412	1.3412
0.800	1.2545	1.2545	1.2545	1.2545	1.2545	1.2545	1.2545	1.2545	1.2545	1.2545	1.2545	1.2545
0.900	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828
1.000	1.1221	1.1221	1.1221	1.1221	1.1221	1.1221	1.1221	1.1221	1.1221	1.1221	1.1221	1.1221

How to complete this matrix?

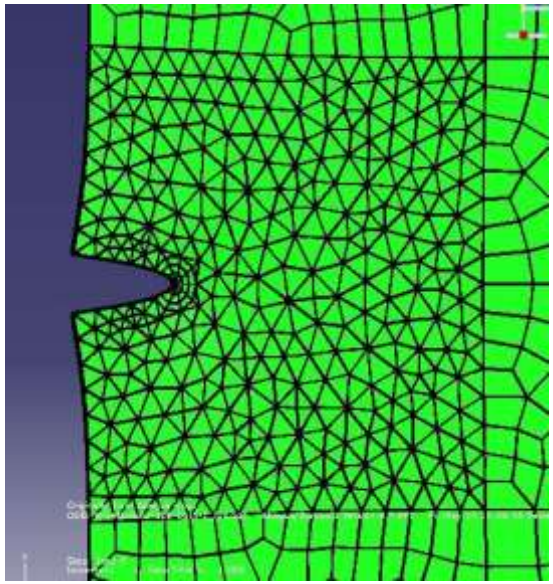
Finite Element Modeling

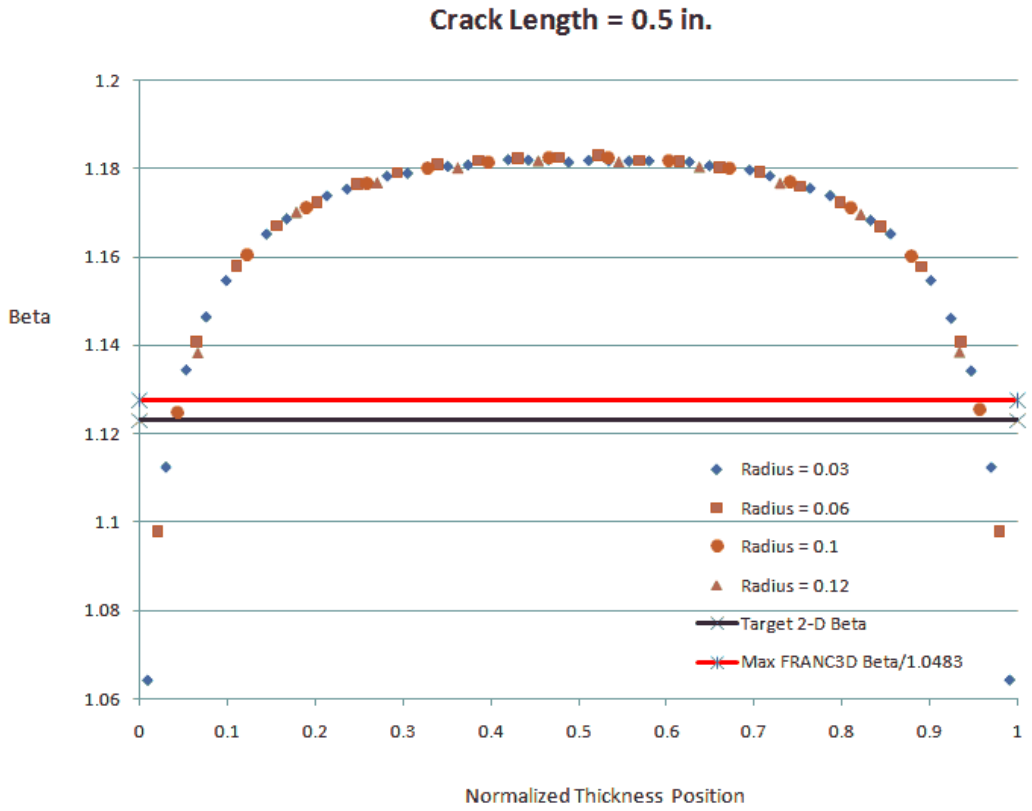


ABAQUS w/ FRANC3D/NG

Verification/Validation

(Classic 2-D Edge Crack)





$$K_{Plane\ Stress} = \sqrt{GE}$$

$$K_{Plane\ Strain} = \sqrt{\frac{GE}{(1 - \upsilon^2)}}$$

$$\frac{K_{Plane\ Strain}}{K_{Plane\ Stress}} = \frac{1}{\sqrt{(1 - \upsilon^2)}}$$

Model Dimensions/Boundary Conditions

Dimensions:

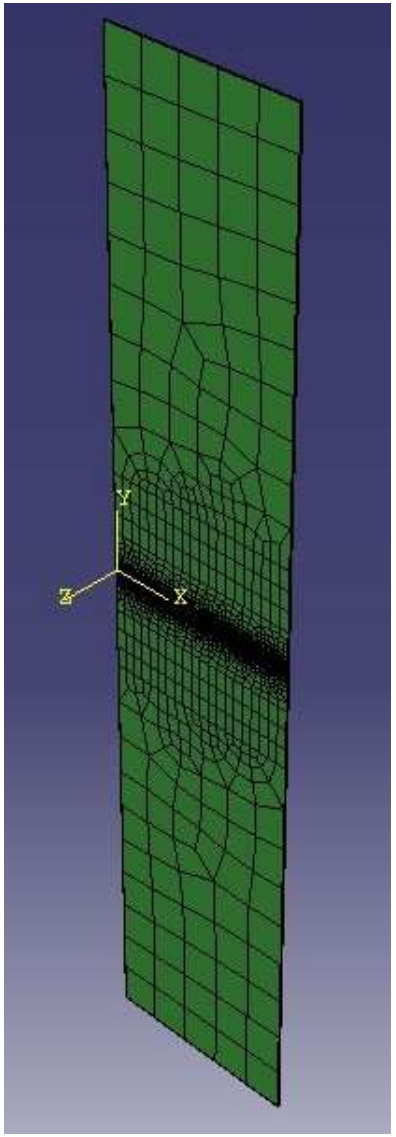
Width = 100 * N
Thickness = 1
Total Height = 500 * N

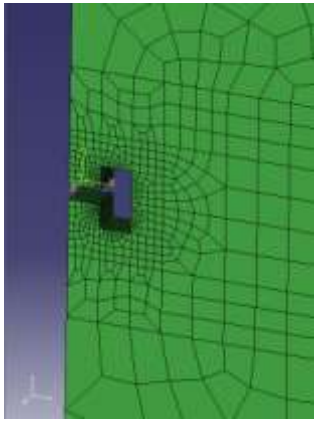
Loading:

Top Edge: Pressure Loading

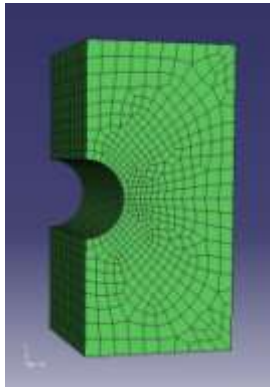
Boundary Conditions:

Bottom Edge : Fixed in Y-Direction
Mid-Plane in Z-Direction
Right Edge: Fixed in X-Direction
(Counteracts In-Plane Bending)



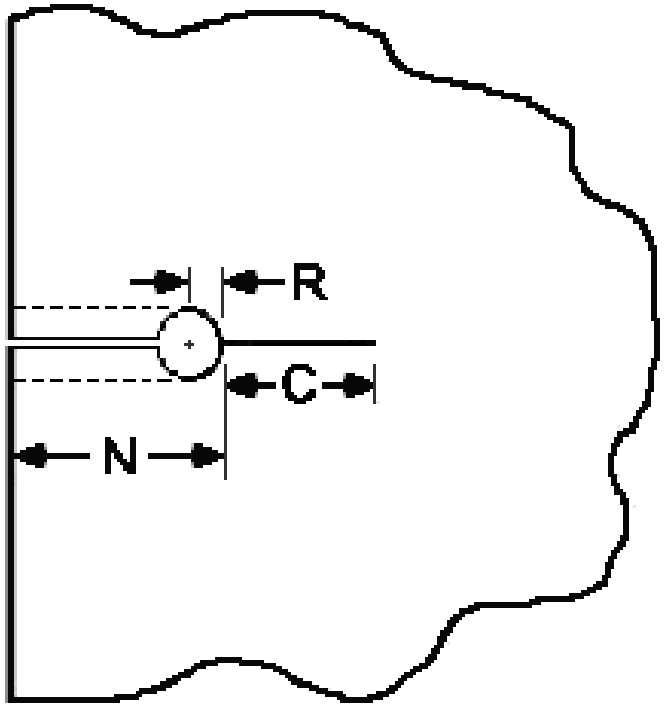


Global Model



Local Model

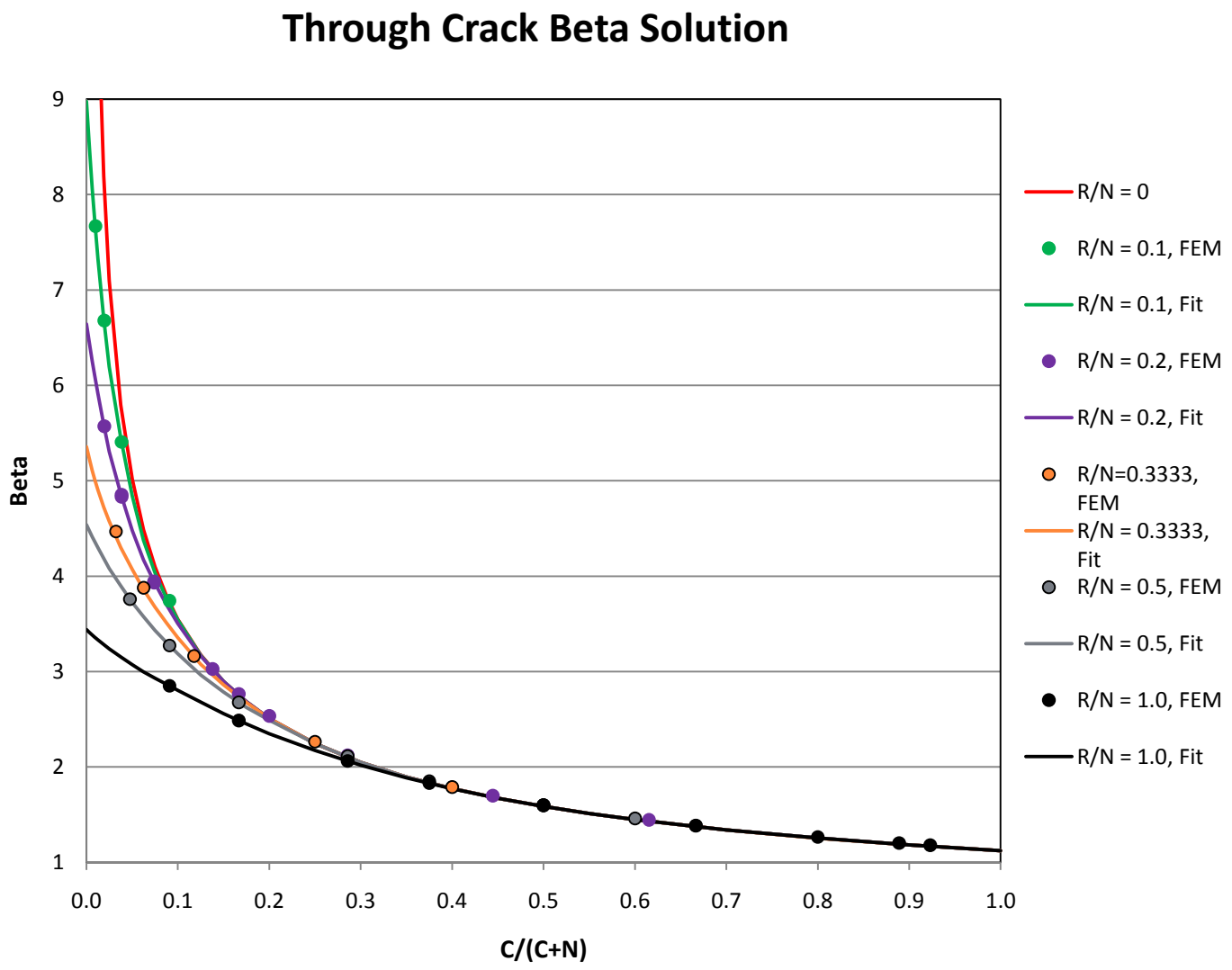
Wide Plate Models



R/N: 0.1, 0.2, 1/3, 0.5, 1.0

e/D: 4.5, 2.0, 1.0, 0.5, 0.0

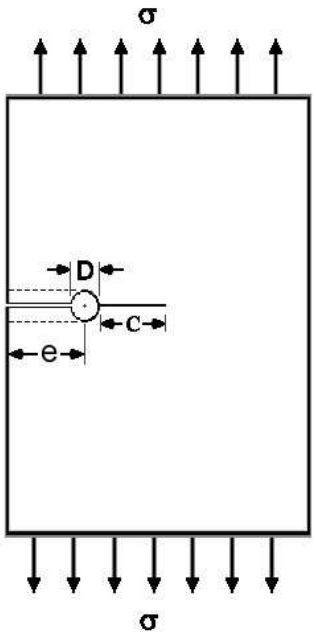
Wide Plate Results



U-Shaped Notch Solution

(Thru-Crack/Semi-Infinite Plate)

$K_T * 1.1221$



$N = e + R$

$R = D/2$

Semi-Infinite Plate Betas												
	R/N											
C/(C+N)	0.01	0.02	0.04	0.0625	0.1	0.15	0.2	0.25	0.33333	0.5	0.75	1
0.000	25.9205	18.6830	13.5774	11.1368	8.9768	7.5068	6.6428	6.0425	5.3580	4.5389	3.8600	3.4392
0.003	18.1250	14.8275	11.9950	10.2450	8.5050	7.2365	6.4478	5.8677	5.2290	4.4753	3.8185	3.4108
0.00625	13.9750	12.4500	10.7300	9.4750	8.0625	6.9678	6.2525	5.6950	5.1050	4.4127	3.7750	3.3820
0.009	11.8000	10.9100	9.8650	8.8948	7.7175	6.7545	6.0965	5.5635	5.0095	4.3609	3.7393	3.3583
0.0125	10.0364	9.5000	9.0000	8.2500	7.3220	6.5000	5.9100	5.4100	4.9000	4.2965	3.6950	3.3310
0.01875	8.1947	8.0350	7.7150	7.3100	6.6985	6.0750	5.5900	5.1500	4.7225	4.1855	3.6180	3.2822
0.025	7.0968	7.0968	6.8500	6.6000	6.1920	5.7000	5.3050	4.9250	4.5750	4.0800	3.5450	3.2375
0.0375	5.7945	5.7945	5.7600	5.6700	5.4500	5.1450	4.8650	4.6000	4.3000	3.9000	3.4200	3.1535
0.050	5.0182	5.0182	5.0182	4.9500	4.8400	4.6500	4.4800	4.3125	4.0700	3.7255	3.3150	3.0738
0.0625	4.4884	4.4884	4.4884	4.4884	4.3750	4.2800	4.1700	4.0600	3.8595	3.5750	3.2200	2.9980
0.075	4.0973	4.0973	4.0973	4.0973	4.0350	3.9820	3.9220	3.8350	3.6750	3.4320	3.1300	2.9315
0.0875	3.7934	3.7934	3.7934	3.7934	3.7755	3.7415	3.7000	3.6375	3.5100	3.3050	3.0450	2.8676
0.100	3.5484	3.5484	3.5484	3.5484	3.5484	3.5325	3.5050	3.4600	3.3575	3.1880	2.9650	2.8040
0.125	3.1738	3.1738	3.1738	3.1738	3.1738	3.1680	3.1625	3.1300	3.0750	2.9650	2.8145	2.6800
0.150	2.8972	2.8972	2.8972	2.8972	2.8972	2.8972	2.8950	2.8900	2.8600	2.7850	2.6675	2.5600
0.175	2.6823	2.6823	2.6823	2.6823	2.6823	2.6823	2.6823	2.6800	2.6728	2.6215	2.5400	2.4495
0.200	2.5091	2.5091	2.5091	2.5091	2.5091	2.5091	2.5091	2.5091	2.5070	2.4750	2.4150	2.3490
0.250	2.2442	2.2442	2.2442	2.2442	2.2442	2.2442	2.2442	2.2442	2.2442	2.2375	2.2100	2.1739
0.300	2.0487	2.0487	2.0487	2.0487	2.0487	2.0487	2.0487	2.0487	2.0487	2.0420	2.0300	2.0185
0.350	1.8967	1.8967	1.8967	1.8967	1.8967	1.8967	1.8967	1.8967	1.8967	1.8967	1.8920	1.8870
0.400	1.7742	1.7742	1.7742	1.7742	1.7742	1.7742	1.7742	1.7742	1.7742	1.7742	1.7740	1.7730
0.450	1.6727	1.6727	1.6727	1.6727	1.6727	1.6727	1.6727	1.6727	1.6727	1.6727	1.6727	1.6695
0.500	1.5869	1.5869	1.5869	1.5869	1.5869	1.5869	1.5869	1.5869	1.5869	1.5869	1.5869	1.5869
0.550	1.5130	1.5130	1.5130	1.5130	1.5130	1.5130	1.5130	1.5130	1.5130	1.5130	1.5130	1.5130
0.600	1.4486	1.4486	1.4486	1.4486	1.4486	1.4486	1.4486	1.4486	1.4486	1.4486	1.4486	1.4486
0.700	1.3412	1.3412	1.3412	1.3412	1.3412	1.3412	1.3412	1.3412	1.3412	1.3412	1.3412	1.3412
0.800	1.2545	1.2545	1.2545	1.2545	1.2545	1.2545	1.2545	1.2545	1.2520	1.2545	1.2545	1.2545
0.900	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828
1.000	1.1221	1.1221	1.1221	1.1221	1.1221	1.1221	1.1221	1.1221	1.1221	1.1221	1.1221	1.1221

Edge Crack Beta

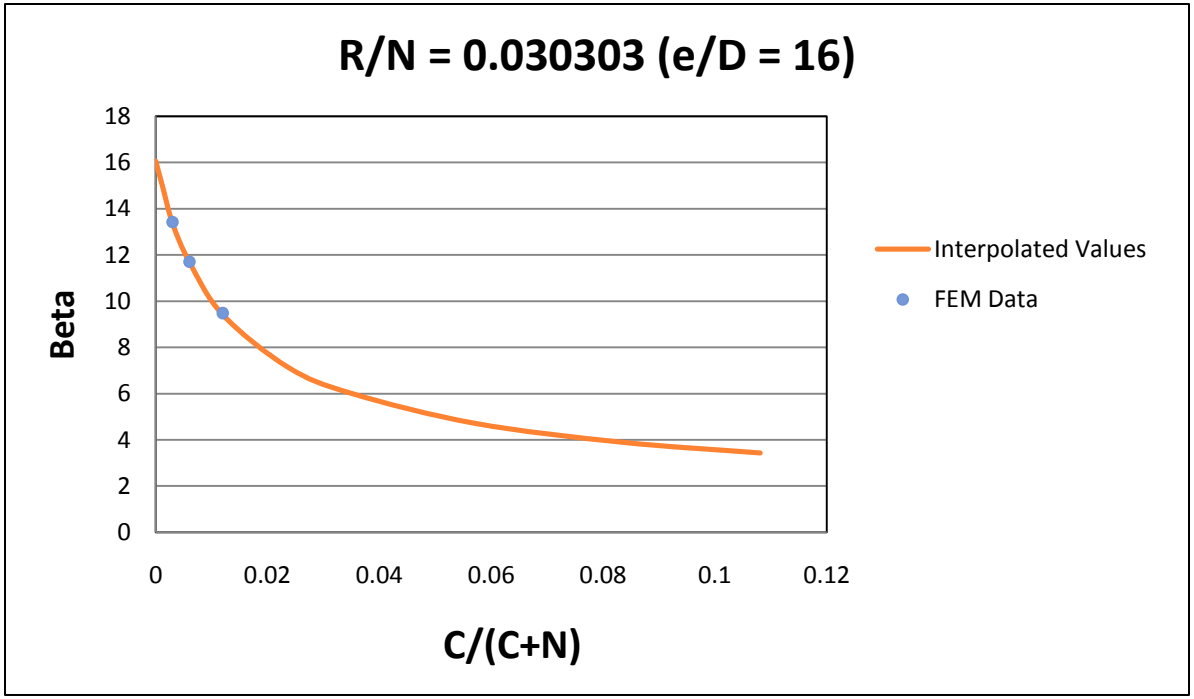
Edge Crack Solution Error

Equivalent Edge Crack Error (%)												
	R/N											
C/(C+N)	0.01	0.02	0.04	0.0625	0.1	0.15	0.2	0.25	0.33333	0.5	0.75	1
0.000	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
0.003	13.03	38.17	70.79	99.97	140.88	183.10	217.73	249.14	291.79	357.77	436.51	500.64
0.00625	1.56	14.00	32.28	49.80	76.04	103.70	127.01	149.23	178.03	221.65	275.99	319.68
0.009	0.24	8.41	19.90	32.98	53.26	75.11	94.01	112.60	136.11	171.23	216.32	252.20
0.0125	0.00	5.65	11.52	21.65	37.07	54.41	69.82	85.52	104.82	133.59	171.62	201.30
0.01875	0.00	1.99	6.22	12.10	22.34	34.89	46.59	59.12	73.52	95.79	126.50	149.67
0.025	0.00	0.00	3.60	7.53	14.61	24.50	33.78	44.10	55.12	73.94	100.19	119.21
0.0375	0.00	0.00	0.60	2.20	6.32	12.62	19.11	25.97	34.76	48.58	69.43	83.75
0.050	0.00	0.00	0.00	1.38	3.68	7.92	12.01	16.36	23.30	34.70	51.38	63.26
0.0625	0.00	0.00	0.00	0.00	2.59	4.87	7.64	10.55	16.29	25.55	39.39	49.71
0.075	0.00	0.00	0.00	0.00	1.54	2.90	4.47	6.84	11.49	19.39	30.91	39.77
0.0875	0.00	0.00	0.00	0.00	0.47	1.39	2.52	4.29	8.07	14.78	24.58	32.28
0.100	0.00	0.00	0.00	0.00	0.00	0.45	1.24	2.55	5.69	11.30	19.68	26.55
0.125	0.00	0.00	0.00	0.00	0.00	0.18	0.36	1.40	3.21	7.04	12.77	18.42
0.150	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.25	1.30	4.03	8.61	13.17
0.175	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.09	0.36	2.32	5.60	9.51
0.200	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	1.38	3.90	6.82
0.250	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.30	1.55	3.23
0.300	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.33	0.92	1.49
0.350	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.51
0.400	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.07
0.450	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.19
0.500	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.550	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.600	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.700	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.800	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.00	0.00	0.00
0.900	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1.000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Notch & Equivalent Edge Crack Solutions Converge When $\mathbf{C \geq R}$

Beta Value Interpolation/Verification

Data were interpolated by cross plotting FEM results along with known beta information to populate the full matrix to provide good resolution for notch depths covering e/D values from 0.0 to 49.5.



Since FEM data were difficult to obtain for the larger notch depths, FEM data for a limited number of crack lengths were compared to data linearly interpolated from the full matrix as shown above

Problem #4

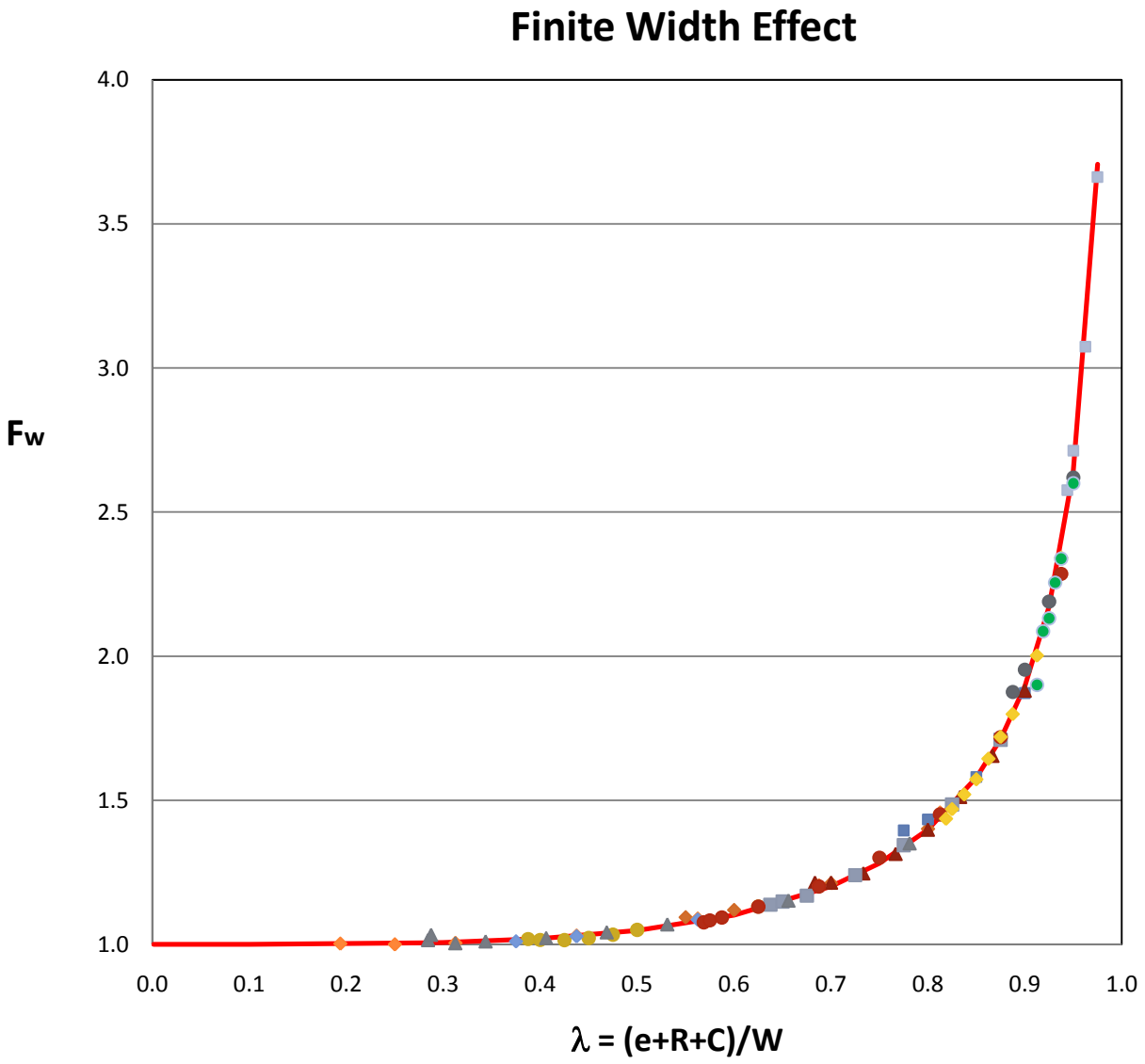
You can never find an infinite plate when you need one.

Finite Width Models

Hole Dia. = 0.5						
	Plate Width					
Offset (e)	0.5	1	1.5	2	4	8
0						
0.5						
0.75						
1						
1.5						
2						
3						
3.5						

Hole Dia. = 0.25						
	Plate Width					
Offset (e)	0.5	1	1.5	2	4	8
0						
0.5						
0.75						
1						
1.5						
2						
3						
3.5						

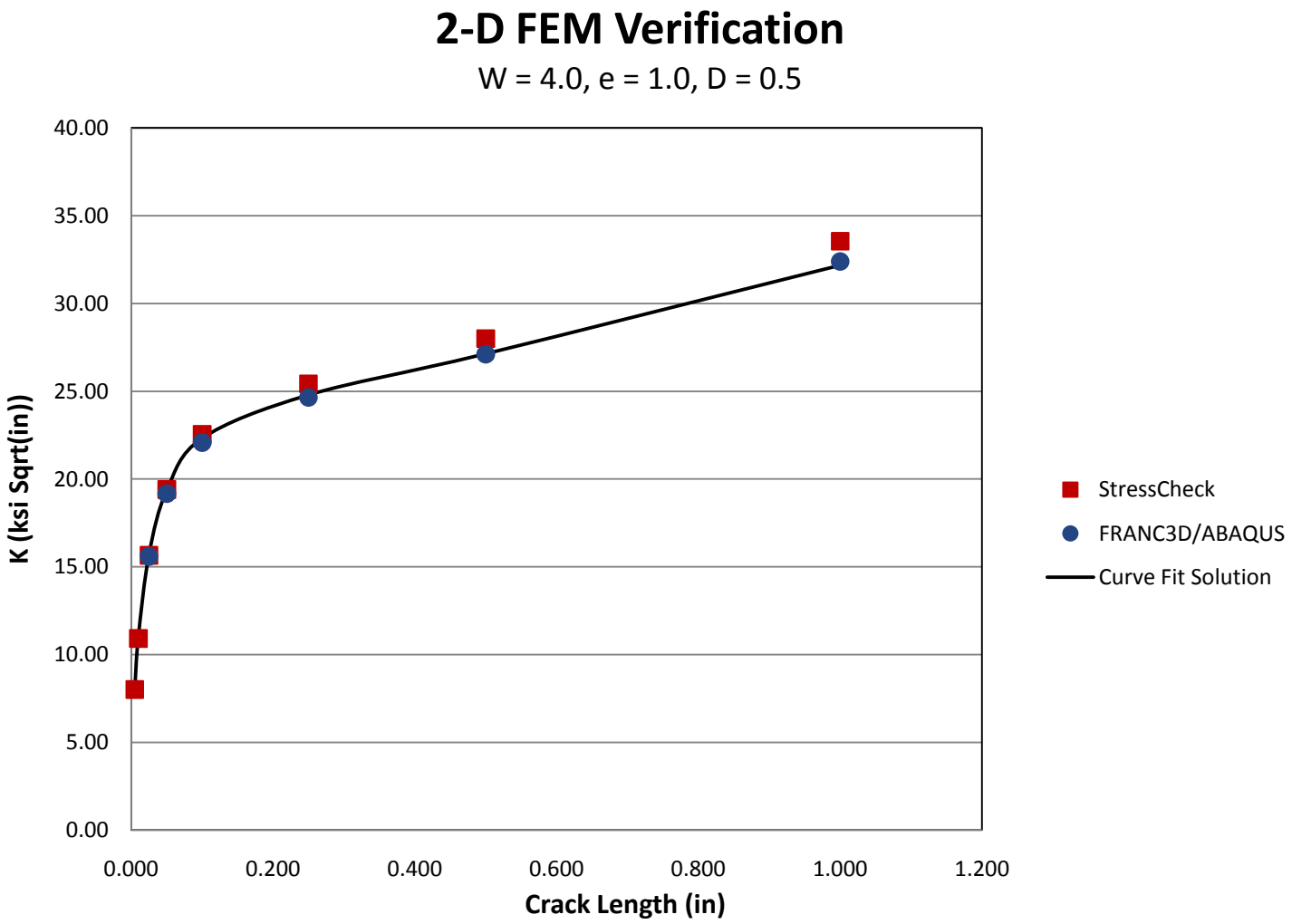
2-D Finite Width Correction



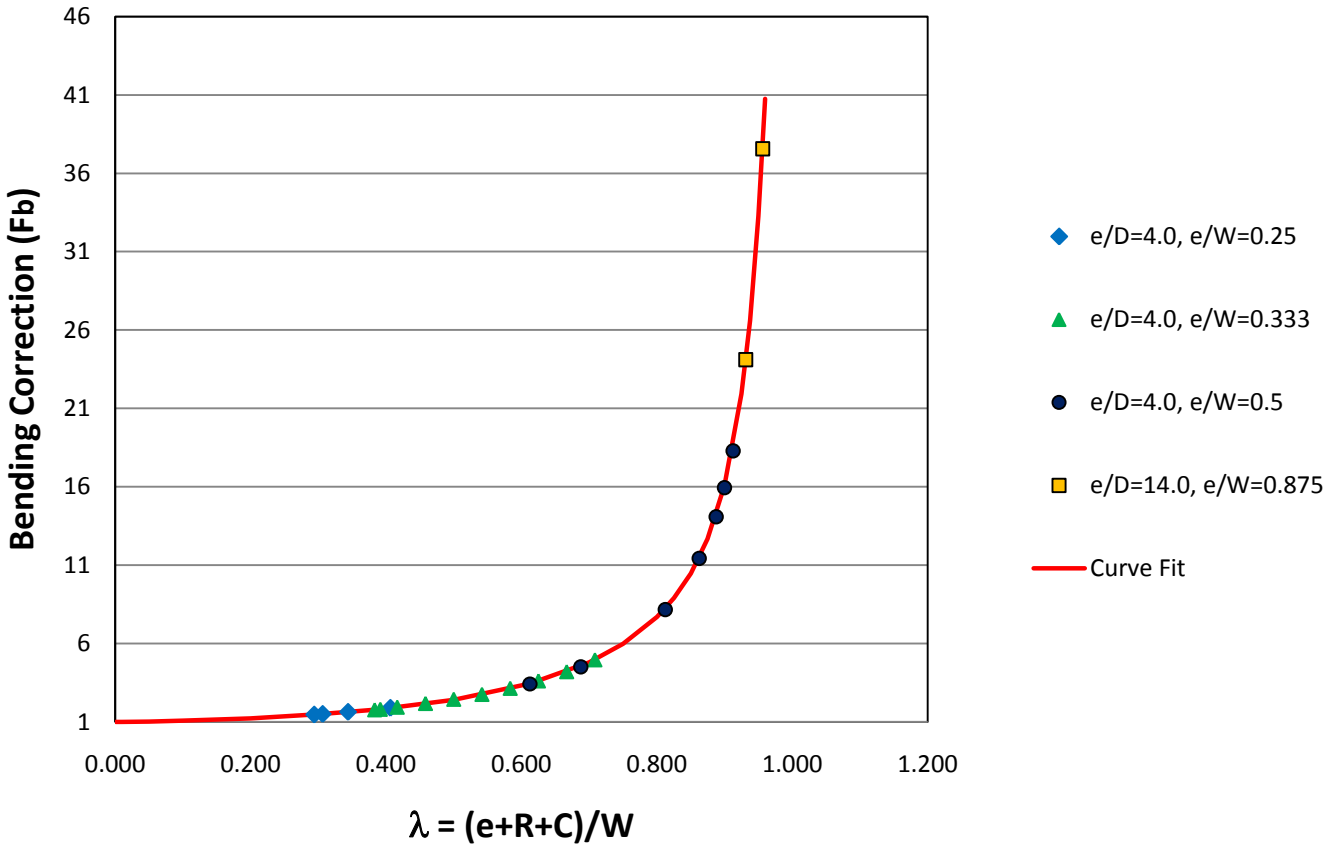
$$F_W = \frac{K_{finite}}{K_{wide}}$$

$$F_W = \sqrt{\sec\left(\frac{\pi}{2}\lambda^{1.875}\right)}$$

2-D Finite Width Verification



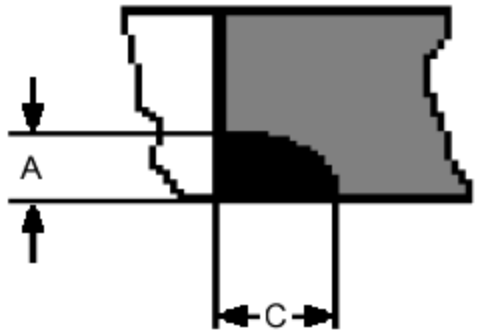
2-D In-Plane Bending Correction



$$F_b = \frac{K_{Bend}}{K_{No\ Bend}}$$

$$F_b = 1 + 0.26\lambda + 4.4\lambda^2 + 11.3\lambda^6 + 8.2\lambda^{12} + 18.5\lambda^{22} + 69\lambda^{39}$$

What About Corner Cracks?



Corner Cracked Hole Framework

(Classic Newman/Raju Solution)

Stress	D	T	C	A	A/T	A/C	Q	SQRT(PI*A/Q)	G2	M1	M2	M3	F(M)	G1	G3	G4	Fphi	K
10	0.25	0.25	0.05	0.05	0.2	1	2.464	0.252487328	1.879721038	1.04	0.201666667	-0.1060606	1.047897	1.114	0.94080794	1	1	5.2124
									2.36505693					1	1.03488873		1	6.4758
10	0.25	0.25	0.1	0.1	0.4	1	2.464	0.357071004	1.473799748	1.04	0.201666667	-0.1060606	1.0695515	1.156	0.9595977	1	1	6.2436968
									1.843968678					1	1.05555747		1	7.4334721
10	0.25	0.25	0.1	0.2	0.8	2	1.4664892	0.654561566	1.473799748	0.7212489	0.0125	-0.006875	0.7264329	1.212	1.02657638	1	1	8.719235
									1.843968678					1	1.12923402		0.707106781	7.001136

G2 – Accounts for the Stress Distribution in the A & C Directions

G3 – Used to Adjust the Notch Solution to Account for A/T and A/C

Otherwise, the Newman/Raju Solution framework was employed

Wide Plate Corner Crack Models

R/N = 0.1 (e/D = 4.5), R/T = 0.25									
	A								
C	0.025	0.05	0.1	0.25	0.3	0.375	0.5	0.75	0.9
0.025									
0.05									
0.1									
0.25									
0.3									
0.5									
0.75									
0.9									
1									

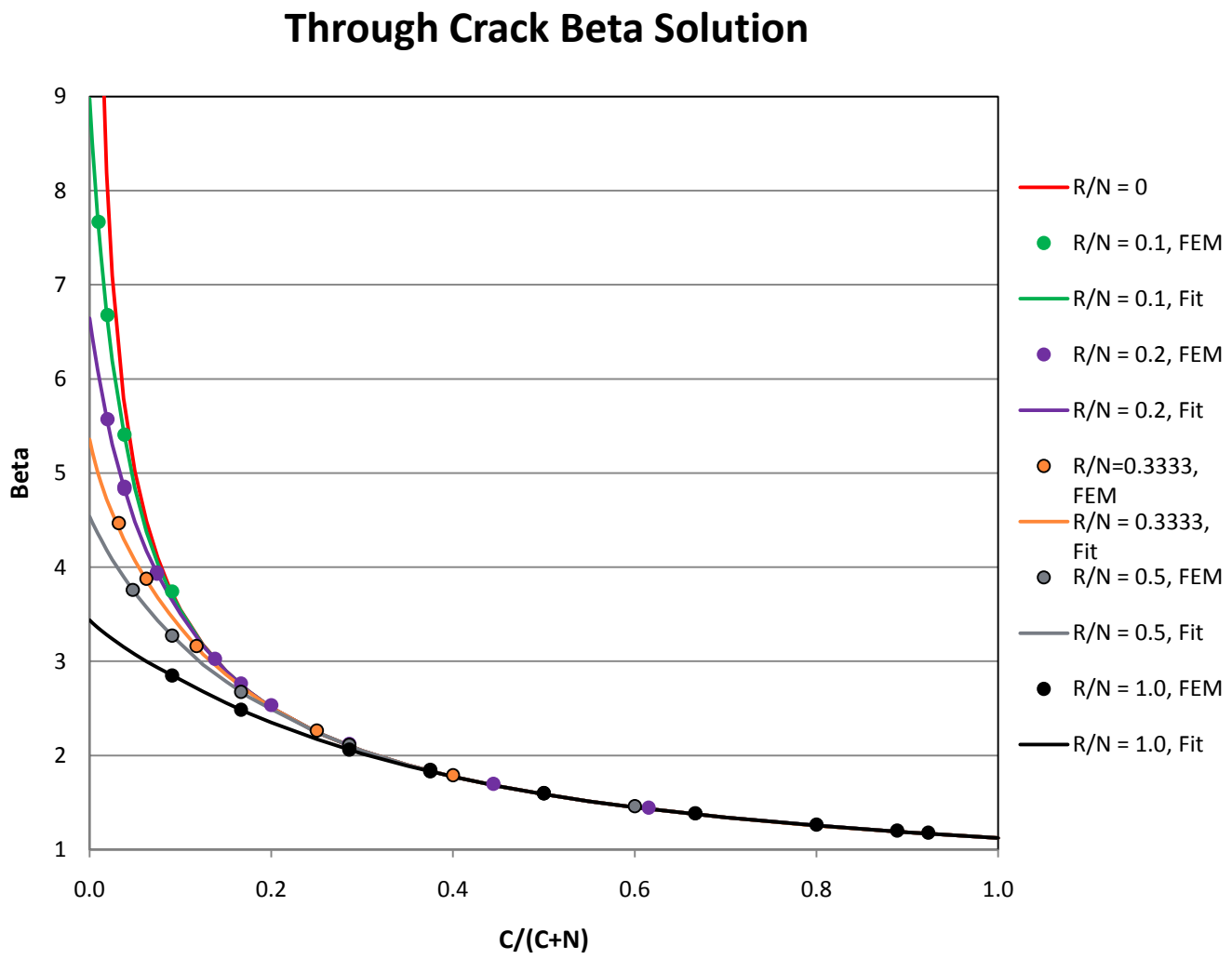
R/N = 0.2 (e/D = 2.0), R/T = 0.25														
	A													
C	0.025	0.05	0.075	0.1	0.125	0.1875	0.2	0.25	0.3	0.375	0.5	0.6	0.75	0.9
0.025														
0.05														
0.1														
0.2														
0.25														
0.3														
0.375														
0.45														
0.5														
0.75														
0.9														

Wide Plate Corner Crack Models

R/N = 0.3333 (e/D = 1.0), R/T = 0.25														
	A													
C	**0.025**	**0.05**	**0.075**	**0.1**	**0.125**	**0.1875**	**0.2**	**0.25**	**0.3**	**0.375**	**0.5**	**0.6**	**0.75**	**0.9**
0.025														
0.05														
0.1														
0.2														
0.25														
0.375														
0.45														
0.5														
0.6667														
0.75														
0.9														

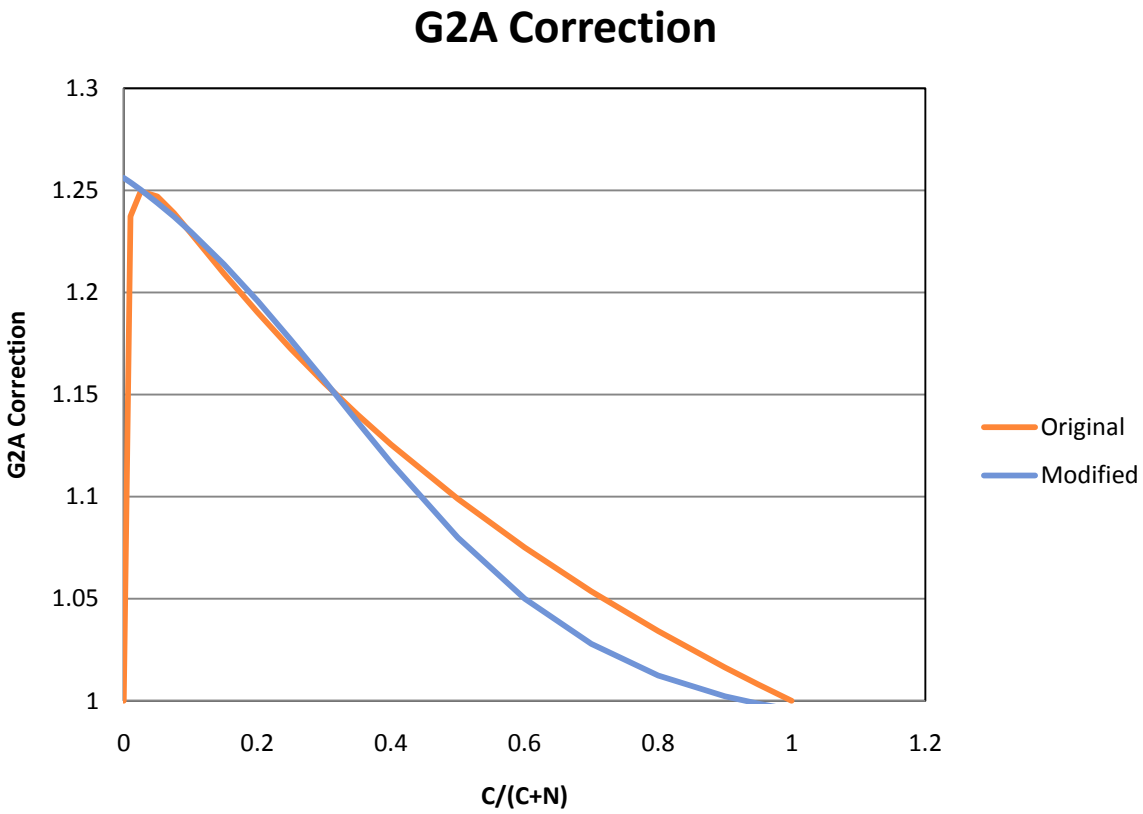
R/N = 1 (e/D = 0.0), R/T = 0.25											
	A										
C	0.025	0.05	0.1	0.1875	0.2	0.25	0.375	0.5	0.6	0.75	0.9
0.025											
0.05											
0.1											
0.2											
0.25											
0.375											
0.45											
0.5											
0.6667											
0.75											
0.9											

Corner Cracked Notch Parameters



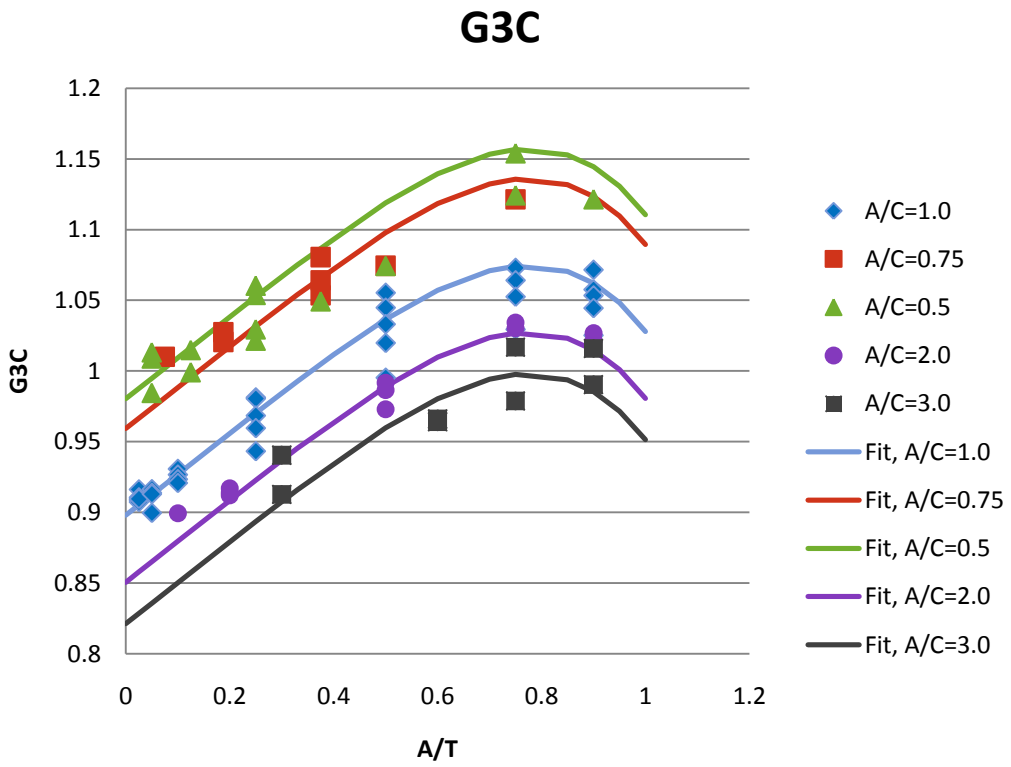
G2C = (2-D Beta) / 1.1221

Corner Cracked Notch Parameters



$$G2A = G2C\left[0.985 + \left[\frac{(1.31 - 0.985)}{1 + e^{-(0.323 - \frac{C}{C+N})/0.2}}\right]\right]$$

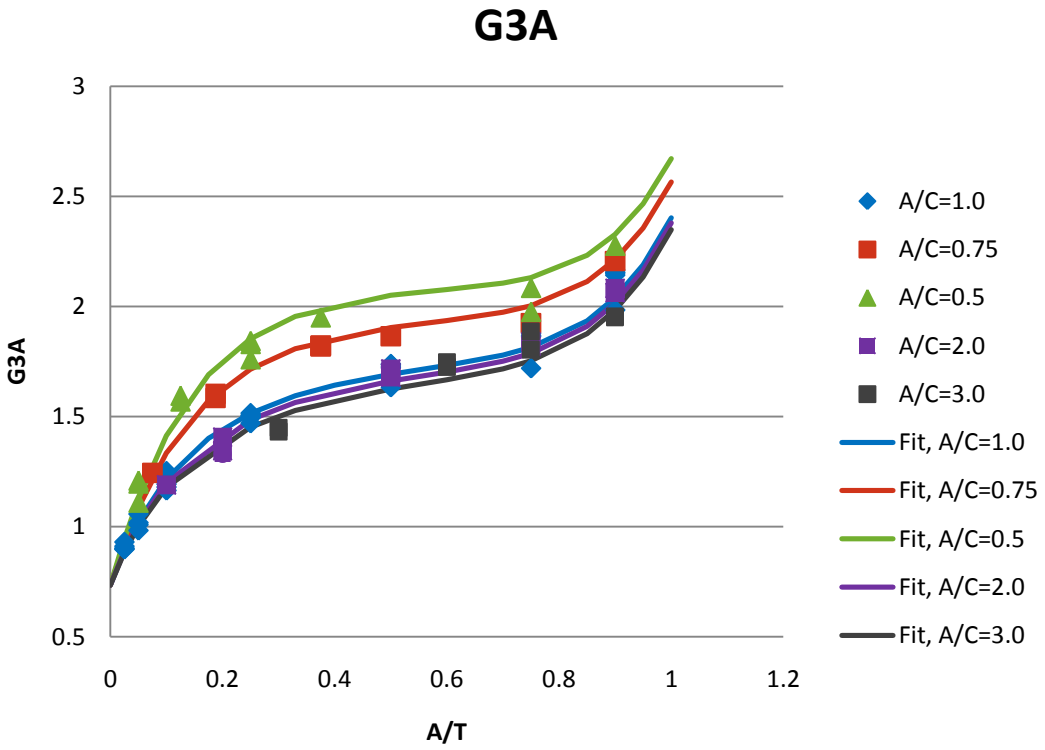
Corner Cracked Notch Parameters



$$G3C = 0.985 - 0.087\left(\frac{A}{C}\right)^{4.25} + 0.29\left(\frac{A}{T}\right) - 0.11\left(\frac{A}{T}\right)^{4.25} - 0.05\left(\frac{A}{T}\right)^{6.0}, \text{ for A/C >= 1.0}$$

$$G3C = 1.558 - 0.66\left(\frac{A}{C}\right)^{0.1} + 0.29\left(\frac{A}{T}\right) - 0.11\left(\frac{A}{T}\right)^{4.25} - 0.05\left(\frac{A}{T}\right)^{6.0}, \text{ for A/C < 1.0}$$

Corner Cracked Notch Parameters



$$G3A = 0.735 + \left[\frac{\left(1.24 - 0.015\left(\frac{A}{C}\right)^{1.5}\right)\left(\frac{A}{T}\right)}{0.175 + \left(\frac{A}{T}\right)^{\left(1.12 - 0.02\left(\frac{A}{C}\right)\right)}}\right] + 0.625\left(\frac{A}{T}\right)^{8}$$, for A/C >= 1.0

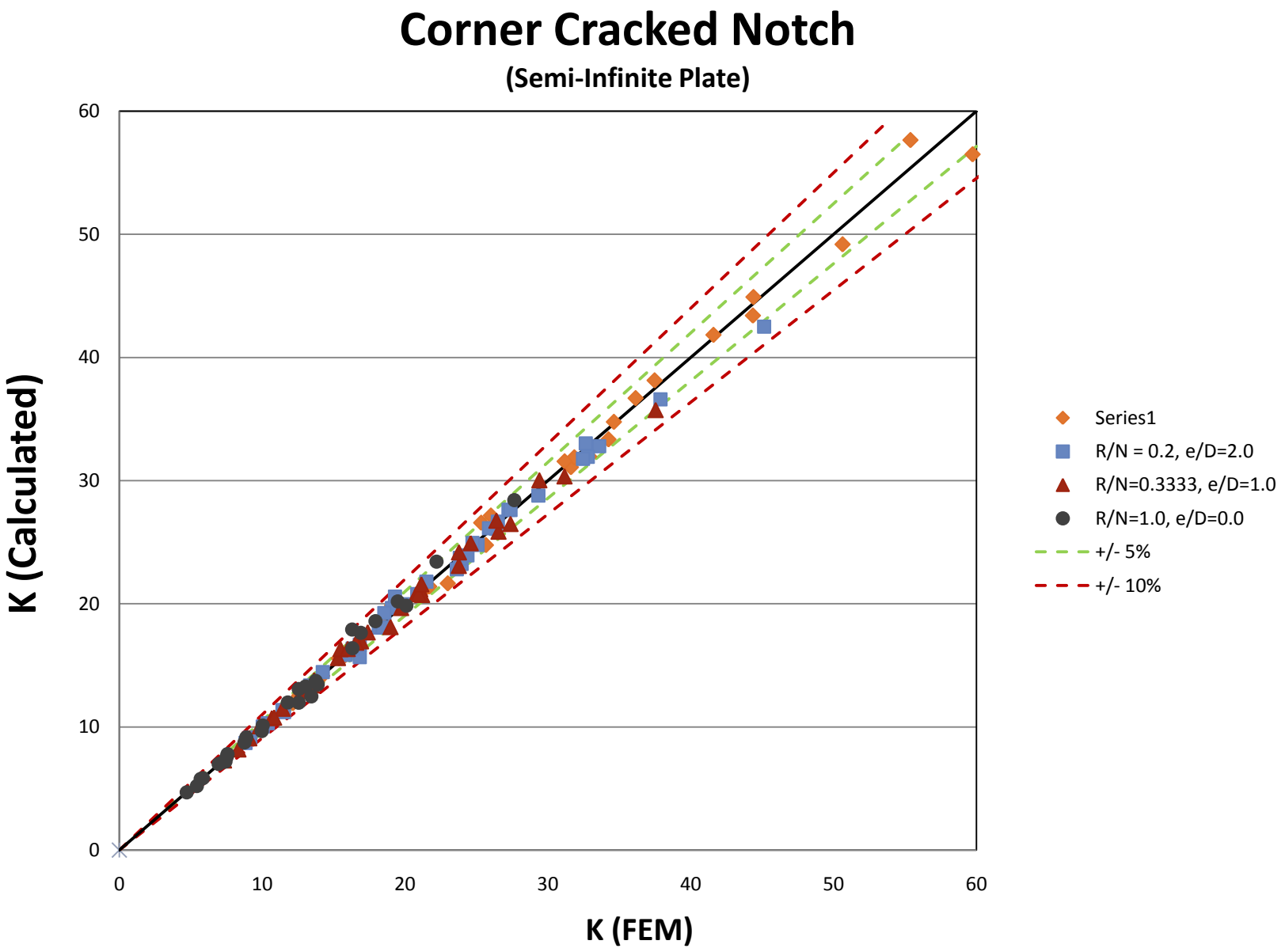
$$G3A = 0.735 + \left[\frac{\left(1.625 - 0.4\left(\frac{A}{C}\right)^{2.25}\right)\left(\frac{A}{T}\right)}{0.175 + \left(\frac{A}{T}\right)^{\left(1.32 - 0.22\left(\frac{A}{C}\right)^{2.5}\right)}}\right] + 0.625\left(\frac{A}{T}\right)^{8}$$, for A/C < 1.0

Corner Cracked Notch Parameters

A minor adjustment to the G1 parameter in the A-direction (G1A) was also required to ensure that the stress intensity factors in each growth direction become equal as the corner crack lengths converge to zero.

$$G1A = 1.0 + 0.0699e^{-150\left(\frac{A}{T}\right)}$$

Results

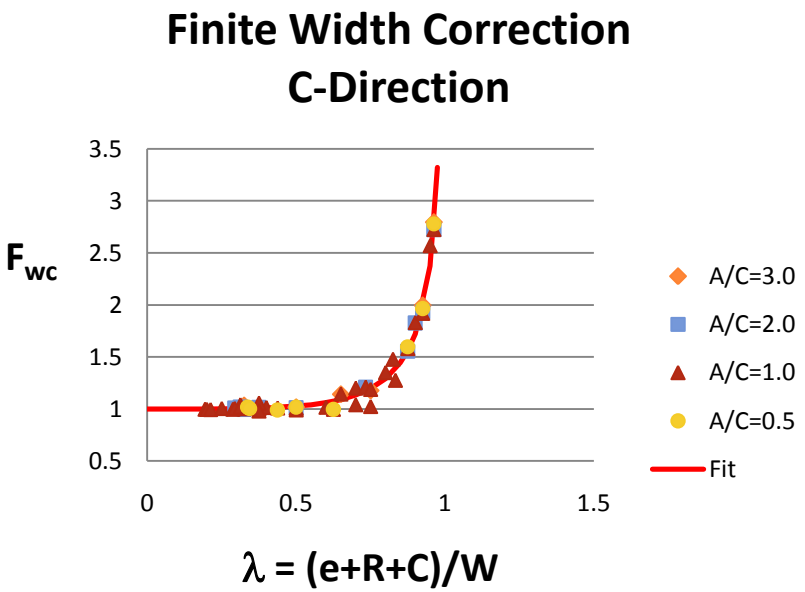

Corner Cracked Notch
(Semi-Infinite Plate)
K (Calculated)
K (FEM)
0
10
20
30
40
50
60
Series1
R/N = 0.2, e/D=2.0
R/N=0.3333, e/D=1.0
R/N=1.0, e/D=0.0
+/- 5%
+/- 10%

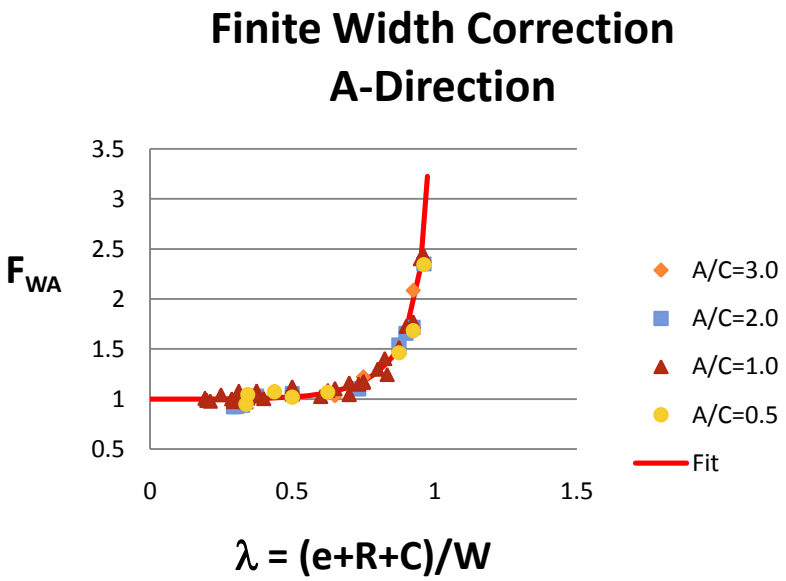
Corner Crack Beta Interpolation/Verification

Plate Width	Hole Dia.	Notch Depth	e/D	R/T
50	0.5	8.25	16	0.25

C	A	A/C	Kc (Fit)	Kc (FEM)	% Diff	Ka (Fit)	Ka (FEM)	% Diff
0.025	0.025	1.000	2.202	2.228	-1.167	2.486	2.500	-0.560
0.050	0.050	1.000	2.750	2.787	-1.328	3.516	3.420	2.807
0.100	0.100	1.000	3.193	3.271	-2.385	4.758	4.574	4.023
0.100	0.300	3.000	4.316	4.538	-4.892	4.656	4.443	4.794
0.250	0.250	1.000	3.774	3.803	-0.763	6.561	6.342	3.453
0.250	0.500	2.000	4.947	4.980	-0.663	6.417	6.297	1.906
0.500	0.500	1.000	4.476	4.548	-1.583	7.643	7.762	-1.533
0.500	0.250	0.500	2.672	2.681	-0.336	7.367	7.359	0.109
0.900	0.900	1.000	5.696	5.745	-0.853	9.720	10.255	-5.217

Finite Width Correction for Corner Cracks

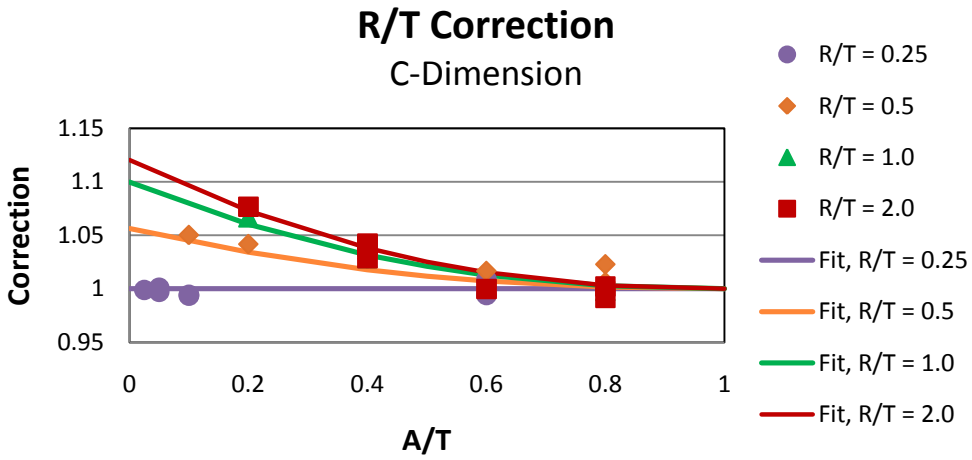




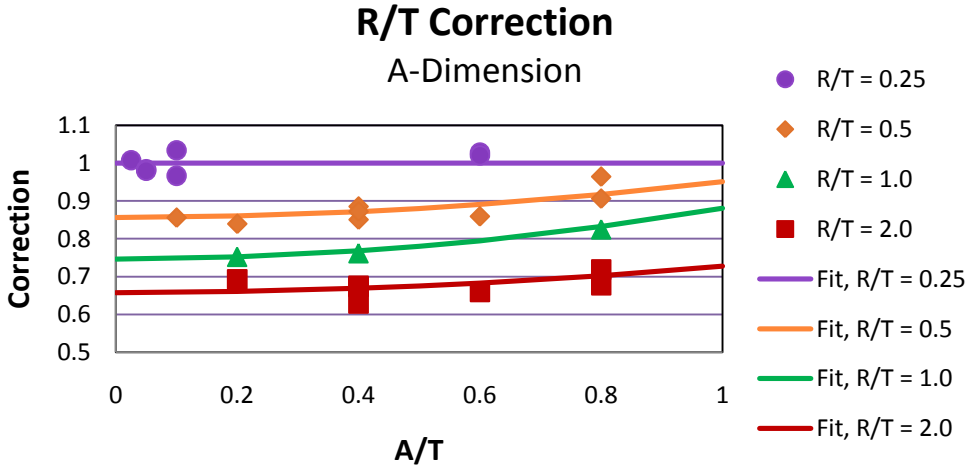
$$Fw_C = \sqrt{sec\left(\frac{\pi}{2}\lambda^{2.35}\right)}$$

$$F_{W_A} = \sqrt{sec\left(\frac{\pi}{2}\lambda^{2.5}\right)}$$

Effect of Hole Radius to Thickness Ratio

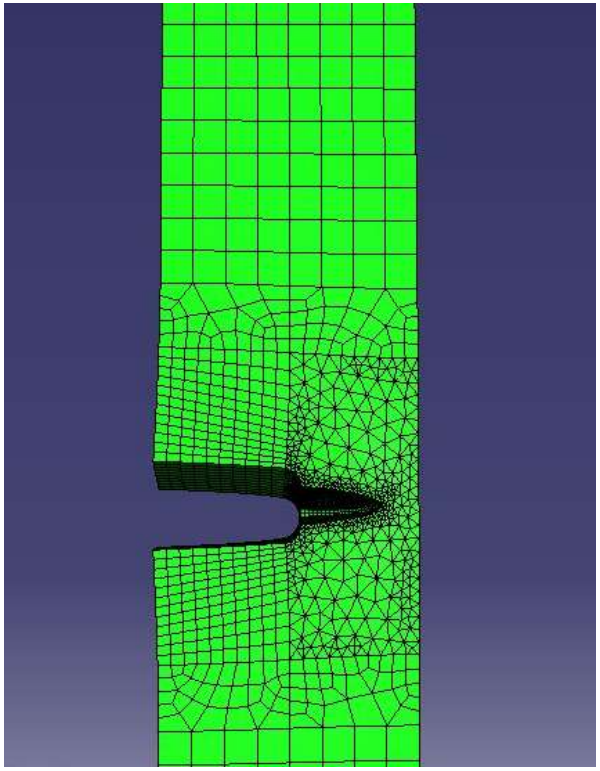


$$G5C = 1 + \left[\frac{1.54\left(\frac{R}{T}\right)^{2.5}}{\left(1 + 10.445\left(\frac{R}{T}\right)^{2.4}\right)} - 0.035\right]\left(1 - \frac{A}{T}\right)^{2.25}$$

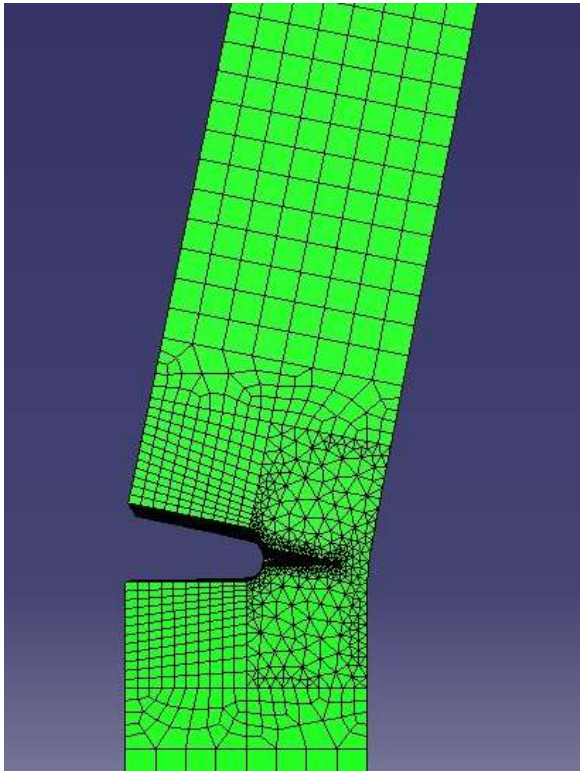


$$G5A = \left[0.5 + \frac{\left(\frac{R}{T}\right)^{1.225}}{\left(0.091 + 1.1\left(\frac{R}{T}\right)\right)}\right]^{-1} + \left(0.8e^{-1.5\left(\frac{R}{T}\right)}\right)\left(\frac{R}{T} - 0.25\right)\left(\frac{A}{T}\right)^{2}$$

In-Plane Bending



In-Plane Bending Constraint
(Ux = 0.0 along the right edge)



No Bending Constraint

In-Plane Bending Correction

2-D Case $F_b = 1 + 0.26\lambda + 4.4\lambda^2 + 11.3\lambda^6 + 8.2\lambda^{12} + 18.5\lambda^{22} + 69\lambda^{39}$

λ =(e + R + C)/W – Width Fraction

In the corner cracked case, a portion of the ligament along the hole bore provides additional bending constraint as a function of A/T. As the C-dimension approaches the neutral axis of the un-cracked net section, the bending effect is significantly reduced.

C-Dimension

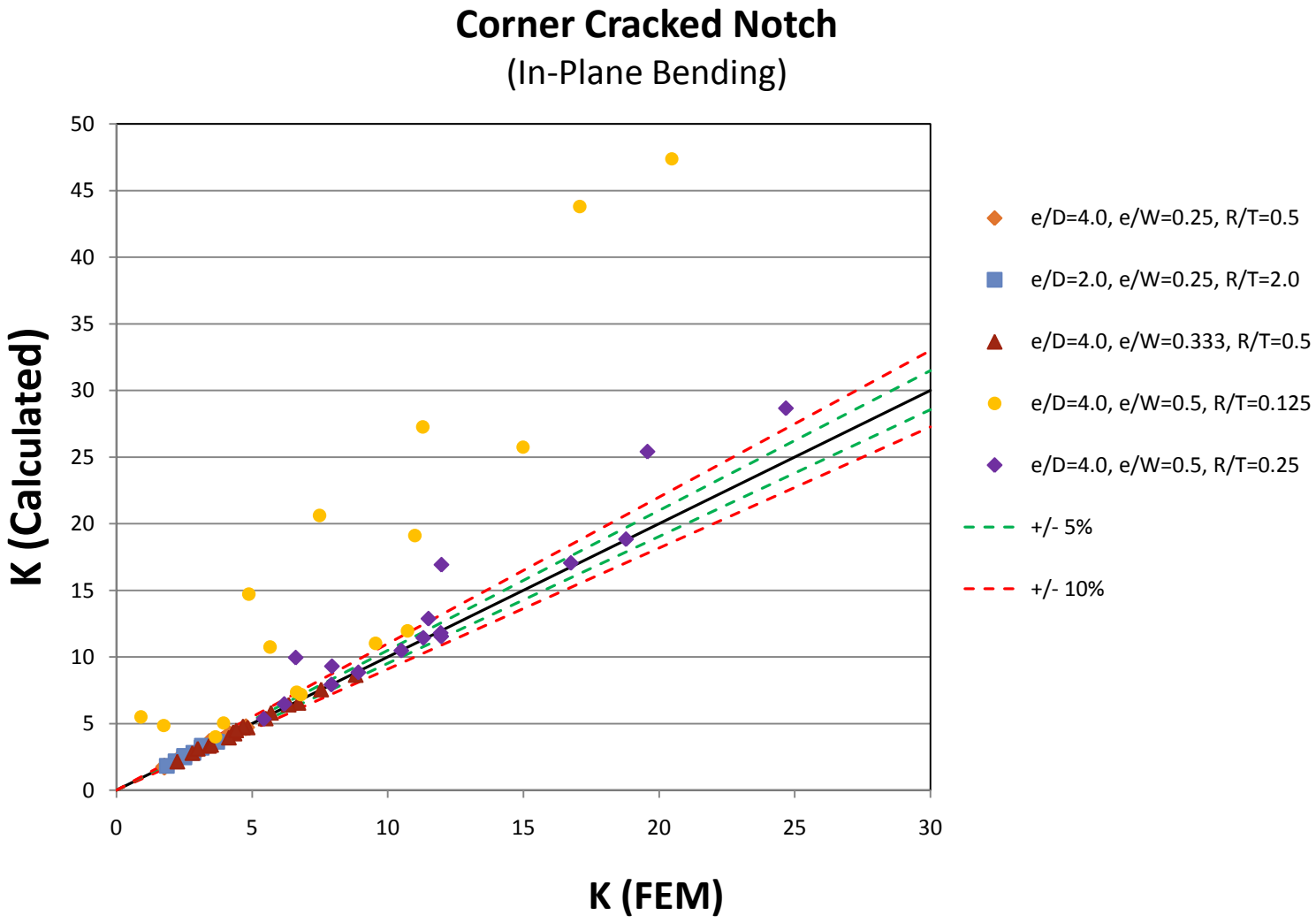
$$F_{bC} = F_b(\lambda_A) + \left(\frac{A}{T}\right)^2 [F_b(\lambda_C) - F_b(\lambda_A)]$$

A-Dimension

$$F_{bA} = F_b(\lambda_A) + \left(\frac{A}{T}\right) [F_b(\lambda_C) - F_b(\lambda_A)]$$

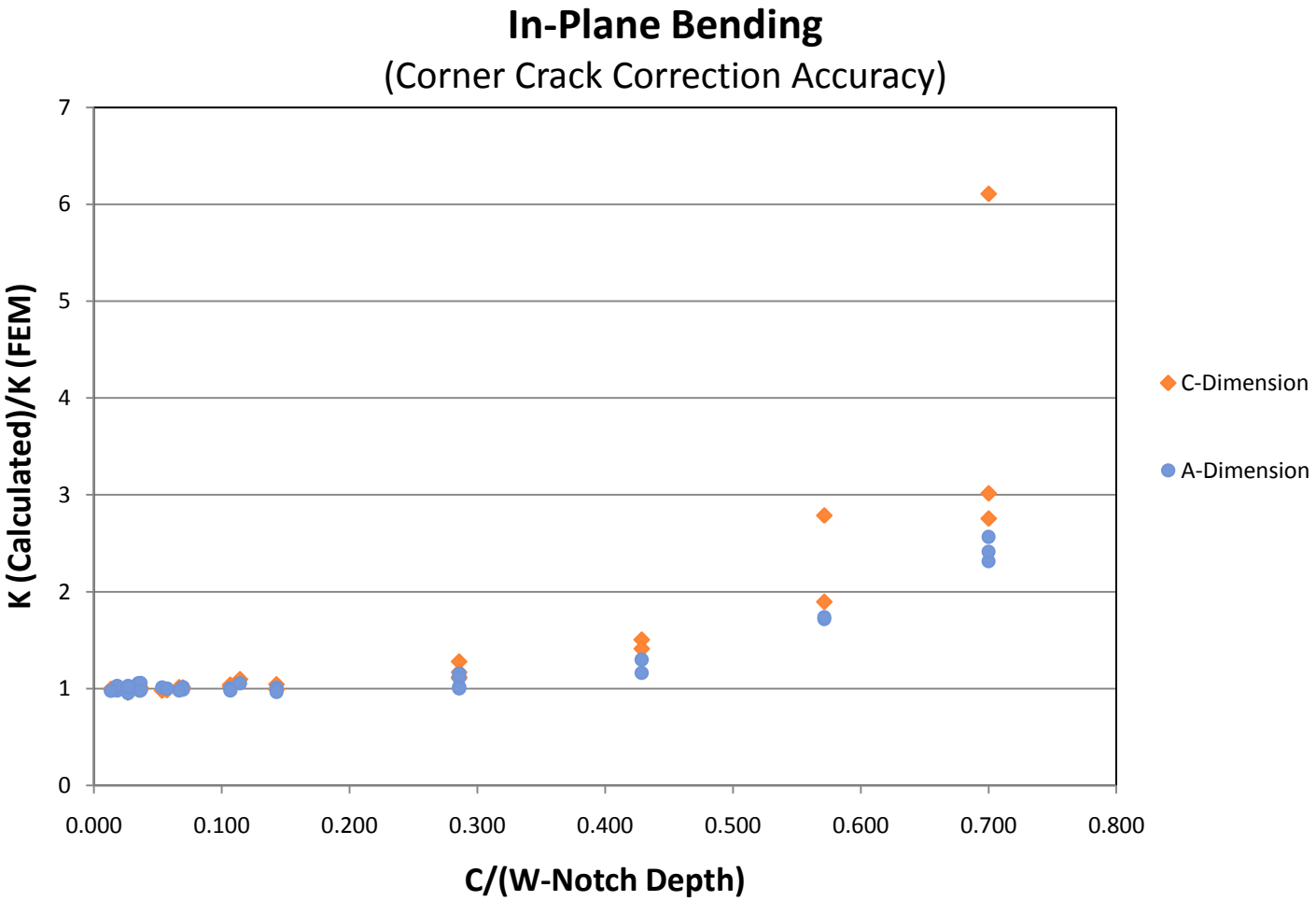
λ_A =(e + R)/W λ_C =(e + R + C)/W

Results



As the C-Dimension Approached the Neutral Axis of the Un-Cracked Net Section, the Calculated Values Became Increasingly Conservative

Limits of Applicability



The Solution Accuracy Appears to be Within Approximately 5% When the C < 20% of the Un-Cracked Net Section

Comments/Questions?